المناقشة الاولى

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Q1) Consider a function $(x, y) = \frac{1}{\sqrt{9-x^2-y^2}}$, answer the following:

(a)Find f(0,0).

(b) Describe the domain (sketch).

(C) Find the range of the function $f(x, y)$.

Q2) Evaluate:

(i) $\lim_{(x,y)\to(0,,0)} \frac{x^2+y^2}{xy+x+1}$.

(ii) $\lim_{(x,y)\to(\sqrt{2},,\sqrt{2})} \frac{x^3y^3-8}{x^3y^3-4xy}$.

(iii) $\lim_{(x,y)\to(1,1)} \frac{x-y+2\sqrt{x}-2\sqrt{y}}{\sqrt{x}-\sqrt{y}}$.

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Q3)If $W(x, y, z) = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$, then show that $x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} + z\frac{\partial w}{\partial z} = 0$.

Q4) Find the critical point of $f(x, y) = x \sin y$.

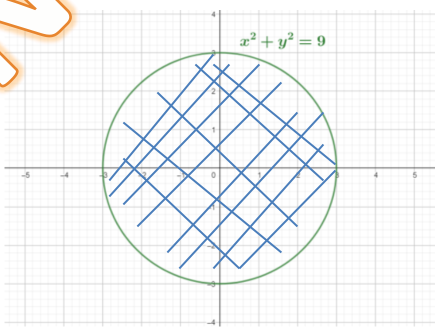
Q5) Find the critical points of $Z = e^y - ye^x$, and say what type each critical point is (local maximum or local minimum or saddle point).

حل اسئلة المناقشة

Q1) Solution: $a)f(0,0)=\frac{1}{\sqrt{9-0^2-0^2}}=\frac{1}{\sqrt{9}}=\frac{1}{3}$

b)The domain of $f(x,y)$ **is** $\{(x,y)\in R^2|9-x^2-y^2>0\}=\{(x,y)\in R^2|x^2+y^2<9\}$

so, the domain is inside $x^2+y^2=9$ **and it is not on** $x^2+y^2=9$



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c) The range is $\left[\frac{1}{3},\infty\right)$ $or\ z\geq\frac{1}{3}.$

Q2)

$i)\ \lim_{(x,y)\to(0,0)}\frac{x^2+y^2}{xy+x+1}=\frac{0^2+0^2}{00+0+1}=\frac{0}{1}=0.$ لاحظ ان المقام لا يساوي صفر لذلك نعوض مباشر

ii) $\lim_{(x,y)\to(\sqrt{2},\sqrt{2})}\frac{x^3\,y^3-8}{x^3\,y^3-4xy}\xrightarrow{\text{لايجوز التعويض مباشر بالمقام}}\lim_{(x,y)\to(\sqrt{2},\sqrt{2})}\frac{(xy-2)(x^2y^2+2xy+4)}{xy(x^2y^2-4)}$

$\lim_{(x,y)\to(\sqrt{2},\sqrt{2})}\frac{(xy-2)(x^2y^2+2xy+4)}{xy(xy-2)(xy+2)}=\lim_{(x,y)\to(\sqrt{2},\sqrt{2})}\frac{(x^2y^2+2xy+4)}{xy(xy+2)}=\frac{12}{8}=\frac{3}{2}$

iii) هناك اكثر من طريقة للحل

$\lim_{(x,y)\to(1,1)}\frac{x-y+2\sqrt{x}-2\sqrt{y}}{\sqrt{x}-\sqrt{y}}\xrightarrow{\text{لايجوز التعويض المباشر بالمقام}}\lim_{(x,y)\to(1,1)}\frac{(\sqrt{x}-\sqrt{y})(\sqrt{x}+\sqrt{y})+2(\sqrt{x}-\sqrt{y})}{\sqrt{x}-\sqrt{y}}$

$\lim_{(x,y)\to(1,1)}\frac{(\sqrt{x}-\sqrt{y})[(\sqrt{x}+\sqrt{y})+2]}{\sqrt{x}-\sqrt{y}}=\lim_{(x,y)\to(1,1)}\sqrt{x}+\sqrt{y}+2=4$

Q3) $\frac{\partial w}{\partial x}=\frac{1}{y}-\frac{z}{x^2},\ \frac{\partial w}{\partial y}=\frac{-x}{y^2}+\frac{1}{z},\ and\ \frac{\partial w}{\partial z}=\frac{-y}{z^2}+\frac{1}{x}$

$x\frac{\partial w}{\partial x}+y\frac{\partial w}{\partial y}+z\frac{\partial w}{\partial z}\xrightarrow{\text{بالتعويض}}x\left(\frac{1}{y}-\frac{z}{x^2}\right)+y\left(\frac{-x}{y^2}+\frac{1}{z}\right)+z\left(\frac{-y}{z^2}+\frac{1}{x}\right)$

$\left(\frac{x}{y}-\frac{xz}{x^2}\right)+\left(\frac{-yx}{y^2}+\frac{y}{z}\right)+\left(\frac{-zy}{z^2}+\frac{z}{x}\right)=\left(\frac{x}{y}-\frac{z}{x}\right)+\left(\frac{-x}{y}+\frac{y}{z}\right)+\left(\frac{-y}{z}+\frac{z}{x}\right)=0$

Q4) $\frac{\partial f}{\partial x} = \sin y$ *and* $\frac{\partial f}{\partial y} = x\cos y$

Now, $\frac{\partial f}{\partial x} = 0 \Rightarrow \sin y = 0 \Rightarrow y = \sin^{-1} 0 = n\pi, where\ n = 0, \mp 1, \mp 2, \cdots$

Also, $\frac{\partial f}{\partial y} = 0 \Rightarrow \mathrm{x} \cos y = 0 \Rightarrow x = 0$ or $\cos y = 0$

Note that $\cos y \neq 0$ because $y = n\pi, where\ n = 0, \mp 1, \mp 2, \cdots$

Thus, x=0 and the critical point of $f(x, y)$ is $(0, n\pi), where\ n = 0, \mp 1, \mp 2, \cdots$

Q5) $\frac{\partial z}{\partial x} = -ye^x$ *and* $\frac{\partial z}{\partial y} = e^y - e^x$

Now, $\frac{\partial z}{\partial x} = 0 \Rightarrow -ye^x = 0 \Rightarrow y = 0\ or\ e^x = 0$, *but* $e^x \neq 0$ *why!*

Also, $\frac{\partial z}{\partial y} = 0 \Rightarrow e^y - e^x = 0 \Rightarrow e^y = e^x \xrightarrow{\text{بتعويض } y=0} e^0 = e^x \Rightarrow e^x = 1 \xrightarrow{\text{باخذ } \ln} x = \ln 1$

Thus, x=0 and the critical point of $z(x, y)$ is $(0, 0)$

$Z_{xx} = -ye^x$, $Z_{yy} = e^y$ *and* $Z_{xy} = Z_{yx} = -e^x$

$z(x, y)$ has saddle point at $(0, 0, 0)$ because :

$$Z_{xx}(0,0)Z_{yy}(0,0) - [Z_{xy}(0,0)]^2 < 0$$

$$(-0e^0)(e^0) - [-e^0]^2 < 0$$

$$0-1 < 0$$

$$-1 < 0$$

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