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محاضرات الاحصاء ٢

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Power of ASTATISTICAL TEST

We gave several tests of a fairly common statistical hypotheses in such a way that described the significance level α and the p-value of each . of course those tests were based on good sufficient) statics of the parameters, when the latter exist. In this lecture, we consider the probability of Making the other type of error accepting I the null hypothesis H_0 when the alternative a hypothesis $H_1$is true. This consideration leads to ways to find Most Powerful tests of the null hypothesis H_0against the alternation hypothesis H_1 . The first example introduces a new concept. using a test about P , the probability of success The Sample size is kept Small so that Table II in Appendix B Can be used to find probabilities. The application is one that you can actually Perform

<u>Example:</u>

Assume that when given a name lag, a person puts it on either the right or left Side Let f equal the probability that the name tag is placed on the right side. we shall test the null hypothesis, H_0:p=$^1/_2$ against the composite alternative the hypothesis H_1:p< $^1/_2$(Included with the null. hypothesis are those values of P which are a greater than $^1/_2$,,that is, we could think of H_0 as H_0 :P≥112 we shall give name tags to a random sample n=20 people. denoting the placements of their name • tags with Bernoulli random variables, $X_1, X_2, \dots \dots \dots X_{20}$ where $X_{i=1}$ if a person places the name tag on the right and $X_i = 0$" if person places the name tag on the left, for our test statistic, we can then use Y= $\sum_{i=1}^{20} X_i$ which has the binomial distribution b (20,P). Say the critical region is defined by C= {Y: Y≤6} or equivalently, by $\{(X_1, X_2, \dots, X_{20})$:$\sum_{i=1}^{20} X_i \leq 6\}$ since y is b (20,$^1/_2$) if P=$^1/_2$ the significance level of the corresponding test is

ϭ=P(Y≤6: P=$^1/_2$) = $\sum_{y=0}^{6} \binom{20}{y} \left(\frac{1}{2}\right)^{20}$=0.0577 from Table II in Appendix B. of course, the probability ß of a Type II error has different value, with different values of P select from the Composite alternative hypothesis $H_1: p < {}^1/_2$ for example::, with P=$^1/_4$, β=P(7≤Y≤20; P=$^1/_4$) =

$\sum_{y=7}^{20}\binom{20}{y}\left(\frac{1}{14}\right)^{y}\left(\frac{3}{4}\right)^{20-y}$ = 0.2142 whereas with P=$^{1}/_{10}$, β=P(7≤Y≤20; P=$^{1}/_{10}$) =$\sum_{y=7}^{20}\binom{20}{y}\left(\frac{1}{10}\right)^{y}\left(\frac{a}{10}\right)^{20-y}$

=0.0024

Instead of considering the probability β of accepting H_0whenH_1, is true, we could compute the probability K of rejecting H_0 when $H_1$is true After all, β and K=1-β Provide the same information since k is a function of P, we denote this explicitly. by writing K(p). The probability

K(P) =$\sum_{Y=0}^{6}\binom{20}{Y}P^{Y}(1-P)^{20-Y}$,0< P≤$\frac{1}{2}$

is called the power function of the test of course, 6=K($^{1}/_{2}$) = 0.0577, 1-K($^{1}/_{4}$)=0.2142, and 1-k($^{1}/_{10}$) =0.0024. The value of the power function at a specified p. is called the power of the test at that point. For instance, K($^{1}/_{4}$)=0.7858 and K($^{1}/_{10}$) = 0.9976 are the powers at P=$^{1}/_{4}$ and P=$^{1}/_{10}$, respectively. An accepta able power function assumes Small Values when H_0 is true and larger values when P differs much from P=$^{1}/_{2}$.