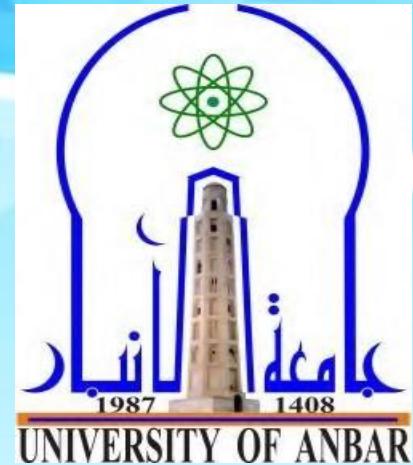


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University of Anbar



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محاضرات الاحصاء ٢

**مدرس المادة : الاستاذ المساعد الدكتور
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CONFIDENCE INTERVAL FOR VARIANCE σ^2

In this topic we have two cases

(a): The case of μ is Unknown

Let $X^2 = \frac{(n-1)s^2}{\sigma^2}$ chi-squared distribution with

$n - 1$ degrees of freedom where $s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$

and s^2 is a point estimator of σ^2

$$P(\chi^2_{1-\alpha/2} < X^2 < \chi^2_{\alpha/2}) = 1 - \alpha$$

$$P(\chi^2_{1-\alpha/2} < \frac{(n-1)s^2}{\sigma^2} < \chi^2_{\alpha/2}) = 1 - \alpha$$

$$P\left(\frac{(n-1)s^2}{\chi^2_{\alpha/2}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}\right) = 1 - \alpha$$

THEOREM : If s^2 is the variance of a random sample of size n from a normal population, a $100(1 - \alpha)\%$ confidence interval for σ^2 is

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}$$

(b): The case of μ is Known

$$P\left(\frac{\sum_{i=1}^n (x_i - \mu_0)^2}{\chi^2_{\alpha/2}} < \sigma^2 < \frac{\sum_{i=1}^n (x_i - \mu_0)^2}{\chi^2_{1-\alpha/2}}\right) = 1 - \alpha$$

ESTIMATING THE RATIO OF TWO VARIANCES

The statistic $\frac{s_1^2}{s_2^2}$ is called an estimator of $\frac{\sigma_2^2}{\sigma_1^2}$.

THEOREM : If σ_1^2 and σ_2^2 are the variances of normal populations, we can establish an interval estimate of $\frac{\sigma_2^2}{\sigma_1^2}$ by using the statistic the random

variable F has an F-distribution with
 $r_1 = m - 1$ and $r_2 = m - 1$ degrees of freedom.

$$F = \frac{\sigma_2^2 S_1^2}{\sigma_1^2 S_2^2}$$

$$P\left(f_{1-\alpha/2}(r_1, r_2) < F < f_{\alpha/2}(r_1, r_2)\right) = 1 - \alpha$$

where $f_{1-\alpha/2}(r_1, r_2)$ and $f_{\alpha/2}(r_1, r_2)$ are the values of the F-distribution with r_1 and r_2 degrees of freedom, leaving areas of $1 - \alpha/2$ and $\alpha/2$, respectively

$$P\left(f_{1-\alpha/2}(r_1, r_2) < \frac{\sigma_2^2 S_1^2}{\sigma_1^2 S_2^2} < f_{\alpha/2}(r_1, r_2)\right) = 1 - \alpha$$

Multiply by $\frac{S_2^2}{S_1^2}$

$$P\left(\frac{S_1^2}{S_2^2} \frac{1}{f_{\alpha/2}(r_1, r_2)} < \frac{\sigma_2^2}{\sigma_1^2} < \frac{S_1^2}{S_2^2} \frac{1}{f_{1-\alpha/2}(r_1, r_2)}\right) = 1 - \alpha$$

replace the quantity $f_{1-\alpha/2}(r_1, r_2)$ by $\frac{1}{f_{1-\alpha/2}(r_1, r_2)}$ Therefore

$$P\left(\frac{S_1^2}{S_2^2} \frac{1}{f_{1-\alpha/2}(r_1, r_2)} < \frac{\sigma_2^2}{\sigma_1^2} < \frac{S_1^2}{S_2^2} f_{\alpha/2}(r_1, r_2)\right) = 1 - \alpha$$

Confidence Interval for $\frac{\sigma_2^2}{\sigma_1^2}$.

THEOREM: if S_1^2 and S_2^2 are the variances of independent samples of sizes n and m , respectively, from normal populations, then a $100(1 - \alpha)\%$ confidence interval for $\frac{\sigma_2^2}{\sigma_1^2}$ is

$$\left(\frac{S_1^2}{S_2^2} \frac{1}{f_{1-\alpha/2}(r_1, r_2)} < \frac{\sigma_2^2}{\sigma_1^2} < \frac{S_1^2}{S_2^2} f_{\alpha/2}(r_1, r_2)\right)$$

where $f_{\alpha/2}(r_1, r_2)$ is an f-value with $r_1 = n - 1$ and $r_2 = m - 1$ degrees of freedom, leaving an area of $\alpha/2$ to the right, and $f_{\alpha/2}(r_1, r_2)$ is a similar f-value with $r_2 = m - 1$ and $r_1 = n - 1$ degrees of freedom.

Example: An optical form purchases for making lenses. A Scum that the refractive index of 20 pieces of glass have variance of 1.20×10^{-9} construct a 95% C.I for the population variance.

Solution :

$$n = 20, s^2 = 10^{-4} \times 1.2$$

$$n-1 = 14$$

$$1 - \alpha = 0.95 \Rightarrow \alpha = 0.05$$

$$\frac{\alpha}{2} = 0.05 \Rightarrow 1 - \frac{\alpha}{2} = 0.475$$

$$\lambda_2_{\frac{x}{2}} = 32.585 \quad (n = 19 \text{ من الجدول حيث})$$

$$\lambda_2_{1-\frac{\alpha}{2}} = 8.9066 \quad (n = 19 \text{ من الجدول حيث } \frac{n s^2}{\lambda_2_{\frac{\alpha}{2}}}, \frac{n s^2}{\lambda_2_{1-\frac{\alpha}{2}}})$$

$$= (7.304 \times 10^{-5}, 2.694 \times 10^{-5})$$

Example: $n = 12$ taken from $N(\mu, \sigma^2)$, $X^l = 10$, $S^2 = 9$. Find : a 90 % C.I. for σ^2 .

Solution

$$n = 12, X^l = 10, S^2 = 9$$

$$1 - \alpha = 0.9 \Rightarrow \alpha = 0.1 \Rightarrow \frac{\alpha}{2} = 0.05$$

$$\Rightarrow 1 - \frac{\alpha}{2} = 0.025$$

$$\lambda_2_{\frac{\alpha}{2}(0.05)} = 19.675, \lambda_2_{\frac{\alpha}{2}(0.025)} = 4.575 \text{ then } (\frac{n s^2}{\lambda_2_{\frac{\alpha}{2}}}, \frac{n s^2}{\lambda_2_{1-\frac{\alpha}{2}}}) = (5.48, 23.606)$$

Example: Construct a 95% C.I for σ' with unknown mean using the following sample :

$$4.5, 10.2, 10.5, 9.8, 13.6, 19.2, 15.5, 13.3, 10.8, 16.4.$$

Solution

$$S^2 = \frac{\sum (Xi - X')}{n}$$

$$1 - \alpha = 0.95 , n = 10$$

$$\Rightarrow n - 1 = 9$$

$$X' = \frac{\sum Xi}{n} = \frac{123.2}{10}$$

$$X' = 12.23$$

$$S^2 = \frac{152.42}{10}$$

$$S^2 = 15.292$$

$$\frac{\alpha}{2} = 0.05$$

$$\frac{1 - \alpha}{2} = 0.95$$

$$\lambda^{(p)}_{(0.05)} = 16.919$$

$$\lambda^{(p)}_{(0.95)} = 3.325$$

$$\left(\frac{n s^2}{\lambda^2 \frac{\alpha}{2}}, \frac{n s^2}{\lambda^2 \frac{1-\alpha}{2}} \right) = \left(\frac{10 (15.292)}{16.919}, \frac{10 (15.292)}{3.325} \right) = (9.038, 45.90)$$

Xi	X - X'	(X - X')²
4.5	-7.73	59.75
10.2	-2.03	4.12
10.5	-1.73	2.99
9.8	-20.43	5.9
13	0.77	0.59
19.2	6.97	48.5
15.5	3.27	10.69
13.3	1.02	1.04
10.8	-1.43	2.04
16.4	4.17	17.3
123.2		152.92

اذا كانت μ معلوم

$$\left(\frac{n s^2}{\lambda^2 \frac{\alpha}{2}}, \frac{n s^2}{\lambda^2 \frac{1-\alpha}{2}} \right)$$

ملاحظة : نأخذ قيمة من الجدول بـ n

مثال : اذا علمت ان تباين عينة عشوائية ذات حجم ٢٥ مسحوبة من $N(10, \sigma^2)$ وكان $S^2 = 9$ جد بمعامل ثقة ٩٥% لتباين هذا المجتمع

// الحل

$$1 - \alpha = 0.95 \Rightarrow \alpha = 0.05 \Rightarrow \frac{\alpha}{2} = 0.025$$

$$1 - \frac{\alpha}{2} = 0.975$$

$$\lambda^2_{\frac{25}{0.025}} = 13.1197 , \lambda^2_{\frac{25}{0.975}} = 40.6465$$

$$\left(\frac{n s_2}{\lambda^2 \frac{\alpha}{2}}, \frac{n s_2}{\lambda^2 1 - \frac{\alpha}{2}} \right)$$

$$= (5.5355, 17.1498)$$

Example: A r.v of size 21 ~ N (μ , σ^2) with $S^2 = 9$. Determine 90% C.I. for σ^2

Solution

$$n = 21, S^2 = 9$$

$$1 - \alpha = 0.90 \Rightarrow \alpha = 0.1 \Rightarrow \frac{\alpha}{2} = 0.05$$

$$1 - \frac{\alpha}{2} = 0.95$$

$$n = 21 \Rightarrow n-1 = 20$$

$$\lambda^2 \frac{20}{\frac{\alpha}{2}} = 31.410, \lambda^2 \frac{20}{0.95} = 10.831$$

$$\left(\frac{n s_2}{\lambda^2 \frac{\alpha}{2}}, \frac{n s_2}{\lambda^2 1 - \frac{\alpha}{2}} \right) = \left(\frac{21(9)}{31.410}, \frac{21(9)}{10.831} \right) = (6.017, 17.4)$$

تقدير $(\mu_1 - \mu_2)$ عندما تكون σ^2 مشتركة وغير معلومة

$$((X'_1 - X'_2) \pm t_{\frac{\alpha}{2}}^{(n_1+n_2)-2} SP \sqrt{\frac{1}{n_1} + \frac{1}{n_2}})$$

حيث :

$$SP \sqrt{\frac{n_1(s_1)^2 + n_2(s_2)^2}{(n_1+n_2)-2}}$$

Example: $n_1 = 32, X'_1 = 72, S_1 = 8, n_2 = 32, X'_2 = 72, S_2 = 8$. Constrict a 99% C. I. from the difference of mean (Assume S.D are equal)

$$\text{Solution: } SP \sqrt{\frac{n_1(s_1)^2 + n_2(s_2)^2}{(n_1+n_2)-2}} = SP \sqrt{\frac{(32)(64) + 32(36)}{62}}$$

$$1 - \alpha = 0.99 \Rightarrow \alpha = 0.01 = 7.18 \text{ where } \frac{\alpha}{2} = 0.005$$

$$= [(72-70) \pm (2.660)(7.18) \sqrt{\frac{1}{64}}] = [2 \pm 19.0988(0.156)] = [2 \pm 0.298]$$

$$= [1.702, 2.298]$$