

**MATHEMATICS DEPARTMENT COLLEGE OF
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Lecture Note

Advance Calculus

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Chapter 1

Partial Differentiation

1.1 Functions of several variables

In real life, there are many formulas that depend on more than one variable. For example: Area of a rectangular $A = xy$, so A is a function of the two variables x and y . So, if $Z = f(x, y)$, then Z is a function of two variables x and y . Similarly, $W = f(x, y, z)$, a function of variables x, y and z . Also, $U = f(x_1, x_2, \dots, x_n)$, a function of variables x_1, x_2, \dots, x_n .

Example: Evaluating the following functions?

- $h(x, y, z) = \ln(x^2 + y + z^2)$ at the point $(-1, 2, 1)$.
- $g(r, s, t) = \sqrt{r^2 + s^2 + t^2}$ at the point $(3, 0, 4)$.
- $f(x, y, z) = e^{\frac{(x+y)}{z}}$ at the point $(\ln 2, \ln 4, 3)$.
- $T(r, \theta) = \cos(\sqrt{r^2\theta^2 - 1})$ at the point $(-1, -1)$.

Solution:

- The value of $h(x, y, z) = \ln(x^2 + y + z^2)$ at the point $(-1, 2, 1)$ is
$$h(-1, 2, 1) = \ln\left((-1)^2 + 2 + (1)^2\right) = \ln 4.$$
- The value of $g(r, s, t) = \sqrt{r^2 + s^2 + t^2}$ at the point $(3, 0, 4)$ is
$$g(3, 0, 4) = \sqrt{(3)^2 + (0)^2 + (4)^2} = 5.$$

- The value of $f(x, y, z) = e^{\frac{(x+y)}{z}}$ at the point $(\ln 2, \ln 4, 3)$ is

$$f(\ln 2, \ln 4, 3) = e^{\frac{(\ln 2 + \ln 4)}{3}} = e^{\frac{\ln 8}{3}} = e^{\frac{3 \ln 2}{3}} = 2.$$

- The value of $T(r, \theta) = \cos(\sqrt{r^2 \theta^2 - 1})$ at the point $(-1, -1)$ is

$$T(-1, -1) = \cos\left(\sqrt{(-1)^2(-1)^2 - 1}\right) = \cos(0) = 1.$$

1.2 Limit of a Function of Two Variables

We say that a function $f(x, y)$ approaches the limit a as (x, y) approaches (x_0, y_0) and write

$$\lim_{(x,y) \rightarrow (x_0,y_0)} f(x, y) = a.$$

Note the following rules hold if $a, b \in \mathbb{R}$ and

$$\lim_{(x,y) \rightarrow (x_0,y_0)} f(x, y) = a \text{ and } \lim_{(x,y) \rightarrow (x_0,y_0)} g(x, y) = b$$

1.

$$\lim_{(x,y) \rightarrow (x_0,y_0)} f(x, y) \pm \lim_{(x,y) \rightarrow (x_0,y_0)} g(x, y) = a \pm b$$

2.

$$\lim_{(x,y) \rightarrow (x_0,y_0)} f(x, y)g(x, y) = ab$$

3.

$$\lim_{(x,y) \rightarrow (x_0,y_0)} \frac{f(x, y)}{g(x, y)} = \frac{a}{b}, \text{ where } b \neq 0$$

4.

$$\lim_{(x,y) \rightarrow (x_0,y_0)} [f(x, y)]^n = a^n$$

Example: Find the limits of the functions if possible ?

1.

$$\lim_{(x,y) \rightarrow (\sqrt{2}, 0)} (x^2 + xy)$$

2.

$$\lim_{(x,y) \rightarrow (0,0)} (\sin(xy) - \cos(y^2))$$