المحاضرة الثانية

*** The generalization of (Ampere's law), Displacement current.**

* تعميم قانون أمبير ، تيار الازاحة.

- The relation derived from Ampere's law:

$$\nabla \times H = J \;............\; (1)$$

Is a special form of the more general relation given by:

$$\nabla \times H = J + \frac{\partial D}{\partial t} \;................\; (2)$$

The more general equation has an additional term involving the displacement current density. The introduction of the second term $\frac{\partial D}{\partial t}$ represent of Maxwell major contribution to the electromagnetic theory.

العلاقة (1) مشتقة من قانون أمبير وهي حالة خاصة لعلاقة عامة كما في العلاقة (2) و التي تتضمن كثافة تيار الازاحة

$\frac{\partial D}{\partial t}$. ان اضافة الحد الثاني للمعادلة (2) تمثل احد اضافات ماكسويل الرئيسة للنظرية الكهرومغناطيسية.

Suppose we add an unknown term (G) to equation (1), we get:

$$\nabla \times H = J + G \;.........\; (3)$$

By taking the divergence for equation (3) we get:

$$\nabla \,.\, (\overset{0}{\cancel{\nabla} \times H}) = \nabla \,.\, (J \times G)$$

$$\therefore \nabla \,.\, J + \nabla \,.\, G = 0 \Longrightarrow \nabla \,.\, J = -\nabla \,.\, G \;............\; (4)$$

But from equation continuity, we have:

لكن لدينا من معادلة الاستمرارية

$$\nabla \,.\, J + \frac{\partial \rho}{\partial t} = 0 \Rightarrow \nabla \,.\, J = -\frac{\partial \rho}{\partial t} \;.......\; (5)$$

By equal equation (4, 5) we get:

$$\nabla \,.\, G = \frac{\partial \rho}{\partial t} \;.........\; (6)$$

But from Gauss's law we have:

لكن لدينا من قانون كَاوس

$$\nabla \,.\, D = \rho \;.............\; (7)$$

Sub. Equation (7) in (6) we get:

$$\nabla \cdot G = \frac{\partial}{\partial t}(\nabla \cdot D)$$

$$\cancel{\nabla} \cdot G = \cancel{\nabla} \cdot \left(\frac{\partial D}{\partial t}\right)$$

$$\therefore G = \left(\frac{\partial D}{\partial t}\right) \dots\dots\dots\dots\dots (8)$$

Sub. (8) in (3) we get:

$$\boxed{\therefore \nabla \times H = J + \frac{\partial D}{\partial t}} \dots\dots\dots\dots\dots (9)$$

However a displacement current is present only for time changing fields so that for steady fields.

تصح العلاقة (9) لظاهرة المجالات المستقرة والتي تتوافق مع معادلة الاستمرارية للحالات التي تعتمد على الزمن.

*** The four Maxwell's equations are differential equations it's convenient to convert the differential form into their integral equivalent which are:**

* الصيغ التكاملية لمعادلات ماكسويل:

1) $\int_c H \cdot dl = \int_s (J + \frac{\partial D}{\partial t}) \cdot ds$ [Ampere's circuital law]

2) $\int_c E \cdot dl = -\int_s \frac{\partial B}{\partial t} \cdot ds$ [Faraday's law]

3) $\int_c D \cdot ds = \int_v \rho \, dv$ [Gauss's law for electric field]

4) $\int_c B \cdot ds = 0$ [The magnetic flux through any close surface equal zero. Gauss's law for magnetic field]

* Potential of electrostatic field:

* مفهوم الجهد الكهروستاتيكي:

- We can define the vector $\vec{B}$ by vector potential $\vec{A}$ by:

من الملائم ان نعبر عن الحث المغناطيسي $\vec{B}$ بدلالة متجه الجهد $\vec{A}$

$\vec{B} = \vec{\nabla} \times \vec{A}$...............(1)

- And define the vector $\vec{E}$ by scalar potential **φ** by:

كما يمكن تعريف المتجه $\vec{E}$ من انحدار الجهد العددي **φ** و بأشارة سالبة

$E = -\nabla . \varphi$ .... (2)

- وهنا يمكننا حساب المجالات الكهربائية الى درجة كبيرة عندما ادخلنا مفهوم الجهد الكهروستاتيكي ولايجاد علاقة تحتوي على الجهد المغناطيسي $\vec{A}$ والجهد العددي φ .

- By adding unknown term to equation (2) becomes:

بأضافة حد غير معرف (N) الى المعادلة (2) نحصل على:

$E = -\nabla . \varphi + N$ .... (3)

By taking the curl of each side to equation (3):

باخذ التفاف طرفي المعادلة (3) نحصل على:

$$\nabla\times E = \underbrace{\cancel{\nabla\times(-\nabla\varphi)}}_{0} + \nabla\times N$$

$\nabla \times E = \nabla \times N$ ....(4)

But $\nabla \times E = -\frac{\partial B}{\partial t}$ ....(5) (معادلة ماكسويل الثانية)

By equal the equations (4) and (5) we get:

$\nabla \times N = -\frac{\partial B}{\partial t}$ ....(6)

Sub. Equation (1) in equation (6) we get:

$$\nabla \times N = -\frac{\partial}{\partial t}(\nabla\times A) \Rightarrow \cancel{\nabla} \times N = -\cancel{\nabla}\times\frac{\partial A}{\partial t} \Rightarrow N = -\frac{\partial A}{\partial t} \text{ ....(7)}$$

Sub. Equation (7) in (3) we get:

$$\boxed{E = -\nabla\varphi - \frac{\partial A}{\partial t}} \text{ ....(8)}$$

- If the time variation is harmonic (8) becomes:

$$E = -\nabla\varphi - j\,\omega\,A$$

$$A = A_0\, e^{j\omega t}$$

$$\frac{\partial A}{\partial t} = j\,\omega\, A_0\, e^{j\omega t} = j\,\omega\, A$$

- when the vector potential A and the scalar potential φ are unknown, the electric and magnetic field can obtained under static or time varying situation from the relation:

$$E = -\nabla\varphi - \frac{\partial A}{\partial t} \qquad (v\,m^{-1})$$

$$B = \nabla \times A \qquad (T)$$

$$\varphi = \frac{1}{4\pi\epsilon_0}\int_v \frac{\rho}{r}\,dv \qquad (v) \,........(10)$$

$$A = \frac{\mu}{4\pi}\int_v \frac{J}{r}\,dv \qquad (\omega\, b\, m^{-1})$$

- The electric field in equation (8) composed of two parts:

- $\nabla\varphi$ is due to charge distribution (ρ)

- $-\frac{\partial A}{\partial t}$ is due to time varying (J)

- The electric scalar potential for volume charge distribution given by equation (10) and the vector potential $\vec{A}$ given by equation (11).

*** The wave equation for $\vec{A}$ is derived by:**

اشتقاق معادلة الموجه بدلالة متجه الجهد $\vec{A}$

We have

$$\nabla \times H = J + \frac{\partial D}{\partial t} \,..........(1)$$

We have also

$$\vec{B} = \nabla \times \vec{A} \,.....................(2)$$

And $$E = -\nabla\varphi - \frac{\partial A}{\partial t} \,..........(3)$$

Where $\left[\begin{array}{l} B = \mu H \\ D = \epsilon E \end{array}\right]$ .... (4)

Sub. (4) in (1) we get:

$$\nabla \times \frac{B}{\mu} = J + \frac{\partial D}{\partial t} \Rightarrow \nabla \times B = \mu J + \mu \frac{\partial D}{\partial t} \text{ .... (5)}$$

Sub. Equation (2) (4) in equation (5) we get:

$$\nabla \times \nabla \times A = \mu J + \epsilon \mu \frac{\partial E}{\partial t} \text{ ....(6)}$$

Sub. Equation (3) in equation (6) we get:

$$\nabla \times \nabla \times A = \mu J + \epsilon \mu \frac{\partial E}{\partial t}(-\nabla \varphi - \frac{\partial A}{\partial t}) \text{........(7)}$$

By using the identity:

بأستخدام المتطابقة التالية

$$\nabla \times \nabla \times A = \nabla (\nabla . A) - \nabla^2 A \text{ .... (8)}$$

Sub. Equation (8) in equation (7) we get:

$$\nabla (\nabla . A) - \nabla^2 A = \mu J - \nabla (\epsilon \mu \frac{d\varphi}{dt}) - \epsilon \mu \frac{d^2 A}{dt^2})$$

و بترتيب هذه المعادلة نحصل

$$\nabla^2 A - \epsilon \mu \frac{d^2 A}{dt^2} = -\mu J + \nabla [(\cancel{\nabla . A}) + \epsilon \mu \frac{d\varphi}{dt})] \text{...........(9)}$$

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By using lorentz condition:

بأستخدام شرط لورنتز

$$\nabla . A + \epsilon \mu \frac{d\varphi}{dt} = 0$$

∴ Equation (9) becomes:

تصبح المعادلة (9) كالاتي

$$\boxed{\nabla^2 A - \epsilon \mu \frac{d^2 A}{dt^2} = -\mu J} \text{ ....(10)}$$

معادلة (10) هي معادلة الموجه بدلالة $(\vec{A})$ و تمثل معادلة غير متجانسة وذلك لوجود الحد الذي يحوي (J) والذي يمثل مصدر المجال.

eq^n (10) is the wave equation in term of $(\vec{A})$ and represent a hetrogeneous equation because there is a term containing (J) that represent the field source.