Republic of Iraq Ministry of Higher Education & Research

University of Anbar

College of Education for Pure Sciences

Department of Mathematics

محاضرات الاحصاء 2

مدرس المادة : الاستاذ المساعد الدكتور

فراس شاكر محمود

A Statistical Test of Hypothesis

A Statistical of hypothesis consists of five parts:

1. The null hypothesis denoted by $H_\circ$.
2. The alternative hypothesis denoted by H_1.
3. The test statistic and its P_value.
4. The rejection region.
5. The conclusion.

Definition:

(1) The Two competing hypothesis are the alternative hypothesis H_1 generally the hypothesis that the researcher wishes to support and the null hypothesis $H_\circ$ a contradiction of the alternative hypothesis.

(2) Test statistic :a single number calculated from the sample data .

(3) P_value :a probability calculated using the test statistic .

Example :. a random sample of 100 California carpenters , you wish to show that the average hourly witness of carpenters in the state of California is different from 14$,which is the national average .

Solution :.

(1) This is alternative hypothesis $H_1 : \mu \neq 14$

The null hypothesis $H_\circ : \mu = 14$

(2) Test statistic let $\bar{X} = 15$ lies

S.E$= \frac{s}{\sqrt{n}} = \frac{2}{\sqrt{100}} \Longrightarrow Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} = \frac{15-14}{2/10} = 5$

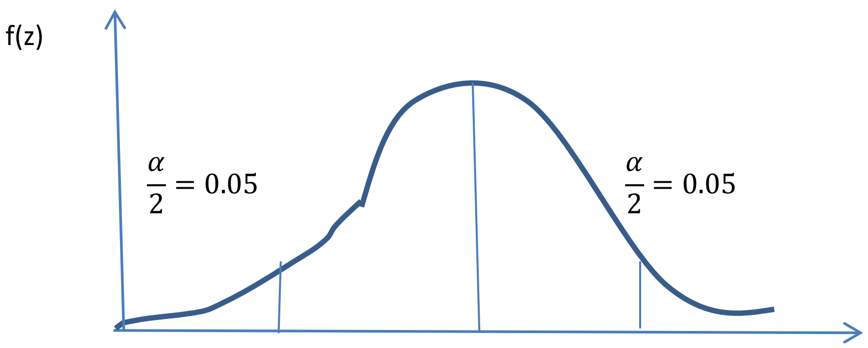
(3) P_value $= P(Z < 5) + P(Z > 5) \cong 0$

Small P_value Large P_value

Definition:

A Type I error for a statistical test is the error of rejecting the null hypothesis when it is true .The level of significance (significance level) for a statistical test of hypothesis is

α =p(Type I error)=p(falsely rejecting $H_\circ$)=p(rejecting $H_\circ$ when it is true)



-2.58 $\quad 0 \quad$ 2.58 $\quad Z$

$Rejection\ region \longrightarrow|$ $\quad$ $|\longleftarrow Rejection\ region$

Large sample statistical Test For μ

(1)Null hypothesis $H_\circ: \mu = \mu_\circ$

(2)Alternative hypothesis $H_1: \mu \neq \mu_\circ$ (Two Tailed test)

$H_1: \mu < \mu_\circ \; or \; H_1: \mu > \mu_\circ$ (one Tailed test)

(3)Test statistic : Z= $\frac{\bar{x}-\mu_\circ}{\sigma/\sqrt{n}}$ estimated as Z= $\frac{\bar{x}-\mu_\circ}{s/\sqrt{n}}$

(4)Rejection region :Reject $H_\circ$ when(one Tail test) $Z > Z_\alpha \; or \; Z < -Z_\alpha$ when alternative hypothesis $H_1: \mu < \mu_\circ$

(Two Tailed test)$Z > Z_{\frac{\alpha}{2}} \; or \; Z < -Z_{\frac{\alpha}{2}}$

Calculating the P_value

<u>Definition:</u> The P_value or observed significance level of a statistical test is the smallest value of α for which $H_\circ$ can be rejected . It is the a ctual risk of committing a Type I error ,if $H_\circ$ is rejected based on the observed value of the test statistic .The P_value measures the strength of the evidence against $H_\circ$.

<u>Definition:</u> If the P_value is less than or equal to α preassigned significance level α , then the null hypothesis can be rejected , and you can report the results are statistically significant at level α .

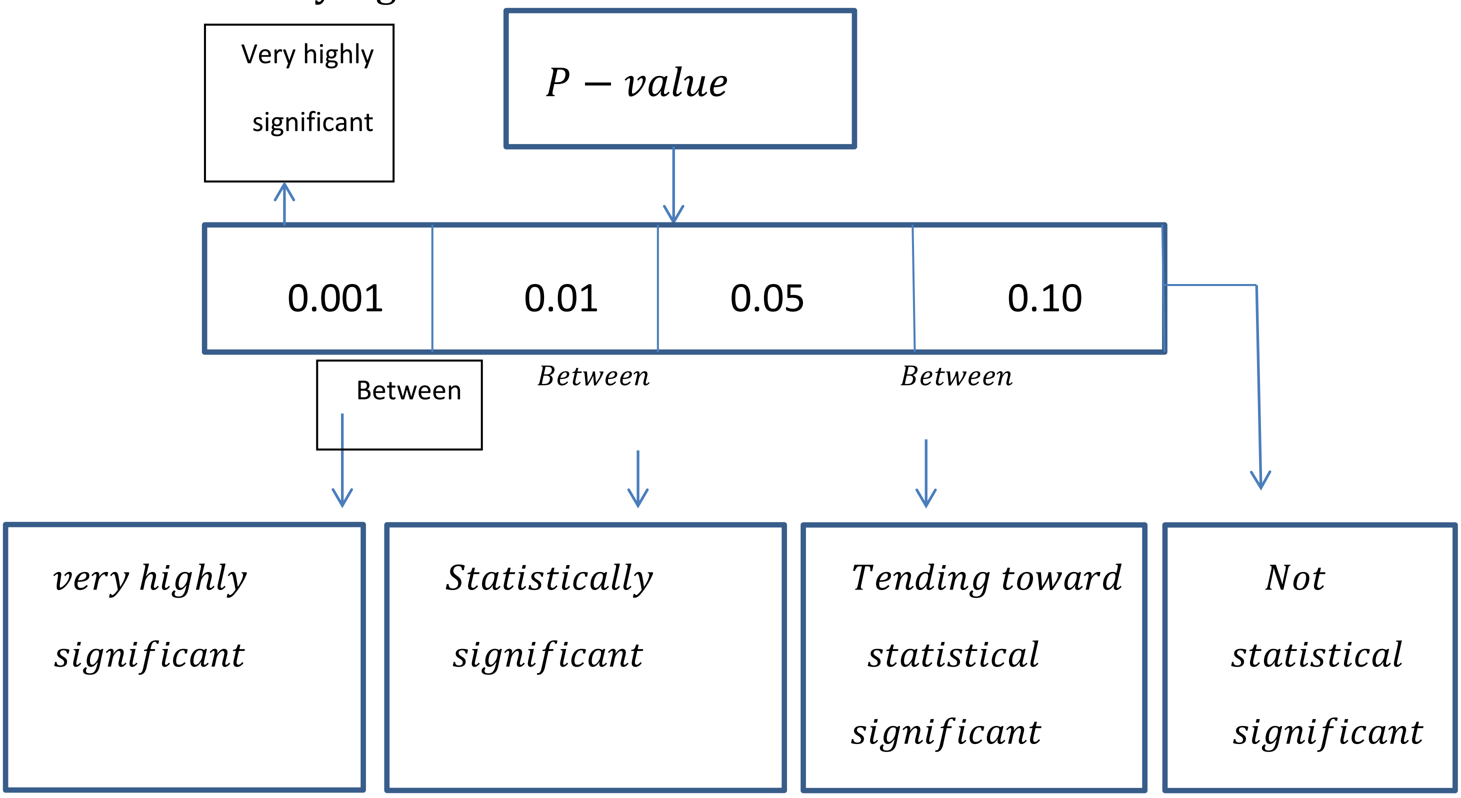
(1)if P_value is less than 0.01,$H_\circ$ is rejected or between 0.01 and 0.001 ,$H_\circ$ is rejected . The result are highly significant

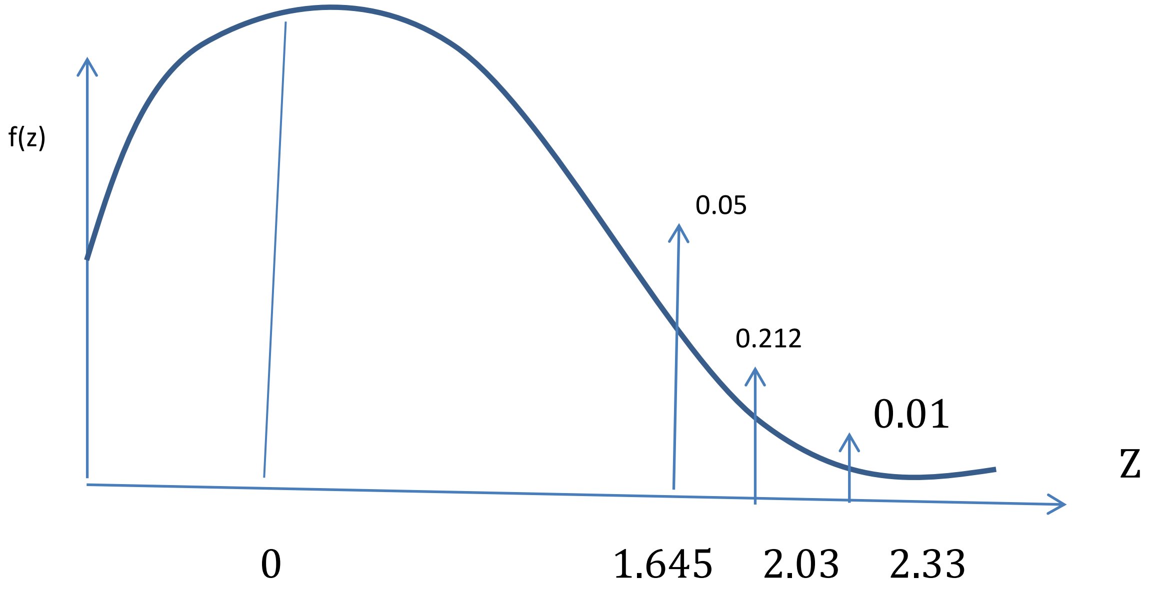
(2) if P_value between 0.01 and 0.05 ,$H_\circ$ is rejected .The result are statistically significant

(3)if P_value is greater than 0.001,the result are very highly significant

(4)if the P_value is between 0.05 and 0.10, $H_\circ$ is usually not rejected .The result are only tending toward statistical significance.

(5)if the P_value greater than 0.10 , $H_\circ$ is not rejected the result are not statistically significant.





Standard normal test Z

Two Type of Errors

<u>Definition:</u> :

(1)A Type I error for a statistical test is the error of rejecting $H_\circ$ when it is true. The probability of making a type I error is denoted by the symbol α .

(2)A Type II error for a statistical test is the error of accepting $H_\circ$ when it is false and some H_1 is true . The probability of making a Type II error is denoted by the symbol β.

(3)The power of a statistical test , given as $1-\beta$ =p(reject $H_\circ$ *when* H_1 *is true*) measures the ability of the test to perform as required .

Large Sample Statistical Test For$(\boldsymbol{\mu_1}-\boldsymbol{\mu_2})$

(1)Null hypothesis :$H_\circ$: $\mu_1-\mu_2=D_\circ$, where $D_\circ$ is some specified difference that you wish to test it .For many test $D_\circ=0$ there is no difference between μ_1 *and* μ_2

(2)Alternative hypothesis:

(one Tailed Test)$H_1{:}\,\mu_1-\mu_2>D_\circ$ *or* $H_1{:}\,\mu_1-\mu_2<D_\circ$

(Two Tailed Test) $H_1{:}\,\mu_1-\mu_2\neq D_\circ$

(3)Test statistic:

$$Z=\frac{(\overline{X}_1-\overline{X}_2)-D_\circ}{S.E}=\frac{(\overline{X}_1-\overline{X}_2)-D_\circ}{\frac{S_1}{\sqrt{n_1}}+\frac{S_2}{\sqrt{n_2}}}$$

(4)Rejection region : Reject $H_\circ$ when

(one Tailed Test)

$Z>Z_\alpha$ or $Z<-Z_\alpha$ when H_1: $(\mu_1-\mu_2<D_\circ)$or when P_value $<\alpha$

(Two Tailed Test)

$Z > Z_{\frac{\alpha}{2}}$ or $Z < -Z_{\frac{\alpha}{2}}$

Large Sample Test of Hypothesis for A Binomial Proportion

(1)Null hypothesis: $H_\circ: P = P_\circ$

(2)Alternative hypothesis:

(One Tailed Test):

$H_1: P > P_\circ$ or $H_1: P < P_\circ$

$(Two\ Tailed\ Test)$:

$H_1: P \neq P_\circ$

(3)Test statistic :

$$Z = \frac{\bar{P} - P_\circ}{S.E} = \frac{\bar{P} - P_\circ}{\sqrt{\frac{P_\circ(1 - P_\circ)}{n}}} \quad with\ \bar{P} = \frac{x}{n}$$

Where x is the number of successes in n binomial trials .

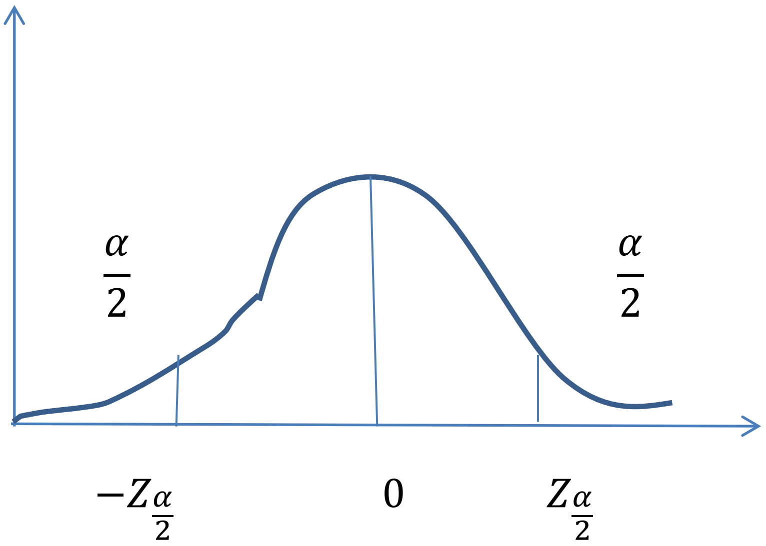
(4)Rejection region : Reject $H_\circ$ when

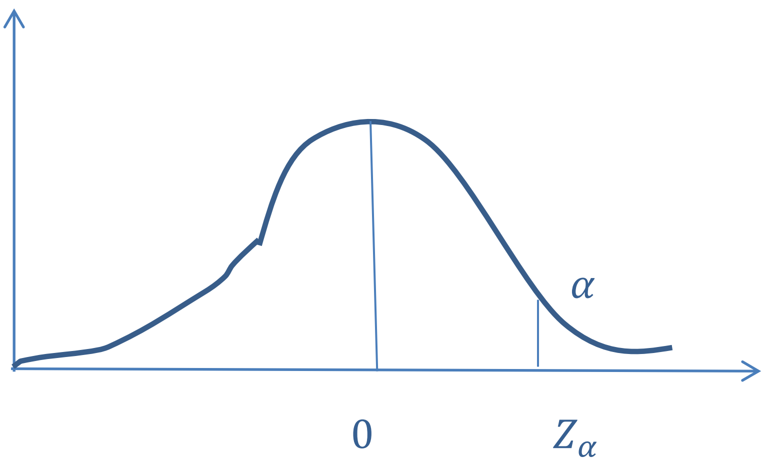
(One Tailed Test)

$Z > Z_\alpha$ or $Z < -Z_\alpha$ $when$ $H_1: P < P_\circ$ $or\ when\ P\_value < \alpha$

$(Two\ Tailed\ Test)$:

$Z > Z_{\frac{\alpha}{2}}$ or $Z < -Z_{\frac{\alpha}{2}}$





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Large Sample Statistical Test For $(P_1 - P_2)$

(1)Null hypothesis : $H_\circ: P_1 - P_2 = 0 \text{ or } H_\circ: P_1 = P_2$

(2)Alternative hypothesis:

(One Tailed Test):

$H_1: P_1 - P_2 > 0 \text{ or } \quad H_1: P_1 - P_2 < 0$

(Two Tailed Test):

$H_1: P_1 - P_2 \neq 0$

(3)Test Statistic:

$$Z = \frac{(\overline{P_1} - \overline{P_2}) - 0}{S.E} = \frac{(\overline{P_1} - \overline{P_2}) - 0}{\sqrt{\frac{P_1(1-P_1)}{n_1} + \frac{P_2(1-P_2)}{n_2}}}$$

where $\overline{P_1} = \frac{x_1}{n_1}$ $and\ \overline{P_2} = \frac{x_2}{n_2}$ $, since\ P_1 = P_2 = P$ used the s.e is unknown, it is estimated by $\hat{P} = \frac{x_1+x_2}{n_1+n_2}$ then the test statistic

$$Z = \frac{(\widehat{P_1} - \widehat{P_2}) - 0}{\sqrt{\frac{\hat{P}(1-\hat{P})}{n_1} + \frac{\hat{P}(1-\hat{P})}{n_2}}} = \frac{(\widehat{P_1} - \widehat{P_2})}{\sqrt{\hat{P}\left(1-\hat{P}\right)(\frac{1}{n_1} + \frac{1}{n_2})}}$$

(4)Rejection Region : Reject $H_\circ$ when

(One Tailed Test):

$Z > Z_\alpha \text{ or } Z < -Z_\alpha \text{ when } H_1: P_1 - P_2 < 0 \quad \text{or when } P\_value < \alpha$

(Tow Tailed Test):

$Z > Z_{\frac{\alpha}{2}} \quad or \quad Z < -Z_{\frac{\alpha}{2}}$

Small Sample Hypothesis Test For μ

(1) Null hypothesis: $H_\circ : \mu = \mu_\circ$

(2) Alternative hypothesis:

(One Tailed Test):

$H_1 : \mu > \mu_\circ \quad or \quad H_1 : \mu < \mu_\circ$

(Two Tailed Test):

$H_1 : \mu \neq \mu_\circ$

(3)Test Statistic :

$$t = \frac{\bar{x} - \mu_\circ}{s/\sqrt{n}}$$

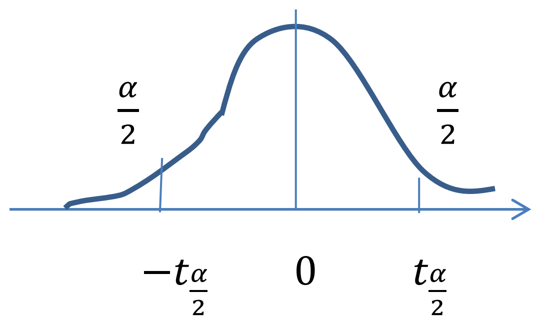
(4)Rejection Region: Reject , $H_\circ$ when

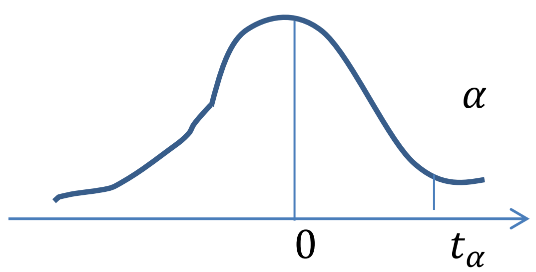
(One Tailed Test):

$t > t_\alpha \quad or \quad t < -t_\alpha \;\; when \; H_1 : \mu < \mu_\circ \quad or \; when \; P\_value < \alpha$

(Two Tailed Test):

$t > t_{\frac{\alpha}{2}} \quad or \quad t < -t_{\frac{\alpha}{2}}$





Test of Hypothesis Concerning The difference between Two means : Independent Random Samples.

(1)Null hypothesis :$H_\circ: \mu_1 - \mu_2 = D_\circ$, where $D_\circ$ is some specified difference that you wish to test it . For many test $D_\circ = 0$ there is no difference between μ_1 and μ_2

(2)Alternative hypothesis:

(one Tailed Test)$H_1: \mu_1 - \mu_2 > D_\circ$ or $H_1: \mu_1 - \mu_2 < D_\circ$

(Two Tailed Test) $H_1: \mu_1 - \mu_2 \neq D_\circ$

(3)Test Statistic :

$$t = \frac{(\overline{x_1} - \overline{x_2}) - D_\circ}{\sqrt{s^2(\frac{1}{n_1} + \frac{1}{n_2})}}$$

where $s^2 = \frac{(n_1-1){s^2}_1 + (n_2-1){s^2}_2}{n_1+n_2-2}$

(4)Rejection region : Reject $H_\circ$ when

(one Tailed Test):

$t > t_\alpha$ or $t < -t_\alpha$ when $H_1 : (\mu - \mu_\circ) < D_\circ$ or when P_value $< \alpha$

(Two Tailed Test):

$$t > t_{\frac{\alpha}{2}} \quad or \quad t < -t_{\frac{\alpha}{2}}$$

Test of hypothesis Concerning a population Variance

(1)Null hypothesis :

$H_\circ : \sigma^2 = {\sigma^2}_\circ$

(2)Alternative hypothesis :

(One Tailed Test):

$H_1: \sigma^2 > {\sigma^2}_\circ$ or $H_1: \sigma^2 < {\sigma^2}_\circ$

(Two Tailed Test):

$\sigma^2 \neq {\sigma^2}_\circ$

(3)Test statistic :

$$x^2 = \frac{(n-1)s^2}{{\sigma^2}_\circ}$$

(4)Rejection Region :Reject $H_\circ$ when

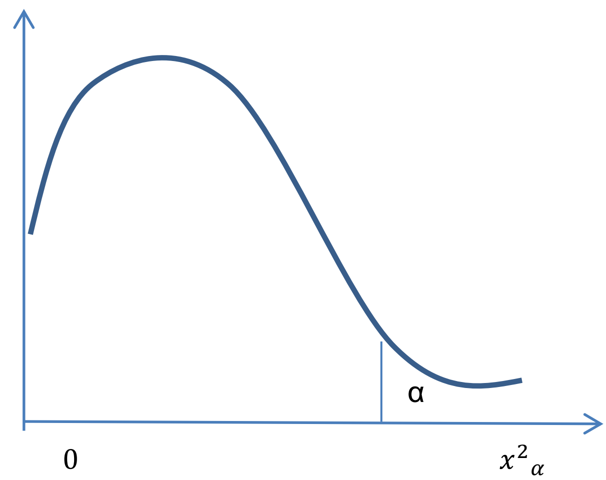
(One Tailed Test):

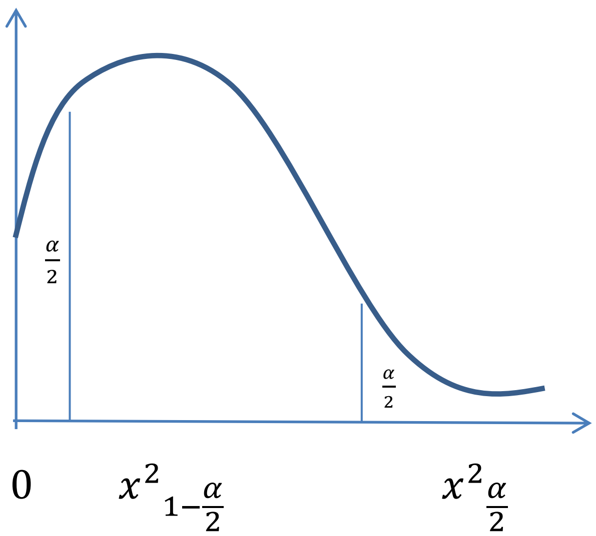
$x^2 > {x^2}_\alpha$ or $x^2 < {x^2}_{(1-\alpha)}$ $when$ $H_1: \sigma^2 < {\sigma^2}_\circ$

(Two Tailed Test):

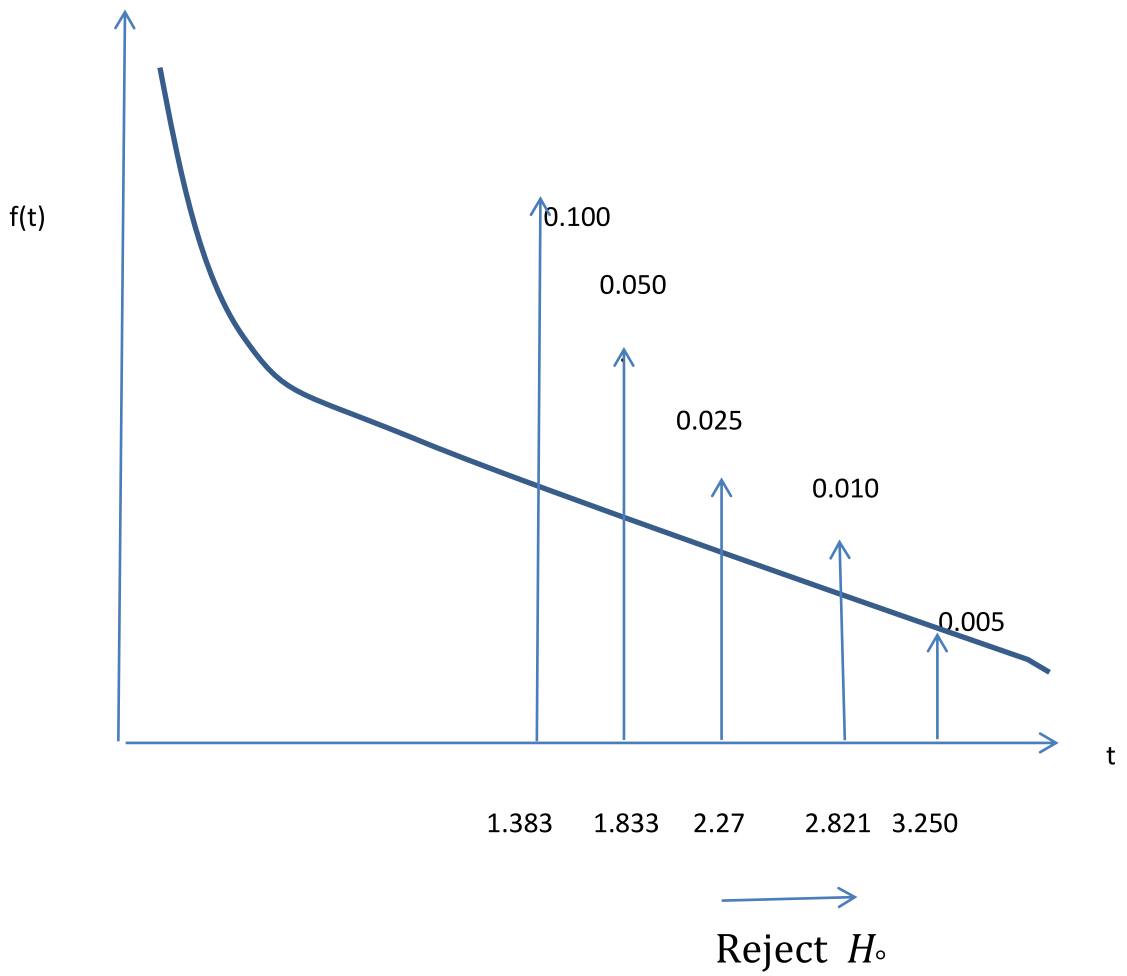
$x^2 > {x^2}_{\frac{\alpha}{2}}$ or $x^2 < {x^2}_{(1-\frac{\alpha}{2})}$

$upper$ $lower$

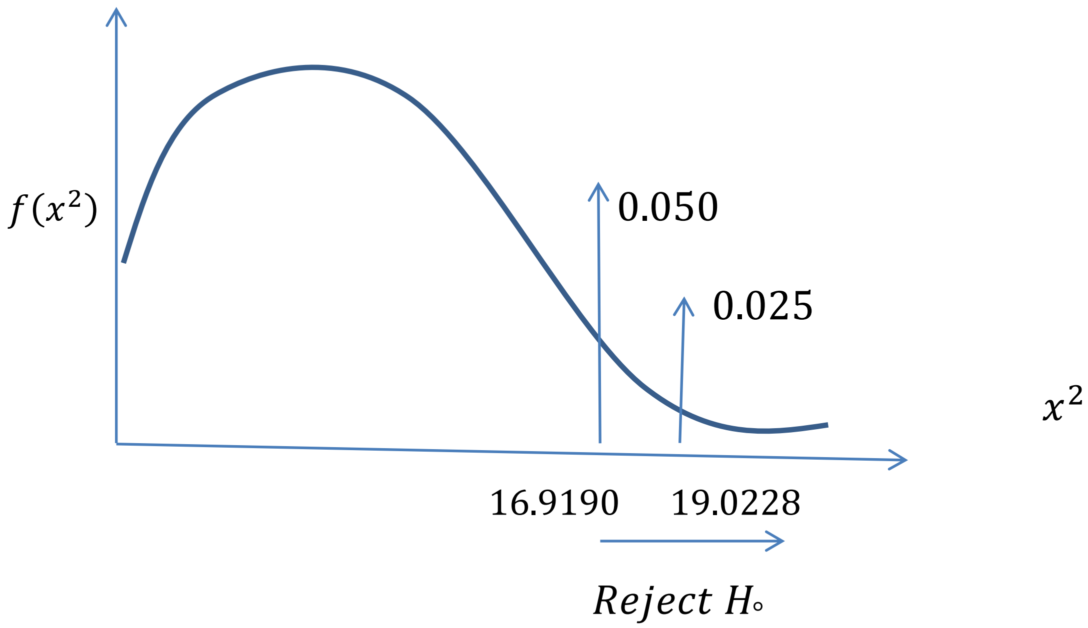




$t - student\ test$



$chi - square\ test$



Test of hypothesis Concerning the equality of two population variances

(1)Null hypothesis :

$H_\circ : \sigma^2{}_1 = \sigma^2{}_2$

(2)Alternative hypothesis :

(One Tailed Test):

$H_1: \sigma^2{}_1 > \sigma^2{}_2$ or $H_1: \sigma^2{}_1 < \sigma^2{}_2$

(Two Tailed Test):

$\sigma^2{}_1 \neq \sigma^2{}_2$

(3) Test Statistic :

(One Tailed Test):

$F_\propto = \frac{s^2{}_1}{s^2{}_2}$

(Two Tailed Test):

$F_{\frac{\propto}{2}} = \frac{s^2{}_1}{s^2{}_2}$

where $s^2{}_1$ is the larger sample variance .

(4) Rejection Region :Reject $H_\circ$ when

(One Tailed Test):

$\mathrm{F} > \mathrm{F}_\alpha$ or when P_value $< \alpha$

(Two Tailed Test):

$F > F_{\frac{\alpha}{2}}$