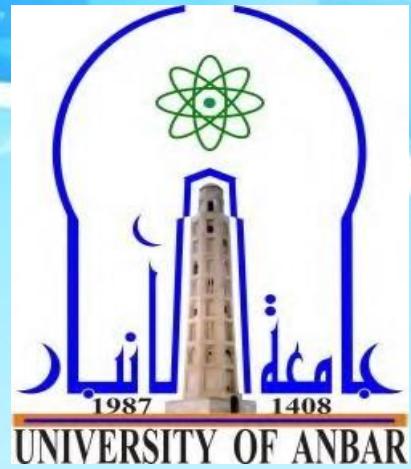


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**College of Education for Pure Sciences**

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## **محاضرات الاحصاء ١**

**مدرس المادة : الاستاذ المساعد الدكتور**

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## Distribution function method

Basically the method of distribution function is as follows If x is a random variable with pdf  $f_x(x)$  and if y is some function of x , then we can find the cdf  $f_y(y) = p(Y \leq y)$  directly by integrating  $f_x(x)$  over the region for which  $(Y \leq y)$  , now by differentiating  $f_y(y)$  , we get the probability density function  $f_y(y)$  of Y . In general , if Y is a function of random variable  $x_1.....x_n$  say  $g(x_1.....x_n)$ , then we can summarize the method of distribution function as follows.

### PROCEDURE TO FIND CDF OF A FUNCTION OF R.V USING THE METHOD OF DISTRIBUTION FUNCTIONS.

- 1- find the region  $(Y \leq y)$  in the  $(x_1, x_2....., x_n)$ space that is find the set of  $(x_1, x_2....., x_n)$ for which  $g((x_1,.....,x_n)) \leq y$ .
- 2- find  $f_Y(y)= p(Y \leq y)$  by integrating  $(x_1, x_2....., x_n)$  over the region  $(Y \leq y)$ .
- 3- find the distribution function  $f_y(y)$  by differentiating  $f_Y(y)$ .

**Example:** let  $x \sim N(0,1)$  using the cdf of x find the pdf of  $y=x^2$

**Solution:**

Note that the pdf of X is

$$f_x(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \quad -\infty < x < \infty$$

then the cumulative distribution function of Y for a given  $y > 0$  is  $f_Y(y)= p(Y \leq y) = p(e^x \leq y)$

$$\begin{aligned} &= p(x \leq \ln y) \\ &= \int_{-\infty}^{\ln y} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx \end{aligned}$$

Hence by differentiating  $f_y(y)$ , we obtain the probability density function as.

$$f(y) = \begin{cases} \frac{1}{y\sqrt{2\pi}} e^{\frac{x^2}{2}} & 0 < y \\ 0 & \text{otherwise} \end{cases}$$

Example: let  $f(x) = \frac{1}{x^2}$ ,  $x \geq 1$  find the p. d. f.,  $Y = e^{-x}$  by using distribution technique ?

Solution:

$$f(x) = \begin{cases} \frac{1}{x^2} & \text{for } x \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$f(y) = p(Y \leq y) \implies p(e^{-x} \leq y)$$

$$f(y) = p(-x \leq \ln y) * -1$$

Example: let  $x \sim N(0,1)$  using the cdf of  $x$  find the pdf of  $y = x^2$

Solution:

Since  $x \sim N(0,1)$

$$\therefore f(x) = \begin{cases} \frac{1}{\sqrt{2\pi}} e^{\frac{-1}{2}x^2} & \text{for } -\infty < x < \infty \\ 0 & \text{otherwise} \end{cases}$$

$$f(y) = p(Y \leq y)$$

$$f(y) = p(x^2 \leq y)$$

$$f(y) = p(x \leq \pm\sqrt{y}) \quad -\sqrt{y} < x < \sqrt{y}$$

$$f(y) = \int_{-\sqrt{y}}^{\sqrt{y}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx$$

$$\forall x^2 = 3$$

$$2xd = dy \implies dx = \frac{dy}{2x}$$

$$dx = \frac{dy}{2\sqrt{y}}$$

$$f(y) = 2 \frac{d}{dy} \int_0^{\sqrt{y}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} \frac{1}{2\sqrt{y}} dy$$

$$f(y) = \begin{cases} \frac{1}{\sqrt{2\pi}} (y)^{-\frac{1}{2}} e^{-\frac{y}{2}} & 0 < y < \infty \\ 0 & \text{o.w} \end{cases}$$

$$\begin{aligned} y \& n x_{(1)} && -\infty < x < \infty \\ & && 0 < x < \infty \\ & && 0 < y < \infty \end{aligned}$$

**Example:** let  $x \sim N(0,1)$  using the c. d. f. of  $x$ . find the p. d. f. of  $y = e^x$

**Solution:**

Since  $x \sim N(0,1)$

$$f(y) = \begin{cases} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} & -\infty < x < \infty \\ 0 & \text{o.w} \end{cases}$$

$$f(y) = p(Y \leq y) \implies = p(e^x \leq y)$$

$$f(y) = p(x \leq \ln y)$$

$$f(y) = \int_{-\infty}^{\ln y} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx \implies f(y) = \frac{d}{dy} \int_{-\infty}^{\ln y} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx$$

$$f(y) = \frac{1}{2\pi} e^{\frac{-1}{2}x^2} dx \quad y = e^x$$

$$x = \ln y$$

$$f(y) = \begin{cases} \frac{1}{y\sqrt{2\pi}} e^{\frac{-1(\ln y)^2}{2}} & 0 < y < \infty \\ 0 & \text{o.w.} \end{cases}$$

$$\infty < x < \infty$$

$$e^\infty < e^x > e^\infty$$

$$0 \leq e^x < \infty$$

$$0 < y < \infty$$

**Example:** If  $X \sim \text{Poisson}(y)$  find the cumulative distribution function of  $Y = ax + b$

**Solution:**

Since  $x \sim N(0,1)$

$$\therefore f(y) = \begin{cases} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} & \text{for } x = 0, \dots, \infty \\ 0 & \text{o.w.} \end{cases}$$

$$f(y) = p(Y \leq y) \implies p(ax + b \leq y)$$

$$f(y) = p(ax \leq y - b) \div a$$

$$f(y) = p\left(x \leq \frac{y-b}{a}\right)$$

sine xupo(1)  $\implies$  discrete distribution

$$f(y) = \sum_{x=0}^{\frac{y-b}{a}} \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}$$

$$y = ax + b \quad x = 0, \dots$$

$$\text{if } x=0 \Rightarrow y = 0+b$$

$$\text{if } x=1 \longrightarrow y=a+b$$

$$\text{if } x=2 \longrightarrow y=2a+b$$

$$y=b, a+b, 2a+b, 3a+b, \dots$$

$$\therefore y = na + b \quad \delta, n = 0, \dots \dots \dots$$

$$f(y) = p \left[ x \geq -\ln y \right] \longrightarrow f(y) = 1 - p \left[ x \leq -\ln y \right]$$

$$f(y) = 1 - \int_1^{-1 \ln y} \frac{1}{x^2} dx \longrightarrow f(y) = 1 - \int_1^{-1 \ln y} x^{-2} dx$$

$$f(y) = 1 - \left[ \frac{1}{2} \right]_1^{-\ln y} \longrightarrow f(y) = 1 - \left[ \frac{1}{\ln y} + 1 \right]$$

$$f(y) = \frac{-1}{x}$$

$$f(y) = \frac{5 - (-1)^{\frac{1}{y}}}{(\ln y)^2} \longrightarrow f(y) = \frac{\frac{1}{y}}{(\ln y)^2}$$

$$f(y) = \frac{1}{y(\ln y)^2} \quad 1 \leq x < \infty$$

$$-1 \geq -x > -\infty$$

$$\infty \leq -x < -1$$

$$e^{-\infty} < e^{-x} < e^{-1}$$

$$0 < y < e^{-1}$$

$$\therefore f(x) = \begin{cases} \frac{1}{y(\ln y)^2} & \text{for } 0 < y < e^1 \\ 0 & \text{o.w.} \end{cases}$$