المحاضرة الرابعة

أمثلة (مسائل)

Ex. 1/ Starting from the equation $\nabla \times H = J + \frac{\partial D}{\partial t}$ to obtain continuity's equation $\nabla \cdot J = -\frac{\partial \rho}{\partial t}$

Solution:

$\nabla \times H = J + \frac{\partial D}{\partial t}$ ……………(1)

By taking the divergence of each side to equation (1), we get:

$$\overset{0}{\nabla \cdot \nabla \times H} = \nabla \cdot J + \nabla \cdot \frac{\partial D}{\partial t}$$

$\nabla \cdot J + \nabla \cdot \frac{\partial D}{\partial t} = 0 \Rightarrow \nabla \cdot J = -\frac{\partial}{\partial t} \nabla \cdot D$ ……….(2)

But from (Gauss's law): $\nabla \cdot D = \rho$ ……………..(3)

Sub. Equation (3) in equation (2) we get:

$$\boxed{\nabla \cdot J = -\frac{\partial \rho}{\partial t}}$$

Continuity's equation **معادلة الاستمرارية**

Ex. 2/ A potential distribution is given by **$U = 7y^2 + 12x$** (v), what is the expression for E? What is its vector value (magnitude and direction) at points: (0 , 0) , (5 , 0) , (3 , 0) and (5 , 3),

Solution:

$E = -\nabla U \Rightarrow E = -\left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k\right)(7y^2 + 12x)$

$$E = \left[\frac{\partial}{\partial x}(7y^2 + 12x)\, i + \frac{\partial}{\partial y}(7y^2 + 12x)\, j + \frac{\partial}{\partial z}(7y^2 + 12x)\, k\right]$$

$E = -12\, i - 14y\, j$

1) At point (0 , 0) $\Longrightarrow E = -12\, i - 14y\, j = -12\, i$

2) At point (0 , 3) $\Longrightarrow E = -12\, i - 14y\, j = -12\, i - 42\, j$

3) At point (5 , 0) $\Rightarrow$ E = − 12 i − 14y j = − 12 i

4) At point (5 , 3) $\Rightarrow$ E = − 12 i − 14y j = − 12 i − 42 j

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Ex. 3/ A parallel plate capacitor with plates have the shape of circular disk, radius (r) separated by distance (d). Filled by material a dielectric constant (k) and the coefficient of the electric conductivity (σ). Connect it by varying voltage $v = v_0\ \omega\ t$, let the electric field is homogenous. What is the magnetic field (H) in the capacitor?

Solution:

We have $E = \frac{v}{d}$ …………(1)

$v = v_0 \sin \omega t$ ……………..(2)

Sub. Equation (2) in equation (1) we get:

$E = \frac{v_0 \sin \omega t}{d}$ …………….(3)

And we have: $D = \epsilon E \Rightarrow D = k \epsilon E$, $J = \sigma E$ .... (4)

بأستخدام قانون امبير الدائري

$\int_c H . d\ell = \int_s (J + \frac{\partial D}{\partial t}) . ds$ ……………(5)

Sub. Equations (3), (4) in equation (5) we get:

$$\int_c H . d\ell = \int_s [(\sigma E + \frac{\partial}{\partial t}(k \epsilon E))] . ds$$

و الان نعوض قيمة (E) من (3):

$$\int_c H . d\ell = \int_S \sigma \frac{v_0 \sin \omega t}{d} . ds + \int_s \frac{d}{dt} k \epsilon \frac{v_0 \sin \omega t}{d} . ds$$

$$H . 2 \pi r = \sigma \frac{v_0 \sin \omega t}{d} . \pi r^2 + \frac{d}{dt} k \epsilon \frac{v_0 \sin \omega t}{d} . \pi r^2$$

$\int d\ell = \ell = 2 \pi r$ محيط الدائرة
$\int ds = s = \pi r^2$ مساحة الدائرة

$$H \cdot 2 \not{\pi} \not{r} = \frac{v_0 \not{\pi} r^{\not{2}}}{d} [\sigma \sin \omega t + k \epsilon \cos \omega t] \qquad \frac{d}{dt} \sin \omega t = \omega \cos \omega t$$

$$\therefore H = \frac{v_0 r}{2d} [\sigma \sin \omega t + k \epsilon \cos \omega t]$$

Ex. 4/ (a) state Maxwell's equations in their general differential form. (b) Derive them for harmonically varying field.

Solution:

(a)

1) $\nabla \times H = J + \frac{\partial D}{\partial t}$ (Ampere's law)

2) $\nabla \times E = -\frac{\partial B}{\partial t}$ (Faraday's law)

3) $\nabla \cdot D = \rho$ (Gauss's law in electric)

4) $\nabla \cdot B = 0$ (Gauss's law in magnetic)

(b)

1) $\nabla \times H = (\sigma + j \omega \epsilon) E$

2) $\nabla \times E = - j \omega \mu H$

3) $\nabla \cdot D = \rho$

4) $\nabla \cdot B = 0$

توضيح / من اين اتت المعادلة (1) في الفرع (b)

We have $D = D_0 e^{j\omega t} \Rightarrow \frac{\partial D}{\partial t} = j \omega D_0 e^{j\omega t}$

$= j \omega D$ but $D = \epsilon E$

$\frac{\partial D}{\partial t} = j \omega \epsilon E$ $J = \sigma E$

بتعويض المعطيات السابقة في المعادلة $\nabla \times H = J + \frac{\partial D}{\partial t}$

$\nabla \times H = \sigma E + j \omega \epsilon E$

$\nabla \times H = (\sigma + j\omega\epsilon)\,E$ المعادلة (1) في الفرع (b)

توضيح / من اين اتت المعادلة (2) في الفرع (b)

We have $B = B_0\, e^{j\omega t} \Rightarrow \frac{\partial B}{\partial t} = j\omega\, B_0\, e^{j\omega t}$

$= j\omega\, B$ but $B = \mu\, H$

$\therefore -\frac{\partial B}{\partial t} = -j\omega\mu\, H$

$\nabla \times E = -\frac{\partial B}{\partial t}$ بتعويض هذه المعطيات في المعادلة (2) من الفرع (a)

$\therefore \nabla \times E = -j\omega\mu\, H$ المعادلة (2) في الفرع (b)

Ex. 5/ Give the step by step development of Maxwell's equations from Ampere's law **(a)** integral form and **(b)** using stoke's theorem in differential or point form, **(c)** modify both integral and differential form for harmonically varying field.

Solution:

(a) According to Ampere's law: the line integral around a closed contour is equal to the current enclosed, where both conduction and displacement current are present, this current is the total current.

$\oint_c H \cdot d\ell = \int_s (J_{cond.} + J_{disp.}) \cdot ds$ ....(1)

- The conduction of current through the surface (s) is given by:

$J_{cond.} = \int_s \sigma\, E \cdot ds$ ....(2)

تيار التوصيل خلال سطح مغلق (s) يعطى بالعلاقة (2)

- While the displacement current through the surface (s) is given by:

$J_{disp.} = \int_s \epsilon \frac{\partial E}{\partial t} \cdot ds = \int_s \frac{\partial D}{\partial t} \cdot ds$ ....(3) $D = \epsilon\, E$

تيار التوصيل خلال سطح مغلق (s) يعطى بالعلاقة (3)

Sub. Equations (2), (3) in equation (1) we get:

$$\oint_c H \cdot d\ell = \int_s (\sigma E + \epsilon \frac{\partial E}{\partial t}) \cdot ds \quad \ldots\ldots\ldots\ldots(4)$$

Equation (4) is the complete integral form of Maxwell's equation derived from Ampere's law it is also often written as:

$$\oint_c H \cdot d\ell = \int_s (J + \frac{\partial D}{\partial t}) \cdot ds \ \ldots\ldots\ldots\ldots(5)$$

الصيغة التكاملية لمعادلة ماكسويل المشتقة من قانون أمبير

(b) By an application of stoke's theorem to equation (3) we get:

$$\oint_c H \cdot d\ell = \int_s (\nabla \times H) \cdot ds \ \ldots\ldots\ldots(6)$$

نظرية ستوكس المطبقة على المعادلة (5)

By equal (5) , (6) we get:

بمساواة المعادلة (5) ، (6) نحصل على

$$\int_s (\nabla \times H) \cdot ds = \int_s (J + \frac{\partial D}{\partial t}) \cdot ds$$

$$\therefore \nabla \times H = J + \frac{\partial D}{\partial t} \ \ldots\ldots\ldots\ldots\ldots(7)$$

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(c) For harmonic variation, the phasor forms of Maxwell's integral and differential equation,

If D varies with time as given:

$$D = D_0\, e^{j\omega t} \Rightarrow \frac{\partial D}{\partial t} = j\,\omega\, D_0\, e^{j\omega t}$$

$$= j\,\omega\, D = j\,\omega\,\epsilon\, E \ \ldots\ldots\ (8)$$

And

$$J = \sigma\, E \ \ldots\ldots\ldots\ldots\ldots(9)$$

$$D = \epsilon\, E$$

Sub. Equations (8), (9) in equation (5) we get:

$$\oint_c H \cdot d\ell = (\sigma + j\,\omega\,\epsilon) \int_s E \cdot ds \ \ldots\ldots\ldots(10)$$

الصيغة التكاملية لمعادلة ماكسويل للمجالات المتغيرة مع الزمن

Sub. Equations (8), (9) in equation (7) we get:

Ex. 14/ A potential distribution is given by **$V = 3\,y^{\frac{1}{2}}$** (v), what is the expression for **E**, What is its vector value (magnitude and direction) at points: (0 , 0) , (4 , 0) and (0 , 4).

Solution:

$E = -\nabla U$

$E = -\left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k\right) 3\,y^{\frac{1}{2}}$

$= -3 \times \frac{1}{2} y^{-\frac{1}{2}}\, j$

$= -1.5\, y^{-\frac{1}{2}}\, j = -\frac{1.5}{y^{\frac{1}{2}}}\, j$

1) At point (0 , 0) $\Rightarrow E = \frac{-1.5}{0}\, j = \infty \quad = \frac{\text{كمية}}{\text{صفر}}$

$E = \infty\, j$

2) At point (4 , 0) $\Rightarrow E = \frac{-1.5}{0}\, j = \infty\, j$

3) At point (0 , 4) $\Rightarrow E = \frac{-1.5}{4^{\frac{1}{2}}}\, j = \frac{-1.5}{2}\, j = -0.75\, j$ (v/m).

ملاحظة: لحل كمية اسها سالب في البسط يجب انزالها الى المقام لكي يمكن حسابها وكذلك الكمية التي في المقام واسها سالب يجب رفعها الى البسط لتصبح موجبة و يمكن حسابها مثلا

$-1.5\, y^{-\frac{1}{2}} = -\frac{1.5}{y^{\frac{1}{2}}}$