استمارة انجاز الخطة التدريسية للمادة

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اسم المادة	التبولوجيا العامة (1) --- التبولوجيا العامة (2)
مقرر الفصل	دراسة الفضاءات التبولوجية لفصلين دراسيين
اهداف المادة	1- التأكيد على أهمية موضوع الفضاءات التبولوجية بالنسبة للعلوم الاخرى. 3- أن يتعرف الطلبة على أنواع الفضاءات التبولوجية. 2- تبصير الطلبة بالفضاءات التبولوجية و بديهيات الفصل و الفضاءات المتراصة. 4- أن نبين للطلبة أهم تطبيقات الفضاءات التبولوجية.
التفاصيل الاساسية للمادة	التبولوجيا هو فرع مهم وممتع من فروع الرياضيات حيث يمكن ملاحظة اهمية الفضاءات التبولوجية من خلال تاثيرها الواضح في جميع فروع الرياضيات الاخرى وهذا يجعل دراسة التبولوجيا ذات علاقة مع كل الذين يطمحون ان يصبحوا رياضيون سواء أكان حبهم الأول (الجبر, التحليل, الهندسة, الديناميكا, الرياضيات الصناعية, الميكانيكا الكمية, نظرية التبولوجيا العامة - التبولوجيا الجبرية - العدد, بحوث العمليات أو الأحصاء) والتبولوجيا لها عدة فروع مختلفة مثل التبولوجيا التفاضلية والتبولوجيا الجبرية والتبولوجية الهندسية.
الكتب المنهجية	1- General topology, by: Willard’s. W. Addison Wesley, eading, mass, (1970). 2-Topology a first course, by: Munkres. J. R. (1975).
المصادر الخارجية	➢ General topology, by: J.L., Kelley’s. General topology, by: Bourbaki's.

تقديرات الفصل	الفصل الدراسي	المختبر	الامتحانات اليومية	المشروع	الامتحان النهائي
	%30	-----	%10	-----	60%

يطلب من الطلبة في بعض الأحيان كتابة تقرير في الواجبات التي تعطى لهم خلال الكورس الدراسي

lectures in Topological Spaces-Mathematics department-Fourth stage

Syllabus

1- *Definitions and (Examples) of a Topological Space.*

2- *Types of Topological Spaces.*

3- *Closed subsets of a topological space.* 4- *Neighborhoods.*

5- *Closure of a Set.* 6- *Topologies Induced by Functions.*

7- *Interior of a Set, Exterior of a Set, Boundary of a Set and Cluster Points.*

8- *Dense Subset of the Space. 9- Dense Subset of the Space.*

10- *Continuous Functions.*

11- *Open and Closed mappings*

12- *Homeomorphisms.*

13- *Topological spaces and Hereditary Property.*

14- *Compactness in Topological Spaces.*

15- *Connectedness in Topological Spaces.*

16- *Separation Axioms and study relationships between them.*

المحاضـــــرة التاســـــعة

Results on continuous mapping in Topology spaces

Theorem:

Let (X, T) and (Y, T^*) be topology space let f: x → y then f is continuous iff the in veres image under f of every open set in y is open x.

Proof:-

Let f be continuous and let H be any open satiny if f^{-1}(H) = ∅, it is clearly open so let f^{-1} (H) ≠∅, let x ∈ f^{-1} (H)

Then $f(x)$ ∈ H Bycontinuity of f ,∃ an open set Gin X such that x ∈ G and $f(G)$ ⊆ H. consequently x ∈ G ⊆ f^{-1}(H)

This show that f^{-1} (H) is nhd of each of its points and therefore, it is open in X.

Conversely , let the inverse image under f of every open set in y be open in X, then in order to show that if is continues it is sufficient to show that it is continues at an arbitrary point x ∈ X let H be any open set in y such that $f(x)$ ∈ H. Then x ∈ f^{1} (H) .by hypothesis f^{-1}(H) is an open set in X Now, if we set f^{-1}(H) = G , Then h is an open set in X contain x such that f(G)= f [f^{-1}(H)]⊆ H. This Show that f is continues at each point of X.

Theorem:

Let *(X, T) and (Y, T)* topology space and let f: X→Y Then *F* is continuous iff for each *x* ∈ *X* The inverse image under F of every T – nhd of f *(x)* is T-nhd of X.

Proof:-

Let *f* be continuous and let x ∈ X let M be and T – nhd of *f* (x) then ∋ an open set Hiny Such that *f*(x) ∈ H ⊆ *f* $^{-1}$(M) Since *f* is continuous and His *T* open , So *f* $^{-1}$(H) is T- open this Show that *f* $^{-1}$(M) is a T – nhd of X.

Conversely , let the inverse image under f of every T-nhd of f(x) be a T-nhd of X let H be any open set in Y note is *f* $^{-1}$(H) = ∅, it is clearly open So let *f* $^{-1}$(H) ≠ ∅ and let x ∈ *f* $^{-1}$(H) then *f* (x) ∈ H, This show that His a T- nhd of *f* (x). So by hypothesis ; *f* (H) is aT- nhd of X .Thus *f* $^{-1}$(H) is a T-nhd of each of its points and there its open, so it follows that in verse image under *f* of every open Sub of Y is an open Sub Set of X. Hence *f* is continuous.

Theorem:

Let (X, T) and (Y,T) be topology space and *f*: X ⟶ Y then f is continuous iff the inverse image under *f* of every closed Subset of Y is a closed sub set of X.

Proof:-

let *f* be continuous and let K be any closed sub set of Y then (Y-k) is an open sub set of y so by continuity of *f*, *f* $^{-1}$(Y-k) is an open sub set of X But, *f* $^{-1}$(Y^{-1}(k)) is open and therefore *f* $^{-1}$(k) is closed

Conversely: - let the inverse image under f of every closed sub set Y then (Y-H) is closed and therefore by hypothesis *f* $^{-1}$(Y-H) is closed But *f* $^{-1}$(Y-H) = *f* $^{-1}$(H) = X-*f* $^{-1}$(H). So X. *f* $^{-1}$(H) is closed and therefore *f* $^{-1}$(H) is open. Thus the inverse image under *f* of every open sub set of Y is an open sub set of X. This show that f is continuous.

Theorem:

Let X, Y and Ƶ be any three Top – Spaces and let *f*: X ⟶ Y and g: Y ⟶ Ƶ be cont mappings Then the composite *gof* : X⟶ Ƶ is continuous.

Proof:-

Let H be any open sub set in Ƶ, we must prove that (*gof*) $^{-1}$(H) is open sub set in X. Since *g* is cont ⟶ g^{-1}(H) is open sub set in Y and Since *f* is cont ⟶ $f^{-1}(g^{-1}$(H))) is open sub set in X So, $f^{-1}(g$(H)) = (f^{-1}og^{-1}) (H) = (gof^{-1}) (H) is open in X. Thus The inverse image under (*gof*) of every open sub set of Ƶ is open sub set of X.

Theorem:

Let (X, T) and (Y,T*) be a Topological space and Let *f*: X→ Y Then *f* is continuous iff for every B ⊆ Y; f^{-1}(B) ⊆ $f^{-1}(\overline{B})$

Proof:- **Let *f* be continuous and let B ⊆ Y. Then $\overline{B}$ being closed, by continuity of *f*, $f^{-1}(\overline{B})$ is closed ∴ $f^{-1}(B) = f^{-1}(\overline{B})$. Now, B ⊆ $\overline{B}$ ⟹ f^{-1}(B) ⊆ $f^{-1}(\overline{B})$ ⟹ $f^{-1}(B) \subseteq f^{-1}(\overline{B})$.**

Conversely:- Let $f^{-1}(B) \subseteq f^{-1}(\overline{B})$ for every B ⊆ Y.

Now, let K be a closed sub set of Y so that $\overline{f}$ = K.

Now, by hypothesis, $f^{-1}(K) \subseteq f^{-1}(\overline{K}) = f^{-1}(K)$. But, $f^{-1}(K) \subseteq f^{-1}(K)$

∴ $f^{-1}(K) = f^{-1}$ (K) showing that f^{-1}(K) is closed thus the inverse image under *f* of every closed sub set of Y is a closed sub set of X Hence *f* is continuous.