المحاضرة السادسة

Chapter (3)

Plane waves in dielectric and conducting media.

الموجات المستوية في الاوساط الموصلة و العازلة.

3.1 / phase velocity سرعة الطور

The phase velocity , or velocity of constant - phase point is given by ω/β. That ω/β has the dimension of velocity is given by

$\frac{\omega}{\beta} = \frac{3\pi f}{2\pi/\lambda} = \lambda f$ ....(1)

Where $\omega = 2\pi f$

$\beta = 2\pi/\lambda$ ⇒ phase constant الطور ثابت

$\lambda =$ wave length

f=frequency

and we have the phase velocity

$\frac{\omega}{\beta} = \nu = \frac{1}{\sqrt{\mu\varepsilon}}$ ....(2)

equation (2) gives the phase velocity of a wave in an unbounded medium of permeability (μ) and permittivity (ε) .

For free space (vacuum) the velocity is a well-known constant by (c) and usually called the velocity of light . Thus $c = \frac{1}{\sqrt{\mu_o \ \varepsilon_o}} = \mathrm{Mms^{-1}}$

The SI unit for the permeability of vacuum is $\mu_o = 400\ \pi\ \mathrm{nHm^{-1}}$

therefore the permittivity of vacuum is

$\varepsilon_o = \frac{1}{\mu_o \ c^2} = 8.85\ \mathrm{pFm^{-1}}$

- For other media the phase velocity relative to the velocity of light , or relative phase velocity is

$P = \frac{\nu}{c} = \frac{\sqrt{\mu_o \ \varepsilon_o}}{\sqrt{\mu\ \varepsilon}} = \frac{1}{\sqrt{\mu_r \ \varepsilon_r}}$ ....(3)

Where μ_r =relative permeability of medium.

$\varepsilon_r =$ relative permeability of medium.

3.2 / Index of Refraction. معامل الانكسار

- In optics the index of refraction η is defined as the reciprocal of the relative phase velocity P

.

That is

$\eta = \frac{1}{P} = \frac{1}{v/c} = \frac{c}{v} = \sqrt{\mu_r \ \varepsilon_r}$ ....(4)

For nonferrous media (للأوساط غير الحديدية) μ_r is very nearly unity so that

$\eta = \sqrt{\varepsilon_r}$ ....(5)

Example 1/ Paraffin has a relative permittivity $\varepsilon_r = 2.1$, Find the index of refraction for paraffin and also the phase velocity of a wave in an unbounded medium of paraffin .

solution/ The index of refraction

$$\eta = \sqrt{\varepsilon_r}$$

$$\eta = \sqrt{2.1} = 1.45$$

The phase velocity $\eta = \frac{c}{v} \Rightarrow v = \frac{c}{\eta} = \frac{3 \times 10^8}{1.45} = 207 \text{ Mms}^{-1}$

Example 2/ Distilled water has the constant ≈ 0 , $\varepsilon_r = 81$, $\mu_r = 1$, Find η and v .

solution/ $\eta = \sqrt{\varepsilon_r} \Rightarrow \eta = \sqrt{81} = 9$

$$v = \frac{c}{\eta} = \frac{3 \times 10^8}{9} = 0.111\, c = 33.3 \text{ Mms}^{-1}$$

3.3 / Group velocity. سرعة المجموعة

The group velocity give by the relation

$u = v + \beta \frac{dv}{d\beta}$ ....(6)

or

$u = v - \lambda \frac{dv}{d\lambda}$ ....(6)

Where (u) is the phase velocity of the wave envelope which is usually called the group velocity.

حيث ان (u) سرعة الطور لمغلف الموجه و الذي يدعى بسرعة المجموعة.

- In nondispersive media - للأوساط غير المشتتة : the group velocity is the same as the phase velocity , in free space $u = v = c$

- In dispersive media the phase and group velocities differ.

- A dispersive media is one in which the phase velocity is a function of the frequency (and hence of the free - space wave length).

<u>-Dispersive media are of two type.</u>

1- Normally dispersive - التشتت العادي : In these media the change in phase velocity with wave length positive that is ($dv/d\lambda > 0$) , for these media ($u < v$).

2- Anomalously dispersive - التشتت الشاذ : In these media the change in phase velocity with wave length is negative that is ($dv/d\lambda < 0$) , for these media ($u > v$).

Example / A 1-MHz (300 m wave length) plane wave traveling in a normally dispersive , lossless media has a phase velocity at this frequency of 300 $\mathrm{Mm}s^{-1}$, the phase velocity as a function of wave length is give by $\nu = K\sqrt{\lambda}$, Where K is constant . Find the group velocity.

solution/ The group velocity is $u = \nu - \lambda \frac{d\nu}{d\lambda}$ ....(1)

and we have $\nu = K\sqrt{\lambda} \Longrightarrow \nu = K\lambda^{\frac{1}{2}} \Longrightarrow \frac{d\nu}{d\lambda} = \frac{1}{2}K\lambda^{-\frac{1}{2}}$ ....(2)

Substituted equation (2) in (1)

$$u = \nu - \frac{K}{2}\lambda\lambda^{-\frac{1}{2}}$$

$$u = \nu - \frac{K}{2}\lambda^{\frac{1}{2}}$$

$$\because \ \nu = K\sqrt{\lambda}$$

$$u = \nu - \frac{\nu}{2}$$

$$u = \nu\left(1 - \frac{1}{2}\right) \Longrightarrow u = \frac{\nu}{2} = \frac{300}{2} = 150\ \mathrm{Mm}s^{-1} \ .$$

3.4 / Impedance of dielectric media . الممانعة في الاوساط العازلة

- Intrinsic impedance : refer to the impedance of a plane TEM (transverse electromagnetic wave) travelling through a homogeneous media , the impedance of the wave everywhere in space is equal to the intrinsic impedance , it can also defined as the ratio of the electric field to the magnetic flux density and give by

$Z = \frac{E}{H}$ ....(1)

Where Z is called the intrinsic impedance of the med

ممانعة الموجة الكهرومغناطيسية هي عبارة عن النسبة بين عنصر المجال الكهربائي E و عنصر المجال المغناطيسيH .

اذا كانت الموجة مستوية او تسير في وسط متجانس فأن ممانعة الموجة تساوي الممانعة الذاتية و اذا كان الوسط المحيط هو الفراغ فأن ممانعة الموجة تساوي ممانعة الفراغ.

- For free space (vacuum)

$Z = Z_o = \sqrt{\frac{\mu_o}{\varepsilon_o}} = 376.731 \ \approx 120\pi\ \Omega$ ....(2)

or the intrinsic impedance is defined as the square root of the ratio permeability of the medium and permittivity of the medium.

Example / If the magnitude of H in a plane wave is 1 $m^{-\square}$, Find the magnitude of E for a plane wave in free space.

solution/ $Z = \frac{E}{H} \Longrightarrow E = Z\,H$

$$= 376.7 \times 1 = 376.7\ \mathrm{Vm}^{-1} \ .$$

3.5 / Two plane waves traveling in opposite direction ; standing wave . الموجة الثابتة

- Standing wave : also called stationary wave combination of two waves moving in opposite direction , each having the same amplitude and frequency.

الموجة الواقفة تدعى بالموجة الثابتة تحصل عند تداخل موجتين متحركتين
لهما نفس التردد و السعة متضادتان في الاتجاه.

VSWR definition : The voltage standing wave ratio and define the wave with maximum and minimum electric field intensities . Thus

$\mathrm{VSWR} = \frac{E_{max}}{E_{min}} = \frac{E_o+E_1}{E_o-E_1}$ ....(1)

E_o = amplitude of incident wave .

E_1 = amplitude of reflected wave .

- When the reflected wave is zero ($E_1 = 0$) the VSWR is unity .

- when the reflected wave is equal to the indicate wave ($E_1 = E_o$) the VSWR is infinite .

- Hence for all intermediate value of reflected wave , the VSWR lies between 1 and ∞ .

و بالتالي لكل القيم المتوسطة للموجات المنعكسة تقع VSWR بين 1 و ∞ .

Example 1/ Find the standing wave ratio of the wave with maximum and minimum electric field intensities of 12 and 4 .

solution/ $\mathrm{SWR} = \frac{E_{max}}{E_{min}} = \frac{12}{4} = 3$

Example 2/ Find the standing wave ratio of a wave travelling through the media having intrinsic impedance of 3 and 2 .

solution/ $\mathrm{SWR} = \frac{Z_1}{Z_2} = \frac{3}{2}$

Hint/ The standing wave ratio is given by the ratio of the intrinsic impedance of medium 1 to the intrinsic impedance of medium 2 .

3.6 / Depth of penetration (skin effect) : is a measure of how deep light or any electromagnetic radiation can penetrate into a material . it is defined as the depth at which the intensity of the radiation inside the material falls to 1/e (about 37%) of its original value at (or more properly, just beneath) the surface .

or penetration depth is defined as the distance from the dielectric surface at which the μ_o permeability of free space $= 4\pi \times 10^{-7}$ H/m .

- The depth of penetration decreases with increasing frequency and give by

$\delta = \frac{1}{\sqrt{f\,\pi\,\mu\,\sigma}}$ ....(1)

- The phase constant (β) of a wave give by the relation

$\beta = \frac{1}{\delta}$ ....(2)

- The skin depth of material with attenuation constant give by

$\delta = \frac{1}{a}$ ....(3)

a = attenuation constant .

- To find the wave length λ_c in the conductor give by the relation

$\lambda_c = 2\,\pi\,\delta$ ....(4)

Example 1/ Find the skin depth of the wave having of 3 MHz and a velocity 12 m/s .

solution/ $V = f\,\delta \Longrightarrow \delta = V/f = \frac{12}{3\times 10^6} = 4\ \mu m$.

Example 2/ Find the effective skin resistance of a material with conductivity 120 and skin depth of 2 μm.

solution/ The effective skin resistance given by

$R_\delta = \frac{1}{\delta\,\sigma} = \frac{1}{120\times 2\times 10^{-6}} = 4.16\ K\Omega$.

Example 3/ The attenuation constant is 0.5 unit , Find the skin depth .

solution/ $\delta = \frac{1}{a} = \frac{1}{0.5} = 2$ units .

Example 4/ Calculate the phase constant of a wave with skin depth of 2.5 unit .

solution/ $\delta = \frac{1}{\beta} \Longrightarrow \beta = \frac{1}{\delta} = \frac{1}{2.5} = \frac{2}{5}$ unit .

3.7 / Relaxation time . زمن الاسترخاء

- In the physical sciences.

الاسترخاء : العودة بالنظام المضطرب الى حالة التوازن.

- Relaxation usually means the return of perturbed system into equilibrium .

- The simplest theoretical description of relaxation as a function of time (t) is an exponential law exp(-*t*/τ) .

في دائرة تحتوي على متسعة و مقاومة يكون الجهد بالشكل التالي

$V(t) = V_0\, e^{-t/RC}$

$V(t) = V_0\, e^{-t/\tau} \Longrightarrow \tau = RC$

τ زمن الاسترخاء للدائرة الكهربائية او ثابت المتسعة
زمن الاسترخاء : هو الزمن اللازم لهبوط قيمة السعة الى 1/e من قيمتها الاصلية.
حيث e يمثل اساس اللوغاريتم الطبيعي و يساوي (2.718) اي ان الزمن اللازم لهبوط قيمة السعة من (c) الى 0.368 تمثل زمن الاسترخاء.

Example 1/ For a conductor such as copper , for which $\sigma = 58\ \mu\upsilon m^{-1}$ and $\epsilon = 8.85\ PFm^{-1}$. Find the time τ_r.

solution/ $\tau_r = \frac{\varepsilon}{\sigma} = \frac{8.85\times10^{-12}}{58\times10^{6}} = 1.5\times10^{-19}\ s$.

Example 2/ Find the time constant of a capacitor with capacitance of $2\mu f$ having an internal resistance of $4\mu\Omega$.

solution/ $\tau = RC$.

$$\tau = 4\times10^{6}\times2\times10^{-6} = 8\ \text{second}$$

* To derive the Relaxation Time
The continuity relation between current density and charge density is

$\nabla\cdot J = -\frac{\partial\rho}{\partial t}$ ....(1)

From Maxwell's equation $\nabla\cdot E = \frac{\rho}{\varepsilon}$ ....(2)

But $J = \sigma E$, so that(2) becomes

$\nabla\cdot J = \frac{\rho\,\sigma}{\varepsilon}$ ....(3)

From (1) and (3) it follows that

$\frac{\partial\rho}{\partial t} + \frac{\sigma}{\varepsilon}\rho = 0$ ....(4)

A solution of this equation is

$\rho = \rho_{0\ e^{-(\sigma/\varepsilon)t}}$ ....(5)

let $\tau_r = \frac{\varepsilon}{\sigma}$ ....(6)

τ_r has the dimension time , At - t = 0 , $\rho = \rho_0$, which is the initial charge density.

when = τ_r , the equation (5) becomes

$\rho = \rho_0\, e^{-(t/\tau_r)}$

$\rho = \rho_0\, e^{-1} = \rho\frac{1}{e}$ ....(7)

thus τ_r is the time required for the charge density to decrease to $\frac{1}{e}$ of the initial value , the quantity (τ_r) is called the ((Relaxation time)).

3.8 / Energy relation in a traveling wave .

The energy density at a point in an electric field is

$W_e = \frac{1}{2}\,\varepsilon\,E^2$ ....(1)

The energy density at a point in magnetic field is

$W_m = \frac{1}{2}\,\mu\,H^2$ ....(2)

In a traveling wave in an unbounded , lossless medium

$\frac{E}{H} = \sqrt{\frac{\mu}{\varepsilon}} \Longrightarrow \frac{E^2}{H^2} = \frac{\mu}{\varepsilon} \Longrightarrow H^2 = \frac{E^2\varepsilon}{\mu}$ ....(3)

Substituting for (H) from (3) in (2) we get

$W_m = \frac{1}{2}\,\mu\,H^2 = \frac{1}{2}\,\varepsilon\,E^2 = W_e$ ....(4)

Thus , the electric and magnetic energy densities in a plan traveling wave are equal , and the total energy (W) is the sum of the electric and magnetic energies , thus

$W = W_e + W_m$

$$W = \frac{1}{2}\,\varepsilon\,E^2 + \frac{1}{2}\,\mu\,H^2$$

$W = \frac{1}{2}\,\varepsilon\,E^2 + \frac{1}{2}\,\varepsilon\,E^2$ (From 4)

$\therefore W = \varepsilon\,E^2 + \mu\,H^2$ $(\mathrm{J\,m^{-3}})$