Republic of Iraq Ministry of Higher Education & Research

University of Anbar

College of Education for Pure Sciences

Department of Mathematics

Lecture Note On Mathematical Statistics 1

B.Sc. in Mathematics

Fourth Stage

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Syllabus of Mathematical Statistics 1

- **Chapter 1: Additional Topics in Probability**
- **Special Distribution Functions : The Binomial Probability Distribution , Poisson Probability Distribution , Uniform Probability Distribution , Normal Probability Distribution , Gamma Probability Distribution , Distributions of Functions of random Variables (Transformation technique, Distribution Function technique, Moment generating function technique), Limit Theorems: Chebyshev`s Theorem Law of Large Numbers, Central Limit Theorem.**
- **Chapter 2: Sampling Distributions**
- **Sampling Distributions Associated with Normal Populations, Distribution of $\overline{X}$ and S^2 , Chi-Square Distribution, Student t-Distribution, F-Distribution, Distributions of Order statistics, Large sample Approximations: The Normal Approximation to the Binomial Distribution, Limiting Distribution: Stochastic Convergence, Limiting of moment generating functions, Theorems on Limiting distributions.**
- **Chapter 3: Point Estimation**
- **The Method of Moments, The Method of Maximum Likelihood, Some desirable properties of point estimators, Unbiased Estimators, Sufficiency, Consistency, Efficiency, Minimal Sufficiency and Minimum-Variance Unbiased Estimation, Cramer–Rao procedure to test for efficiency.**

References

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- مقدمة في الاحصاء الرياضي، صباح داود سليم، الطبعة الاولى، جامعة البصرة، ١٩٨٩.
- **Introduction to Mathematical Statistics, R. V. Hogg and A.T. Craig, (4, 5, 6) edition.**
- **Introduction to the Theory of Statistics, Alexander M.Mood, Franklin, A. Graybill, MC- Graw Hill, 1974.**
- **Mathematical Statistics with Applications, K. M. Ramachandran and C. P. Tsokos, 2009.**
- **Mathematical Statistics with Application, Richard L.Schaffer, Seventh Edition, Thomson Brooks, 2008.**
- **Probability and mathematical Statistics, Prasanna Sahoo, University of Louisville,, USA, 2008.**
- **An introduction to Mathematical Statistics and Its Applications, Ricard I. larson and Morris Luois, Fifth edition, Person Education, INC, 2012.**
- **Probability and Statistical Inference, Robert V. Hogg ,Elliot A. Tanis and ale L. Zimmerman, Ninth Edition, Pearson Education, USA,2015.**

Discrete Distributions

Bernoulli

$0 < p < 1$

$$f(x) = p^x(1-p)^{1-x}, \qquad x = 0, 1$$
$$M(t) = 1 - p + pe^t, \qquad -\infty < t < \infty$$
$$\mu = p, \qquad \sigma^2 = p(1-p)$$

Binomial

$b(n, p)$

$0 < p < 1$

$$f(x) = \frac{n!}{x!(n-x)!} p^x(1-p)^{n-x}, \qquad x = 0, 1, 2, \ldots, n$$
$$M(t) = (1 - p + pe^t)^n, \qquad -\infty < t < \infty$$
$$\mu = np, \qquad \sigma^2 = np(1-p)$$

Geometric

$0 < p < 1$

$$f(x) = (1-p)^{x-1}p, \qquad x = 1, 2, 3, \ldots$$
$$M(t) = \frac{pe^t}{1-(1-p)e^t}, \qquad t < -\ln(1-p)$$
$$\mu = \frac{1}{p}, \qquad \sigma^2 = \frac{1-p}{p^2}$$

Hypergeometric

$N_1 > 0, \ \ N_2 > 0$

$N = N_1 + N_2$

$$f(x) = \frac{\binom{N_1}{x}\binom{N_2}{n-x}}{\binom{N}{n}}, \qquad x \le n, x \le N_1, n - x \le N_2$$
$$\mu = n\left(\frac{N_1}{N}\right), \qquad \sigma^2 = n\left(\frac{N_1}{N}\right)\left(\frac{N_2}{N}\right)\left(\frac{N-n}{N-1}\right)$$

Negative Binomial $f(x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}, \qquad x = r, r+1, r+2, \ldots$

$0 < p < 1$

$r = 1, 2, 3, \ldots$ $\qquad M(t) = \dfrac{(pe^t)^r}{[1-(1-p)e^t]^r}, \qquad t < -\ln(1-p)$

$$\mu = r\left(\frac{1}{p}\right), \qquad \sigma^2 = \frac{r(1-p)}{p^2}$$

Poisson $f(x) = \dfrac{\lambda^x e^{-\lambda}}{x!}, \qquad x = 0, 1, 2, \ldots$

$\lambda > 0$

$$M(t) = e^{\lambda(e^t - 1)}, \qquad -\infty < t < \infty$$

$$\mu = \lambda, \qquad \sigma^2 = \lambda$$

Uniform $f(x) = \dfrac{1}{m}, \qquad x = 1, 2, \ldots, m$

$m > 0$

$$\mu = \frac{m+1}{2}, \qquad \sigma^2 = \frac{m^2-1}{12}$$

Continuous Distributions

Beta
$\alpha > 0$
$\beta > 0$

$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \qquad 0 < x < 1$$

$$\mu = \frac{\alpha}{\alpha+\beta}, \qquad \sigma^2 = \frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2}$$

Chi-square
$\chi^2(r)$
$r = 1, 2, \ldots$

$$f(x) = \frac{1}{\Gamma(r/2)2^{r/2}} x^{r/2-1} e^{-x/2}, \qquad 0 < x < \infty$$

$$M(t) = \frac{1}{(1-2t)^{r/2}}, \qquad t < \frac{1}{2}$$

$$\mu = r, \qquad \sigma^2 = 2r$$

Exponential
$\theta > 0$

$$f(x) = \frac{1}{\theta} e^{-x/\theta}, \qquad 0 \le x < \infty$$

$$M(t) = \frac{1}{1-\theta t}, \qquad t < \frac{1}{\theta}$$

$$\mu = \theta, \qquad \sigma^2 = \theta^2$$

Gamma	$f(x) = \dfrac{1}{\Gamma(\alpha)\theta^\alpha} x^{\alpha-1} e^{-x/\theta}, \qquad 0 < x < \infty$
$\alpha > 0$	
$\theta > 0$	$M(t) = \dfrac{1}{(1-\theta t)^\alpha}, \qquad t < \dfrac{1}{\theta}$
	$\mu = \alpha\theta, \qquad \sigma^2 = \alpha\theta^2$
Normal	$f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}, \qquad -\infty < x < \infty$
$N(\mu, \sigma^2)$	
$-\infty < \mu < \infty$	$M(t) = e^{\mu t + \sigma^2 t^2/2}, \qquad -\infty < t < \infty$
$\sigma > 0$	$E(X) = \mu, \qquad \mathrm{Var}(X) = \sigma^2$
Uniform	$f(x) = \dfrac{1}{b-a}, \qquad a \le x \le b$
$U(a, b)$	
$-\infty < a < b < \infty$	$M(t) = \dfrac{e^{tb} - e^{ta}}{t(b-a)}, \qquad t \neq 0; \qquad M(0) = 1$
	$\mu = \dfrac{a+b}{2}, \qquad \sigma^2 = \dfrac{(b-a)^2}{12}$

Table X Discrete Distributions

Probability Distribution and Parameter Values	Probability Mass Function	Moment-Generating Function	Mean $E(X)$	Variance $\mathrm{Var}(X)$	Examples
Bernoulli $0 < p < 1$ $q = 1 - p$	$p^x q^{1-x},\ \ x = 0, 1$	$q + pe^t,$ $-\infty < t < \infty$	p	pq	Experiment with two possible outcomes, say success and failure, $p = P(\text{success})$
Binomial $n = 1, 2, 3, \ldots$ $0 < p < 1$	$\binom{n}{x} p^x q^{n-x},$ $x = 0, 1, \ldots, n$	$(q + pe^t)^n,$ $-\infty < t < \infty$	np	npq	Number of successes in a sequence of n Bernoulli trials, $p = P(\text{success})$
Geometric $0 < p < 1$ $q = 1 - p$	$q^{x-1} p,$ $x = 1, 2, \ldots$	$\dfrac{pe^t}{1 - qe^t}$ $t < -\ln(1 - p)$	$\dfrac{1}{p}$	$\dfrac{q}{p^2}$	The number of trials to obtain the first success in a sequence of Bernoulli trials
Hypergeometric $x \le n, x \le N_1$ $n - x \le N_2$ $N = N_1 + N_2$ $N_1 > 0,\ \ N_2 > 0$	$\dfrac{\binom{N_1}{x}\binom{N_2}{n-x}}{\binom{N}{n}}$		$n\left(\dfrac{N_1}{N}\right)$	$n\left(\dfrac{N_1}{N}\right)\left(\dfrac{N_2}{N}\right)\left(\dfrac{N-n}{N-1}\right)$	Selecting n objects at random without replacement from a set composed of two types of objects
Negative Binomial $r = 1, 2, 3, \ldots$ $0 < p < 1$	$\binom{x-1}{r-1} p^r q^{x-r},$ $x = r, r+1, \ldots$	$\dfrac{(pe^t)^r}{(1 - qe^t)^r},$ $t < -\ln(1 - p)$	$\dfrac{r}{p}$	$\dfrac{rq}{p^2}$	The number of trials to obtain the rth success in a sequence of Bernoulli trials
Poisson $\lambda > 0$	$\dfrac{\lambda^x e^{-\lambda}}{x!},$ $x = 0, 1, \ldots$	$e^{\lambda(e^t - 1)}$ $-\infty < t < \infty$	λ	λ	Number of events occurring in a unit interval, events are occurring randomly at a mean rate of λ per unit interval
Uniform $m > 0$	$\dfrac{1}{m},\ \ x = 1, 2, \ldots, m$		$\dfrac{m+1}{2}$	$\dfrac{m^2 - 1}{12}$	Select an integer randomly from $1, 2, \ldots, m$

Table XI Continuous Distributions

Probability Distribution and Parameter Values	Probability Density Function	Moment-Generating Function	Mean $E(X)$	Variance $\mathrm{Var}(X)$	Examples
Beta $\alpha > 0$ $\beta > 0$	$\dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$, $0 < x < 1$		$\dfrac{\alpha}{\alpha+\beta}$	$\dfrac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2}$	$X = X_1/(X_1+X_2)$, where X_1 and X_2 have independent gamma distributions with same θ
Chi-square $r = 1, 2, \ldots$	$\dfrac{x^{r/2-1}e^{-x/2}}{\Gamma(r/2)2^{r/2}}$, $0 < x < \infty$	$\dfrac{1}{(1-2t)^{r/2}},\ t < \dfrac{1}{2}$	r	$2r$	Gamma distribution, $\theta = 2$, $\alpha = r/2$; sum of squares of r independent $N(0,1)$ random variables
Exponential $\theta > 0$	$\dfrac{1}{\theta}e^{-x/\theta},\ 0 \le x < \infty$	$\dfrac{1}{1-\theta t},\ t < \dfrac{1}{\theta}$	θ	θ^2	Waiting time to first arrival when observing a Poisson process with a mean rate of arrivals equal to $\lambda = 1/\theta$
Gamma $\alpha > 0$ $\theta > 0$	$\dfrac{x^{\alpha-1}e^{-x/\theta}}{\Gamma(\alpha)\theta^{\alpha}}$, $0 < x < \infty$	$\dfrac{1}{(1-\theta t)^{\alpha}},\ t < \dfrac{1}{\theta}$	$\alpha\theta$	$\alpha\theta^2$	Waiting time to αth arrival when observing a Poisson process with a mean rate of arrivals equal to $\lambda = 1/\theta$
Normal $-\infty < \mu < \infty$ $\sigma > 0$	$\dfrac{e^{-(x-\mu)^2/2\sigma^2}}{\sigma\sqrt{2\pi}}$, $-\infty < x < \infty$	$e^{\mu t+\sigma^2 t^2/2}$ $-\infty < t < \infty$	μ	σ^2	Errors in measurements; heights of children; breaking strengths
Uniform $-\infty < a < b < \infty$	$\dfrac{1}{b-a},\ a \le x \le b$	$\dfrac{e^{tb}-e^{ta}}{t(b-a)},\ t \ne 0$ $1,\ t = 0$	$\dfrac{a+b}{2}$	$\dfrac{(b-a)^2}{12}$	Select a point at random from the interval $[a, b]$