
٤- الدالة الأسية المضروبة بدالة أخرى : $\left(f(x,y) = e^{ax+by}.\mathrm{v(x,y)} \right).$

$$Df(x,y) = e^{ax+by}.\mathrm{D}v + vD\,e^{ax+by}. = (D+a)v\,e^{ax+by} .$$

$$\rightarrow D^2 f(x,y) = e^{ax+by}.(D^2+aD) + (D+a)v\,ae^{ax+by} .$$

$$\rightarrow D^2 f(x,y) = (D^2+2aD+a^2)v\,e^{ax+by} .$$

$$\rightarrow D^2 f(x,y) = (D+a)^2 v\,e^{ax+by} .$$

بإستخدام الإستقراء الرياضي نحصل على :

$$\rightarrow D^n f(x,y) = (D+a)^n v\,e^{ax+by} .$$

وبنفس الإسلوب نحصل على :

$$\rightarrow \mathrm{D'}^n f(x,y) = (\mathrm{D'}+b)^n v\,e^{ax+by} .$$

$$\rightarrow f(D,\mathrm{D'})\left(v\,e^{ax+by} \right) = (D+a)^n v\,e^{ax+by} + (\mathrm{D'}+b)^n v\,e^{ax+by} .$$

$$\rightarrow f(D,\mathrm{D'})\left(v\,e^{ax+by} \right) = f(D+a,\mathrm{D'}+b)\ v\,e^{ax+by} .$$

$$\rightarrow Z2 = e^{ax+by}.\frac{1}{f(D+a,\mathrm{D'}+b)}.v .$$

مثال ١ : جد الحل العام للمعادلة :

$$D\mathrm{D'}Z = \mathrm{e}^{2x-3y}\,x\,y.$$

$$\rightarrow \mathrm{m} = 0.(\ subsdiral\ equation\ ) \rightarrow Z1 = \emptyset(y).$$

$$since\ Z2 = e^{ax+by}.\frac{1}{f(D+a,\mathrm{D'}+b)}.v\ .,\ then\ Z2 = e^{2x-3y}.\frac{1}{f(D+2,\mathrm{D'}-3)}.xy.$$

$$\rightarrow Z2 = e^{2x-3y}.\frac{1}{(D+2)(\mathrm{D'}-3)}.xy. = e^{2x-3y}.\frac{1}{(D+2)-3\left(1-\frac{\mathrm{D'}}{3}\right)}.xy.$$

$$\rightarrow Z2 = e^{2x-3y}.\frac{-1}{3(D+2)}\left\{1+\frac{\mathrm{D'}}{3}+\frac{\mathrm{D'}^2}{9}+\cdots\ldots\ldots\right\}.xy.$$

$$\rightarrow\ Z2 = e^{2x-3y}.\frac{-1}{3(D+2)}\left\{xy+\frac{x}{3}+0+\cdots\ldots\ldots.\right\}.$$

$$\rightarrow\ Z2 = \frac{-1}{3}\,e^{2x-3y}.\frac{1}{2\left(1+\frac{D}{2}\right)}\left\{xy+\frac{x}{3}\right\}.$$

$$\rightarrow\ Z2 = \frac{-1}{6}\,e^{2x-3y}.\left\{1-\frac{D}{2}+\frac{D^2}{4}-\frac{D^3}{8}+\cdots\ldots\ldots\ldots\right\}\ (xy+\frac{x}{3}).$$

$$\rightarrow\ Z2 = \frac{-1}{6}\,e^{2x-3y}.\left\{xy+\frac{x}{3}-\frac{y}{2}-\frac{1}{3}\right\}.$$

$$since\ Z = Z1 + Z2.\quad\rightarrow\quad Z = \emptyset(y) + \frac{-1}{6}\,e^{2x-3y}.\left\{xy+\frac{x}{3}-\frac{y}{2}-\frac{1}{3}\right\}$$

مثال 2 : حل المعادلة :

$$\left(D^2 - DD' - D'^2\right)Z = x^2e^{x+2y}$$

$$m^2 - m - 1 = 0.\ (\ subsdiral\ \ equation\ ).\rightarrow m1,2 = \frac{1\ \pm\ \sqrt{5}}{2}.$$

$$\rightarrow\ m1 = \frac{1+\sqrt{5}}{2}.,\ \ m2 = \frac{1-\sqrt{5}}{2}.$$

$$\rightarrow\quad Z1 = \emptyset1\left(y+(\frac{1+\sqrt{5}}{2})x\right) + \emptyset2\left(y-(\frac{1-\sqrt{5}}{2})x\right)$$

$$since\ \ Z2 = e^{ax+by}.\frac{1}{f\ (D+a\ ,D'+b\ )}\ .v\ .,\rightarrow\ \ Z2 = e^{x+2y}.\frac{1}{f\ (D+1\ ,D'+2\ )}\ .x^2.$$

$$\rightarrow\ Z2 = e^{x+2y}.\frac{1}{(D+1)^2-(D+1)(D'+2)-(D'+2)^2}\ .x^2.$$

$$\rightarrow\ Z2 = e^{x+2y}.\frac{1}{(D+1)^2\left\{1-\left(\frac{D'+2}{D+1}+\frac{(D'+2)^2}{(D+1)^2}\right)\right\}}\ .x^2$$

$$\rightarrow\ Z2 = e^{x+2y}.\frac{1}{(D+1)^2}\{x^2+\left(\frac{0}{D+1}+\frac{0}{(D+1)^2}+\cdots\ldots..\right\}.$$

$$\rightarrow\ Z2 = e^{x+2y}.\frac{1}{(1+D)}\{1-D+\ D^2\ -\ D^3\ +\cdots\ldots\ldots..\}\,x^2.$$

$$\rightarrow\ Z2 = e^{x+2y}.\frac{1}{(1+D)}\{x^2-2x+\ 2\ -\ 0\ +\cdots\ldots\ldots..\}.$$

$\rightarrow\ \boldsymbol{Z2 = e^{x+2y}\,.\{\,1 - D + D^2 - D^3 + \cdots\ldots\ldots..\}\,(\,x^2 - 2x + 2\,).}$

$\rightarrow\ \boldsymbol{Z2 = \{\,x^2 - 4x + 2\,.\}}$

$\boldsymbol{since\ Z = Z1 + Z2.}\quad\rightarrow\quad \boldsymbol{Z = \emptyset 1\left(y + (\frac{1+\sqrt{5}}{2})x\right) + \emptyset 2\left(y - (\frac{1-\sqrt{5}}{2})x\right) + x^2 - 4x + 2.}$

تمارين ((جد الحل العام للمعادلات الآتية)) :

1	$\boldsymbol{(D^2 - 2DD')Z = 3x\,.\,e^{2x}}$..
2	$\boldsymbol{(D^2 - 2DD')Z = x^3\,.\,e^{x-y}}$.
3	$\boldsymbol{\left(D^2 + 2DD' - D'^2\right)Z = x^2e^{x-2y}}$.
