

Republic of Iraq Ministry of Higher Education & Research

University of Anbar

College of Education for Pure Sciences

Department of Mathematics



Lecture Note On Mathematical Statistics 2

B.Sc. in Mathematics

Fourth Stage

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Syllabus of Mathematical Statistics 2

Chapter 1: Bayesian Estimation

More Bayesian concepts, Some methods of Estimation, Informative and non informative prior.

Chapter 2: Interval Estimation:

Confidence Intervals for Means, Confidence Intervals for the Difference of Two Means, Confidence Intervals for Proportions sample size, Confidence intervals for Variance and for ratio between two Variances, Confidence intervals for differences of probabilities

Chapter 3: Test Of Hypotheses:

General Concepts, Type of test of Hypothesis, Critical Region, Best of Critical region statistical test, Neyman – Pearson Theorem uniformly most powerful test, Likelihood ratio test, Sequential test.

References

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- مقدمة في الاحصاء الرياضي، صباح داود سليم، الطبعة الاولى، جامعة البصرة، ١٩٨٩ .
- Introduction to Mathematical Statistics, R. V. Hogg and A.T. Craig, (4, 5, 6) edition.
- Introduction to the Theory of Statistics, Alexander M.Mood, Franklin, A. Graybill, MC- Graw Hill, 1974.
- Mathematical Statistics with Applications, K. M. Ramachandran and C. P. Tsokos, 2009.
- Mathematical Statistics with Application, Richard L.Schaffer, Seventh Edition, Thomson Brooks, 2008.
- Probability and mathematical Statistics, Prasanna Sahoo, University of Louisville,, USA, 2008.
- An introduction to Mathematical Statistics and Its Applications, Ricard I. larson and Morris Luoio, Fifth edition, Person Education, INC, 2012.
- Probability and Statistical Inference, Robert V. Hogg ,Elliot A. Tanis and ale L. Zimmerman, Ninth Edition, Pearson Education, USA,2015.

Confidence Intervals

Parameter	Assumptions	Endpoints
μ	$N(\mu, \sigma^2)$ or n large, σ^2 known	$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$
μ	$N(\mu, \sigma^2)$ σ^2 unknown	$\bar{x} \pm t_{\alpha/2}(n-1) \frac{s}{\sqrt{n}}$
$\mu_X - \mu_Y$	$N(\mu_X, \sigma_X^2)$ $N(\mu_Y, \sigma_Y^2)$ σ_X^2, σ_Y^2 known	$\bar{x} - \bar{y} \pm z_{\alpha/2} \sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}$
$\mu_X - \mu_Y$	Variances unknown, large samples	$\bar{x} - \bar{y} \pm z_{\alpha/2} \sqrt{\frac{s_X^2}{n} + \frac{s_Y^2}{m}}$

$\mu_X - \mu_Y$	$N(\mu_X, \sigma_X^2)$ $N(\mu_Y, \sigma_Y^2)$ $\sigma_X^2 = \sigma_Y^2$, unknown	$\bar{x} - \bar{y} \pm t_{\alpha/2}(n+m-2)s_p\sqrt{\frac{1}{n} + \frac{1}{m}},$ $s_p = \sqrt{\frac{(n-1)s_x^2 + (m-1)s_y^2}{n+m-2}}$
$\mu_D = \mu_X - \mu_Y$	X and Y normal, but dependent	$\bar{d} \pm t_{\alpha/2}(n-1)\frac{s_d}{\sqrt{n}}$
p	$b(n, p)$ n is large	$\frac{y}{n} \pm z_{\alpha/2}\sqrt{\frac{(y/n)[1 - (y/n)]}{n}}$
$p_1 - p_2$	$b(n_1, p_1)$ $b(n_2, p_2)$	$\frac{y_1}{n_1} - \frac{y_2}{n_2} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1 - \hat{p}_1)}{n_1} + \frac{\hat{p}_2(1 - \hat{p}_2)}{n_2}},$ $\hat{p}_1 = y_1/n_1, \quad \hat{p}_2 = y_2/n_2$

Tests of Hypotheses

Hypotheses	Assumptions	Critical Region
$H_0: \mu = \mu_0$ $H_1: \mu > \mu_0$	$N(\mu, \sigma^2)$ or n large, σ^2 known	$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \geq z_\alpha$
$H_0: \mu = \mu_0$ $H_1: \mu > \mu_0$	$N(\mu, \sigma^2)$ σ^2 unknown	$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \geq t_\alpha(n-1)$
$H_0: \mu_X - \mu_Y = 0$ $H_1: \mu_X - \mu_Y > 0$	$N(\mu_X, \sigma_X^2)$ $N(\mu_Y, \sigma_Y^2)$ σ_X^2, σ_Y^2 known	$z = \frac{\bar{x} - \bar{y} - 0}{\sqrt{(\sigma_X^2/n) + (\sigma_Y^2/m)}} \geq z_\alpha$
$H_0: \mu_X - \mu_Y = 0$ $H_1: \mu_X - \mu_Y > 0$	Variances unknown, large samples	$z = \frac{\bar{x} - \bar{y} - 0}{\sqrt{(s_x^2/n) + (s_y^2/m)}} \geq z_\alpha$

$$\begin{array}{ll}
H_0: \mu_X - \mu_Y = 0 & N(\mu_X, \sigma_X^2) \\
H_1: \mu_X - \mu_Y > 0 & N(\mu_Y, \sigma_Y^2)
\end{array}$$

$$\sigma_X^2 = \sigma_Y^2, \text{ unknown}$$

$$t = \frac{\bar{x} - \bar{y} - 0}{s_p \sqrt{(1/n) + (1/m)}} \geq t_\alpha(n+m-2)$$

$$s_p = \sqrt{\frac{(n-1)s_x^2 + (m-1)s_y^2}{n+m-2}}$$

$$\begin{array}{ll}
H_0: \mu_D = \mu_X - \mu_Y = 0 & X \text{ and } Y \text{ normal,} \\
H_1: \mu_D = \mu_X - \mu_Y > 0 & \text{but dependent}
\end{array}$$

$$t = \frac{\bar{d} - 0}{s_d / \sqrt{n}} \geq t_\alpha(n-1)$$

$$\begin{array}{ll}
H_0: p = p_0 & b(n, p) \\
H_1: p > p_0 & n \text{ is large}
\end{array}$$

$$z = \frac{(y/n) - p_0}{\sqrt{p_0(1-p_0)/n}} \geq z_\alpha$$

$$\begin{array}{ll}
H_0: p_1 - p_2 = 0 & b(n_1, p_1) \\
H_1: p_1 - p_2 > 0 & b(n_2, p_2)
\end{array}$$

$$z = \frac{(y_1/n_1) - (y_2/n_2) - 0}{\sqrt{\left(\frac{y_1 + y_2}{n_1 + n_2}\right)\left(1 - \frac{y_1 + y_2}{n_1 + n_2}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \geq z_\alpha$$