CH-6

SPHERICAL MIRRORS

A spherical reflecting surface has image forming properties similar to those of a thin lens or of a single reflecting surface. The image from a spherical mirror is in some respects superior to that from a lens ,notably in the absence of chromatic white light. Therefore mirrors are occasionally used in place not so broad as those of lenses because they do not offer aberrations of the image.

Because of the simplicity of the law of reflection as compared to the law of refraction, the quantitative study of image formation by mirrors is easier than in the case of lenses. many features are the same of lenses, and there are some characteristics which are different.

Focal point and focal length

Diagrams showing the reflection of a parallel beam of light by a convex mirror and by a concave mirror .

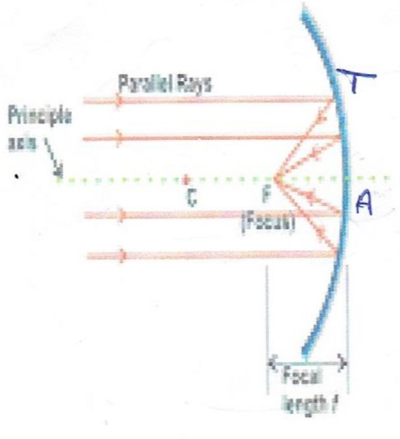


Figure 3a. Concave Mirror

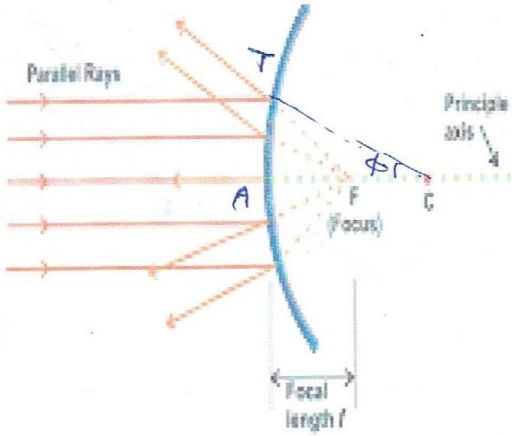


Figure 3b. Convex Mirror

A ray striking the mirror at some point such as T obeys the law of reflection $\phi = \phi$ .All rays are shown as brought to a common focus at F, although this will be strictly true for paraxial rays. The point F is called the focal point and the distance FA the focal length.

In the second diagram the reflected rays diverge as though they come from a common point F. since the angle TCA also equal ϕ, the triangle TCF is an isosceles one, and in general CF=FT, but for very small angles ϕ (paraxial rays),FT approaches equality with FA, hence

$$FA=\frac{1}{2}(CA) \text{ or } \quad f=-\frac{1}{2}\, r$$

and for focal length equals one-half the radius of curvature.

for concave mirror : focal length ⟹ is positive

Positive

radius of curvature ⟹ is ~~positive~~

negative ~~negative~~

for convex mirror: focal length ⟹ is negative

negative

radius of curvature ⟹ is ~~negative~~

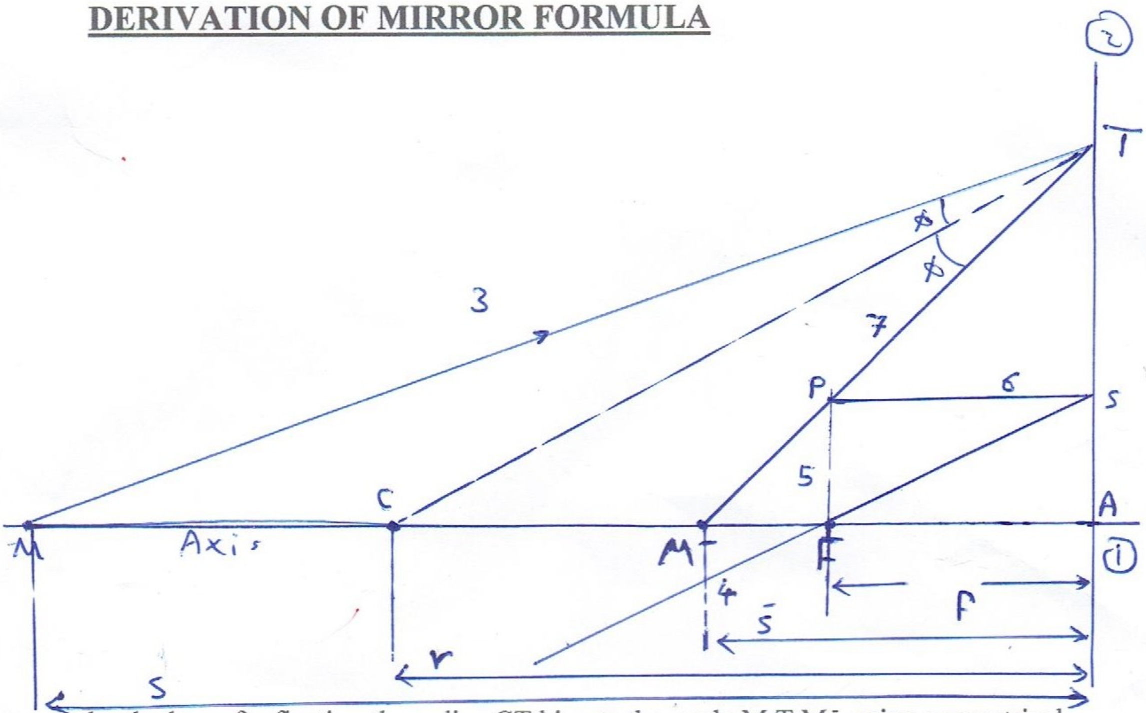
~~Positive~~

Positive

and for mirror there is one focal point.

object distance S and image distance are measured for the object and for image respectively to the vertex. this make both S and S ˉ ,positive and the object and image real when they lie to the left of the vertex, while they are negative and virtual when they lie to the right.

DERIVATION OF MIRROR FORMULA



by the law of reflection the radius CT bisects the angle M T M^{-} ,using geometrical theorem we may can write the proportion:

$$\frac{MC}{MT} = \frac{CM'}{M'T} \quad \text{--------(1)}$$

Now, for paraxial ray, $MT \simeq MA = S$, and $M'T \simeq M'A = S'$,

$$MC = MA - CA = s - (-r) = s + r \quad \text{-----------(2)}$$

$$CM' = CA - M'A = -r - s' = -(s' + r) \quad \text{--------(3)}$$

by substituting eq.(2),and eq.(3) in eq.(1):

$$\frac{s+r}{s} = -\frac{s'+r}{s'}$$

$$\frac{s}{s} + \frac{r}{s} = -\frac{s'}{s'} - \frac{r}{s'}$$

$$1 + \frac{r}{s} = -1 - \frac{r}{s'}$$

$$\frac{r}{s} + \frac{r}{s'} = -2 \Longrightarrow r\left(\frac{1}{s} + \frac{1}{s'}\right) = -2$$

$$\boxed{\frac{1}{s} + \frac{1}{s'} = \frac{-2}{r}} \quad \text{-------(4)} \quad \text{mirror formula}$$

The primary focal point is defined as that axial point object point for which the image is formed at infinity, so substituting $s = f$, $s' = \infty$, in eq.(4),we have:

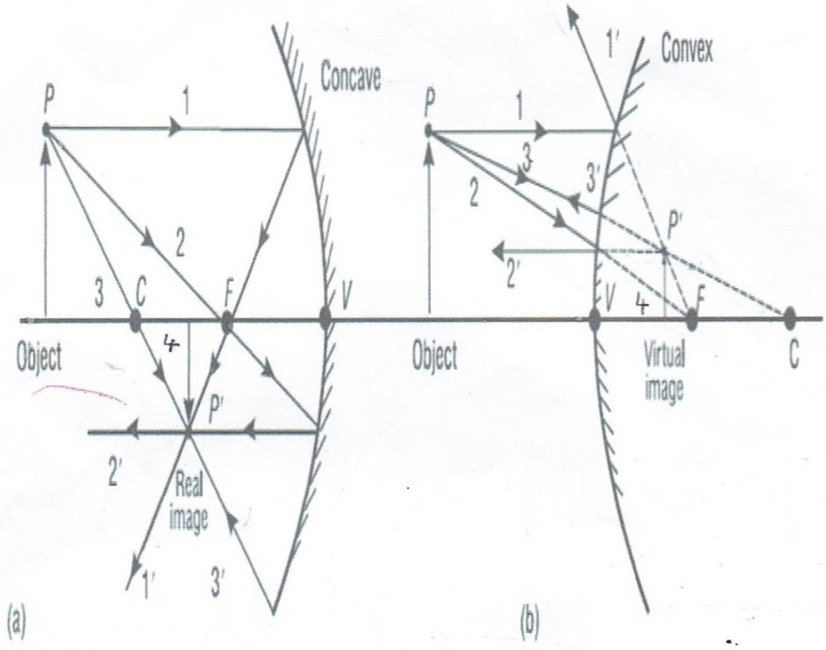
$$\frac{1}{f} + \frac{1}{\infty} = \frac{-2}{r}$$

from which $\dfrac{1}{f} = \dfrac{-2}{r}$ or $f = -\dfrac{r}{2}$ -------(5)

The secondary focal point is defined as the image point of an infinite distant object point. this is, $s' = f'$, $s = \infty$, so that:

$$\frac{1}{\infty} + \frac{1}{f'} = \frac{-2}{r}$$

$$\frac{1}{f'} = \frac{-2}{r} \Longrightarrow f' = -\frac{r}{2} \quad \text{------(6)}$$



concave mirror **convex mirror**

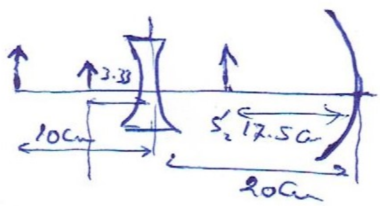
image formation by convex and concave mirror

An object is placed 10 cm to the left of a diverging lens ($f = -5$ cm), A Concave mirror ($R = 20$ cm) is placed 20 cm is placed 20 cm to the right of the lens. Calculate the location of the final image with respect to the lens.

$S = 10$ cm , $f = -5$ cm)
$R = 20$ cm

1- through the lens:-

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$



$$\frac{1}{10} + \frac{1}{s'} = \frac{1}{-5}$$

$$S_1' = -3.33 \text{ cm}$$

2. through the mirror:-

$$F_2 = -\frac{R}{2} = \frac{-20}{2} = 10 \text{ cm}$$

$$S_2 = 3.33 + 20 = 23.33 \text{ cm}$$

3- the lens:- $\frac{1}{S_2} + \frac{1}{S_2'} = \frac{1}{F_2}$

$$\frac{1}{23.33} + \frac{1}{S_2'} = \frac{1}{10}$$

$$S_2' = 17.5 \text{ cm}$$

4- through the lens:-

$$20 - 17.5 = 2.5 \text{ cm}$$

$$\frac{1}{2.5} + \frac{1}{S_3'} = \frac{1}{-5}$$

$$S_3' = -1.67 \text{ cm}$$