MATHEMATICS DEPARTMENT COLLEGE OF EDUCATION FOR PURE SCIENCES

UNIVERSITY OF ANBAR

Lecture Note

Advance Calculus

By

Mimoon Ibrahim Ismael

AL-ANBAR, IRAQ

Contents

Chapter 1

Partial Differentiation

1.1 Functions of several variables

In real life, there are many formulas that depend on more than one variable. For example: Area of a rectangular $A = xy$, so A is a function of the two variables x and y. So, if $Z = f(x, y)$, then Z is a function of two variables x and y. Similarly, $W = f(x, y, z)$, a function of variables x, y and z.
Also, $U = f(x_1, x_2, \ldots, x_n)$, a function of variables $x_1, x_2, \ldots, x_n$.

Example: Evaluating the following functions?

- $h(x, y, z) = \ln(x^2 + y + z^2)$ at the point (-1,2,1).
- $g(r, s, t) = \sqrt{r^2 + s^2 + t^2}$ at the point (3,0,4).
- $f(x, y, z) = e^{\frac{(x+y)}{z}}$ at the point $(\ln 2, \ln 4, 3)$.
- $T(r, \theta) = \cos(\sqrt{r^2\theta^2 - 1})$ at the point (-1,-1).

Solution:

- The value of $h(x, y, z) = \ln(x^2 + y + z^2)$ at the point (-1,2,1) is

  $h(-1, 2, 1) = \ln\left((-1)^2 + 2 + (1)^2\right) = \ln 4.$

- The value of $g(r, s, t) = \sqrt{r^2 + s^2 + t^2}$ at the point (3,0,4) is

  $g(3, 0, 4) = \sqrt{(3)^2 + (0)^2 + (4)^2} = 5.$

- The value of $f(x,y,z) = e^{\frac{(x+y)}{z}}$ at the point $(\ln 2, \ln 4, 3)$ is

  $f(\ln 2, \ln 4, 3) = e^{\frac{(\ln 2+\ln 4)}{3}} = e^{\frac{\ln 8}{3}} = e^{\frac{3\ln 2}{3}} = 2.$

- The value of $T(r,\theta) = \cos(\sqrt{r^2\theta^2 - 1})$ at the point (-1,-1) is

  $T(-1,-1) = \cos\left(\sqrt{(-1)^2(-1)^2 - 1}\right) = \cos(0) = 1.$

1.2 Limit of a Function of Two Variables

We say that a function $f(x,y)$ approaches the limit a as (x,y) approaches (x_0, y_0) and write

$$\lim_{(x,y)\longrightarrow(x_0,y_0)} f(x,y) = a.$$

Note the following rules hold if $a, b \in \mathbb{R}$ and

$$\lim_{(x,y)\longrightarrow(x_0,y_0)} f(x,y) = a \text{ and } \lim_{(x,y)\longrightarrow(x_0,y_0)} g(x,y) = b$$

1. $$\lim_{(x,y)\longrightarrow(x_0,y_0)} f(x,y) \pm \lim_{(x,y)\longrightarrow(x_0,y_0)} g(x,y) = a \pm b$$

2. $$\lim_{(x,y)\longrightarrow(x_0,y_0)} f(x,y)g(x,y) = ab$$

3. $$\lim_{(x,y)\longrightarrow(x_0,y_0)} \frac{f(x,y)}{g(x,y)} = \frac{a}{b}, \text{ where } b \neq 0$$

4. $$\lim_{(x,y)\longrightarrow(x_0,y_0)} \Big[f(x,y)\Big]^n = a^n$$

Example: Find the limits of the functions if possible ?

1. $$\lim_{(x,y)\longrightarrow(\sqrt{2},0)} \left(x^2 + xy\right)$$

2. $$\lim_{(x,y)\longrightarrow(0,0)} \left(\sin(xy) - \cos(y^2)\right)$$