Chapter four

Application of derivative

1. **The slop of curve**

Secant to the curve is a line through two points on a curve.

Slope and tangent lines

1. We start with what we can calculate, namely the slope of the secant through P and a point Q nearby on the curve.
2. We find the limiting value of the secant slope (if it exists) as Q approaches P along the curve.
3. We take this number to be the slope of the curve at P and define the tangent to the curve at P to be the line through P with this slope.

$$\text{The slop } \; m = f'(x) = \frac{dy}{dx}$$

EXAMPLE 1: Write an equation for the tangent line at $x = -1$ of the curve $f(x) = y = 4 - x^2$

Sol:

$$\frac{dy}{dx} = -2x$$

The slope at $x = -1$

$$y'\big|_{x=-1} = (-2x)_{-1}$$

$$= -2 \cdot (-1) = 2$$

$$m = 2$$

$$y = 4 - (-1)^2 = 3$$

The line through $(-1,3)$ with slope $m = 2$

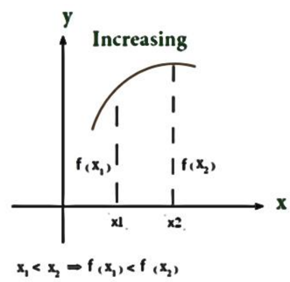
$$y - 3 = (2)(x - (-1))$$

$$y = 2x + 2 + 3$$
$$y = -x + 5$$

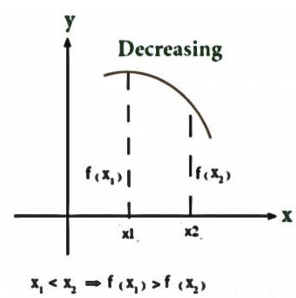
Increasing and decreasing function

Let f be a function defined on an interval I and let x_1 and x_2 be any two points in I.

1. If $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ then f is said to be increasing on I.



2. If $f(x_2) < f(x_1)$ whenever $x_1 < x_2$ then f is said to be decreasing on I.



First Derivative Test

1. If $f'(x) > 0$ at each point $x \in (a, b)$, then f is increasing on $[a, b]$.
2. If $f'(x) < 0$ at each point $x \in (a, b)$, then f is decreasing on $[a, b]$.

Definition Concave Up, Concave Down

The graph of a differentiable function $y = f(x)$ is

(a) Concave up on an open interval I if f' is increasing on I

(b) Concave down on an open interval I if f' is decreasing on I.

Second Derivative Test

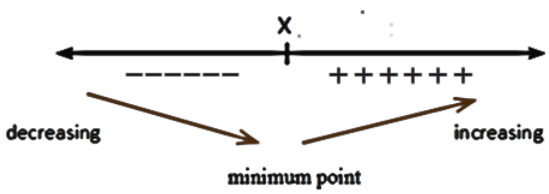
1- If $f''(x) > 0$ on I, the graph of f over I Concave up
2- If $f''(x) < 0$ on I, the graph of f over I Concave down

To find critical point (local maximum point and local minimum), concavity (Concave up and Concave down) and point of inflection point.

لايجاد النقاط الحرجة النهاية العظمى والصغرى والتحدب بنوعيه ونقطة الانقلاب.

1. First derivative test for local extrema f' or y'
2. $y' = 0$ the First derivative is zero at $x = ?$, find the value of x

either



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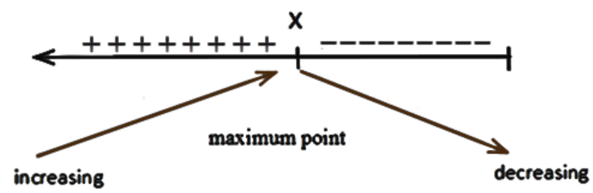
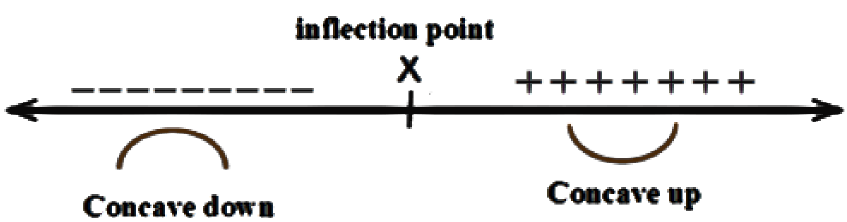


3. Second derivative test for concavity
4. Also $y'' = 0$ the second derivatives is zero that mean find value of x



EXAMPLE 1: Find all critical points, local minimum and maximum, concavity and inflection point. $y = 2x^3 - 3x^2 - 12x + 3$

Sol:

Test 1

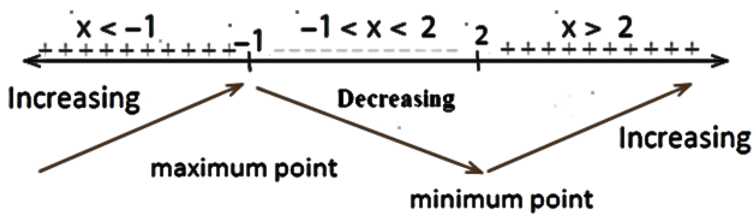
$$y' = 0 \quad \Rightarrow \quad y' = 6x^2 - 6x - 12 = 0$$

$\div 6 \;\Rightarrow\; x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x - 2 = 0 \;\Rightarrow x = 2$

$x + 1 = 0 \;\Rightarrow x = -1$



لكي نجد مناطف التزايد والتناقص نختبر المشتقة الاولى في خط الاعداد باخذ عدد اكبر من 2 ونلاحظ اشارة المشتقة وكذلك عدد في الفترة $(-1,2)$ وكذلك عدد عدد اقل من 1- كما في الرسم اعلاه

1. $\{x : x \in R \;\; , x > 2\}$ $\{x : x \in R \;\; , x < -1\}$ الدالة تكون متزايدة

2. $(-1,2)$ ومتناقصة في الفترة

Sub -1 in $y = 2x^3 - 3x^2 - 12x + 3$

$y = 2(-1)^3 - 3(-1)^2 - 12(-1) + 3 = 10$

$\mathrm{P} = (-1,10)$ **Maximum point** **نفطة عظمى**

Sub 2 in $y = 2x^3 - 3x^2 - 12x + 3$

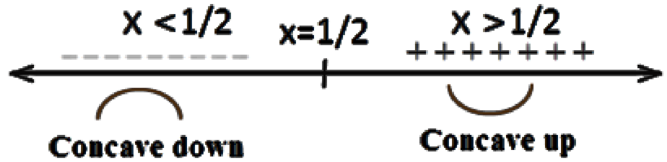
$y = 2(2)^3 - 3(2)^2 - 12(2) + 3 = -17$

$\mathrm{P} = (2,-17)$ **Minimum point** **نقطة صغرى**

Test 2

$y'' = 12x - 6$ $\qquad y'' = 0$

$12x - 6 = 0 \;\Rightarrow x = 1/2$



لكي نجد مناطف التفعر والتحدب نختبر المشتقة الثانية في خط الاعداد باخذ عدد اكبر من 1/ 2 ونلاحط اشارة المشتقة وكذلك عدد اقل من 1/2 ونلاحض اشارة المشتقة كما في الرسم اعلاه

1. $\{x : x \in R \ \ , x > 1/2\}$ **Concave up** منطقة التفعر
2. $\{x : x \in R \ \ , x < 1/2\}$ **Concave down** منطقة النحدب

Sub 1/2 in $y = 2x^3 - 3x^2 - 12x + 3$

$y = 2(1/2)^3 - 3(1/2)^2 - 12(1/2) + 3 = -3.5$ at $x = 1/2$ $y = -3.5$

$P = (1/2, -3.5)$ **Inflection point** **نقطة الانقلاب**

EXAMPLE 2: Find all critical points, local minimum and maximum, concavity and inflection point. $y = (x-4)^2$

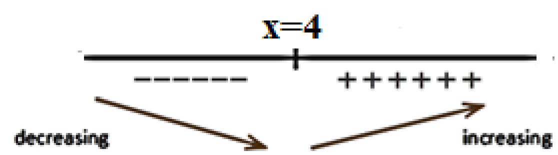
Sol:

$y' = 2(x-4)(1)$

$y' = 2x - 8 \quad \Rightarrow y' = 0$

$2x - 8 = 0 \quad \Rightarrow 2x = 8$

$x = 4$



sub 3 in $y' \Rightarrow \quad y' < 0$ decreasing
sub 5 in $y' \Rightarrow \quad y' > 0$ increasing

$x = 4$ Sub in $y = (x-4)^2$

$y = (4-4)^2$

$y = 0$

$= (4 \ , \ 0)$ **Minimum point**

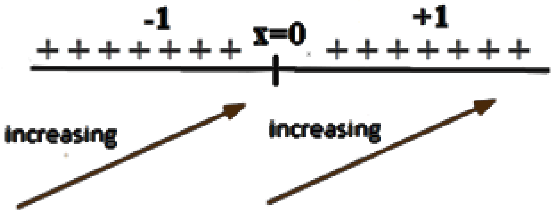
$y'' = 2$

$y'' > 0$ Increasing **Concave up**

Inflection point dose not exit

EXAMPLE 3: Sketch and Find all critical points, local minimum and maximum, concavity, concave up, concave down and inflection point. $y = x^3$

Sol: **Test 1**



$y' = 3x^2 \quad \Rightarrow \quad y' = 0$

$3x^2 = 0 \quad \Rightarrow x = 0$

sub $\begin{matrix}1\\-1\end{matrix} \Rightarrow y' > 0$ increasing

Test 2

$y'' = 6x \quad y'' = 0$

$6x = 0 \quad \Rightarrow \quad x = 0$

also $\begin{matrix}1\\-1\end{matrix}$ in $y'' \quad \Rightarrow$ at $x = 1 \quad \Rightarrow y'' > 0$ **Concave up**

at $x = -1 \quad \Rightarrow y'' < 0$ **Concave down**

$x = 0$ sub in $y = x^3$

$y = 0$ p $(0,0)$ Inflection point

x	$y = x^3$
0	0
-1	-1
1	1
-2	-8
2	8