

The Second Lecture

Some Examples

In this lecture we give some examples about groups, such as $(\mathbb{Z}, +)$, $(\mathbb{Q} \setminus \{0\}, \cdot)$, S_n with the composition operator and Boolean group.

Examples:

- (i) $(\mathbb{Z}, +)$; The set of all integers is an additive abelian group with identity $e = 0$, and with the inverse of an integer n being $-n$.
- (ii) Similarly, one can see that $(\mathbb{Q}, +)$ and $(\mathbb{R}, +)$ are additive abelian groups, where \mathbb{Q} is the set of

rational numbers and \mathbb{R} is the set of real numbers.

(ii) $(\mathbb{Q} \setminus \{0\}, \cdot)$; The set of all nonzero rational numbers, is an abelian group, where (\cdot) is the ordinary multiplication, the number 1 is the identity, and the inverse of r is $1/r$. Similarly, $(\mathbb{R} \setminus \{0\}, \cdot)$ is a multiplicative abelian group.

(iii) Let X be a set. Recall that if A and B are subsets of X , then their symmetric difference is $A \oplus B = (A - B) \cup (B - A)$. The **Boolean group** $P(X)$ is the family of all the subsets of X with addition given by symmetric difference.

(iv) Consider S_n , the set of all permutations of $X = \{1, 2, \dots, n\}$. It is form a group with the composition operation.

Remark: Let G be a group, let $a, b \in G$, and let m and n be (not necessarily positive) integers.

(i) If a and b commute, then $(ab)^n = a^n b^n$.

(ii) $(a^n)^m = a^{nm}$.

(iii) $a^m a^n = a^{m+n}$.