The Earth's Gravitation Field

Lecture 5

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The Earth's Gravitation Field Introduction

- *The* concept of *gravity* is relatively simple, high -precision measurements of the gravity field that are inexpensive and quick, and spatial variations in the gravitational acceleration give important information about the dynamical state of Earth.
- In general the gravity signal has a complex origin : the acceleration due to gravity , denoted g is influenced by ***topography***, a ***spherical variation of density within the Earth***, and ***the Earth's rot*****ation**.

The Earth's Gravitation Field Introduction

- *For* our purposes, gravity can be defined as the force exerted on a mass *m* due to the combination of :

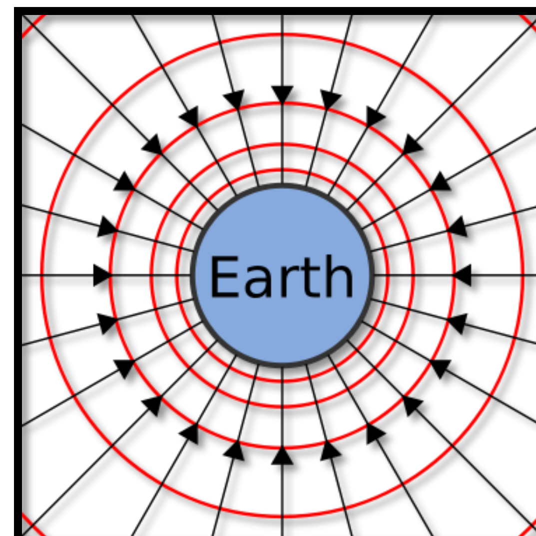
1. **The gravitational attraction of the Earth with mass *M* .**
2. **The rotation of the Earth.**

- The rotation of the Earth has two components:

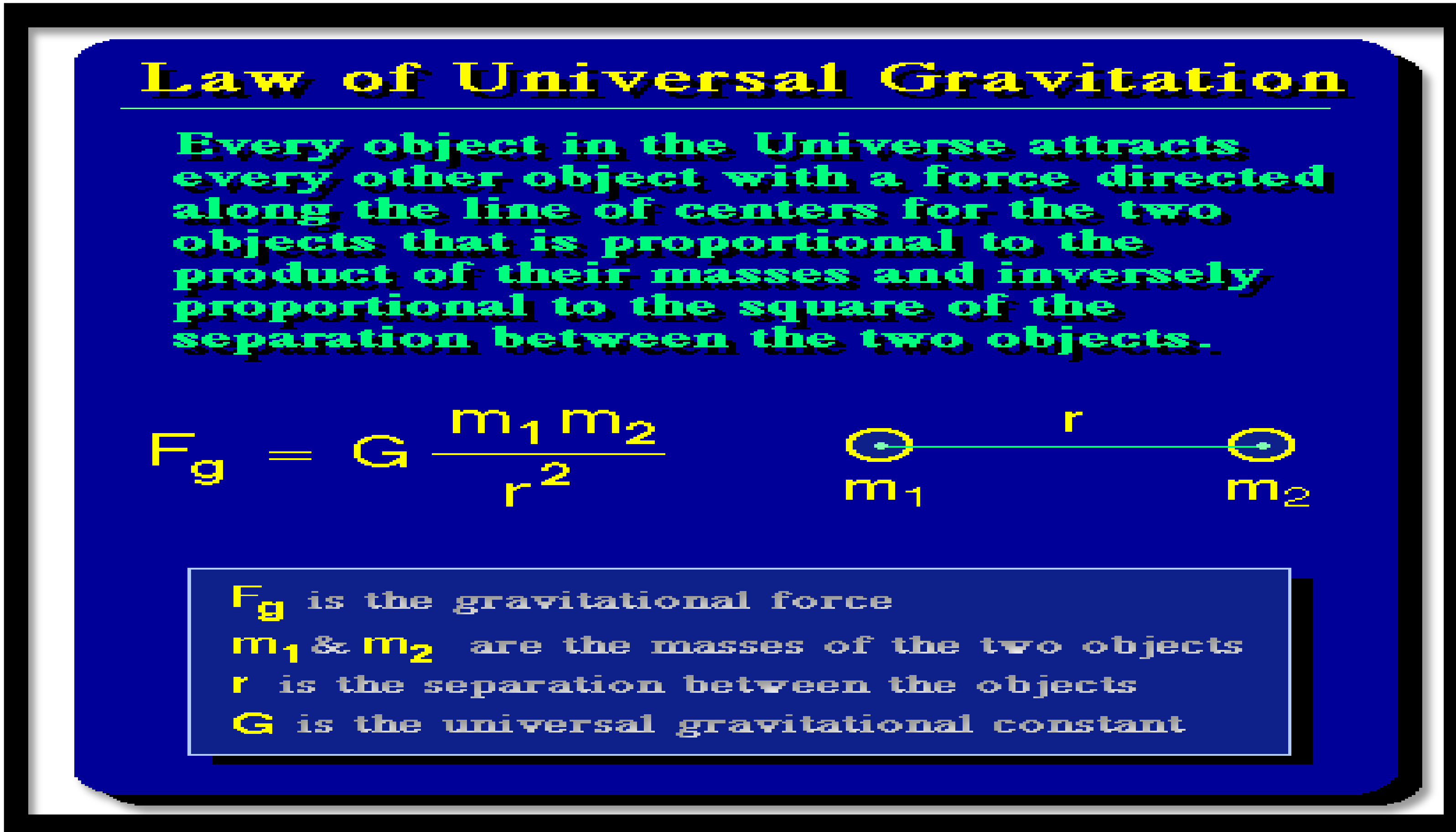
1. **the centrifugal acceleration due to rotation with angular velocity ω.**
2. **the existence of an equatorial bulge that results from the balance between self – gravitational and rotation.**

Definition of Gravitational Field

- *Gravitational field* is defined as the field of force surrounding a body of finite mass in which other body would experience an
- attractive force that is
- proportional to the product
- of the masses and inversely
- proportional to the square
- of the distance between them.



Newton's Law of Universal Gravitation



Gravitational Force

Force of gravity

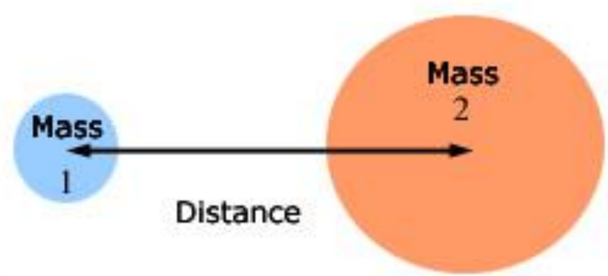
Longhand:

$$\text{Force of gravity} = \text{Gravitational constant} \times \frac{\text{Mass}_1 \times \text{Mass}_2}{\text{Distance}^2}$$

Shorthand:

$$F_g = G \times \frac{m_1 \times m_2}{D^2}$$

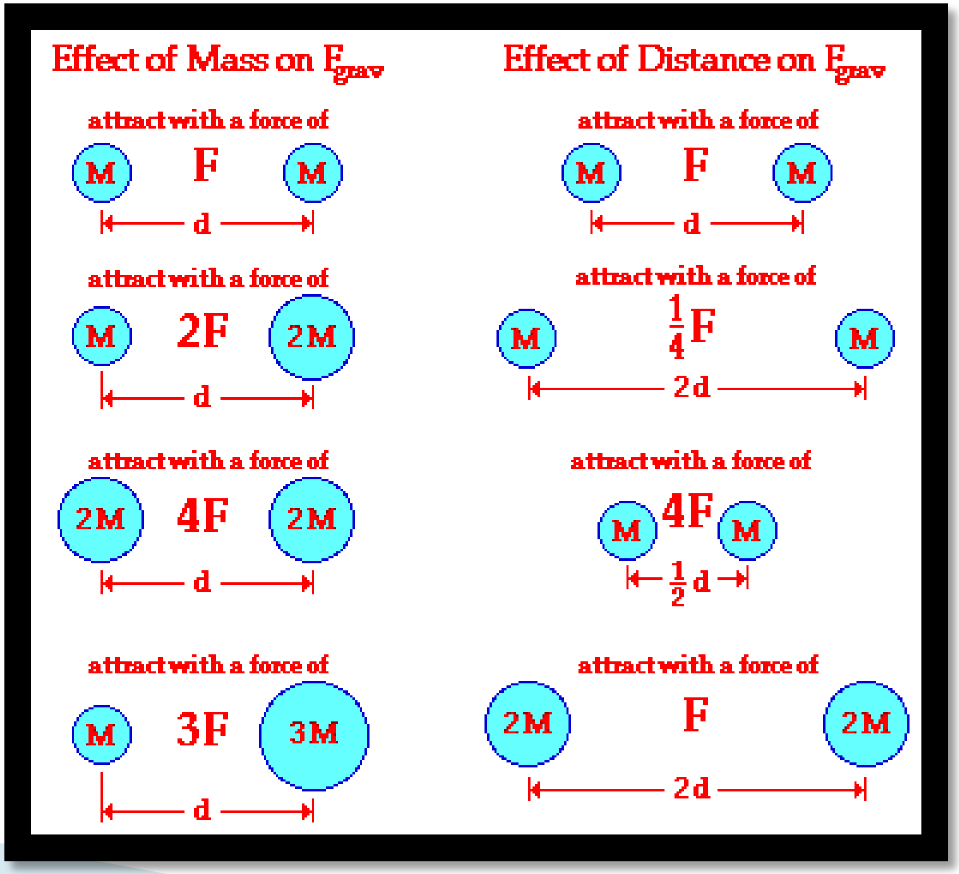
Picture:



M. Yasuda 2002

Gravitational Force

- The following illustration shows how the force of gravity is directly proportional to the product of the two masses and inversely proportional to the square of the distance of separation.



Universal Gravitational Constant

- *The gravitational constant* is the proportionality constant used in Newton's Law of Universal Gravitation, and is commonly denoted by G. This is different from g, which denotes the acceleration due to gravity. In most texts, we see it expressed as:

$$G = 6.673\times10^{-11}\ \mathrm{N\ m^2\ kg^{-2}}$$

- *The* gravitational constant is an empirical value. That is to say, it is proven through a series of experiments and subsequent observations.

Gravitational Acceleration

- *Acceleration* is defined as the rate at which the velocity of a moving object changes with time.
- In its simplest form, Newton's law of force relates the amount of **force on an** object to its **mass and acceleration.**

$$F = m\,a \qquad (1)$$

or force = mass times acceleration. Therefore, to know an acceleration to an object, one must know a force.

Gravitational Acceleration

- One of the most obvious (and the weakest) of all forces in nature is the **gravitational force. Newton's Universal Law of Gravitation describes the gravitational** force *(F_g) as follows:*

$$F_g = Gm_1 \ m_2 \ / r^2 \qquad (2)$$

- This equation states that the force between the two masses m_1 and m_2 is equal to the product of their masses ($m_1 \ m_2$) multiplied by a constant (*G) and divided by the distance* between them squared (*r^2).*

Gravitational Acceleration

- To compute the gravitational force between the Earth and an any object, we substitute the mass of the Earth (M_E) *and the distance from the object to the center of the* Earth (*r*). *When the objects are on or near the Earth's surface, this distance can be* approximated by the value for the radius of the Earth (R_E) so that Equation (2) becomes:

$$F_g = Gm\ M_E / R_E^2 \qquad (3)$$

in which we see that the force only depends on the mass of the object, because *G*, M_E, *and* R_E *are all constants* . This force (measured at the Earth's surface) is called the **weight of** the **object.**

Gravitational Acceleration

- Now looking at Equation (1) and equating *F to the gravitational force (Fg), we see* that:

$$ma = Gm\,M_E\ /R_E^2\ = mg \qquad (4)$$

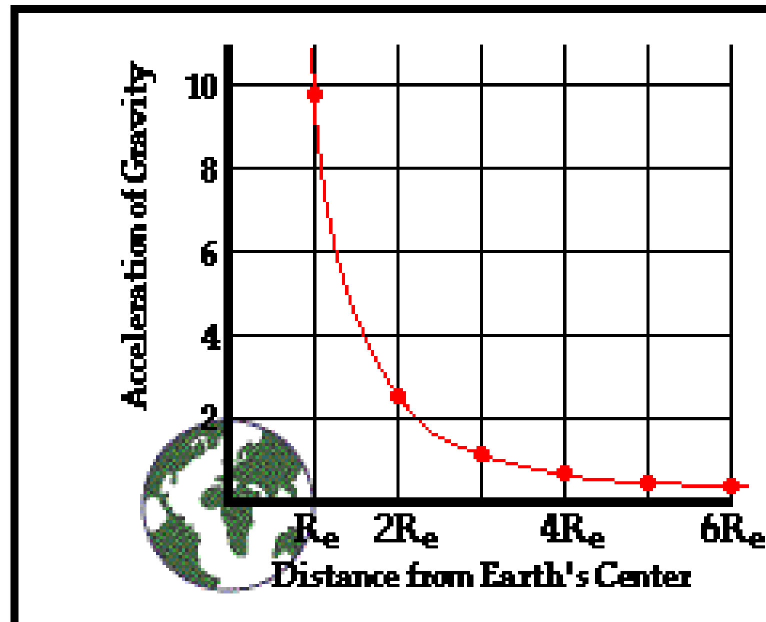
Where

$$g = Gm\,M_E\ /R_E^2 \qquad (5)$$

In this last equation, we see that *g, the **gravitational acceleration, is itself a constant*** because it depends on quantities which do not change with time.

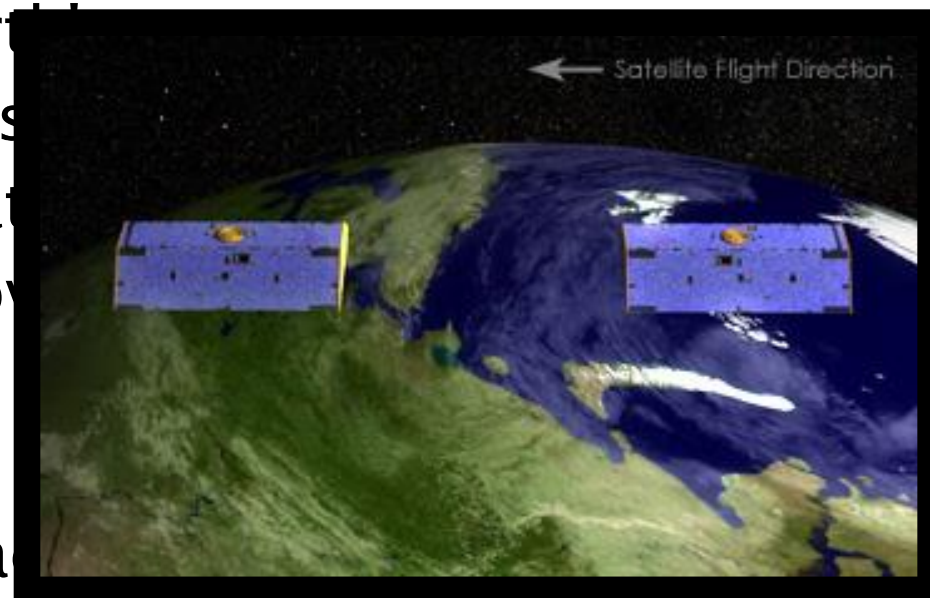
Gravitational Acceleration

- As is evident from the equation (5), the value of **g** varies inversely with the distance from the center of the earth. In fact, the variation in g with distance follows an inverse square law where **g** is inversely proportional to the distance from earth's center. This inverse square relationship is depicted in the graphic at the right.



Mapping the variation in g

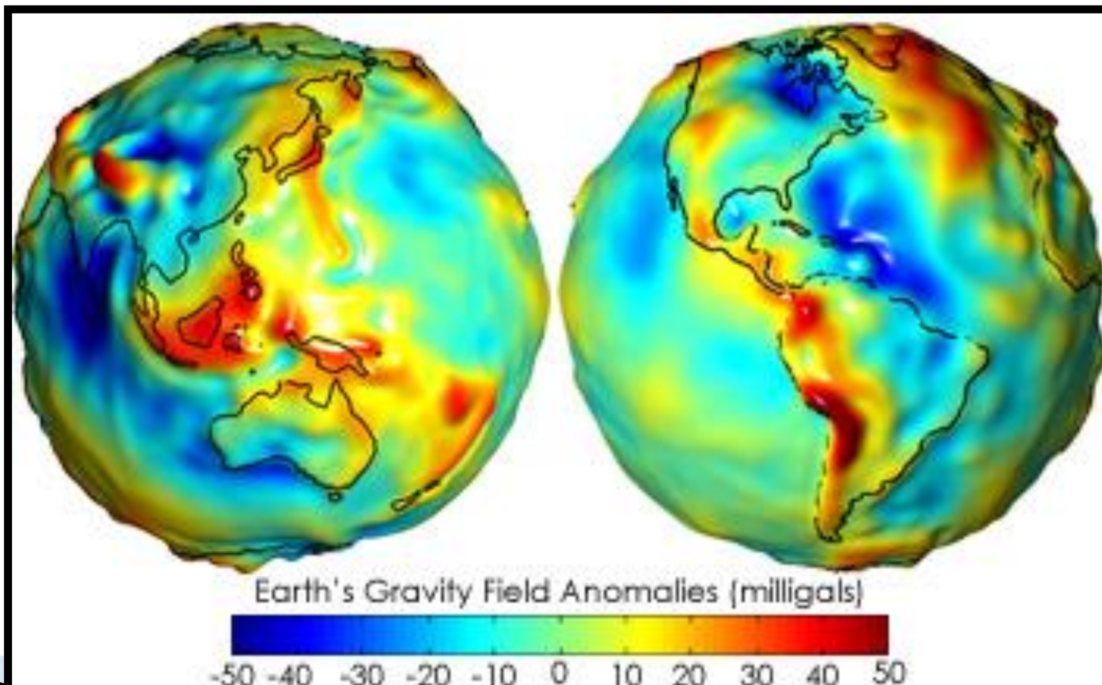
- The variation in the Eart
gravitational field is meas
continuously and accurat
by the Gravitational Reco
And Climate Experiment
(GRACE) satellites.
These satellites follow ea
other in identical orbits but
separated a short distance apart.



(GRACE) satellites measuring the variation in the gravitational field of the Earth.

Mapping the variation in g

- The variation in the Earth's gravitational acceleration is shown in the Figure. The variation in gravity is 9.80665 ms^2 ± 50 milligals (The Galileo is the cgs unit of acceleration, 1 gal = 0.01 m/s^2).



Gravitational Potential

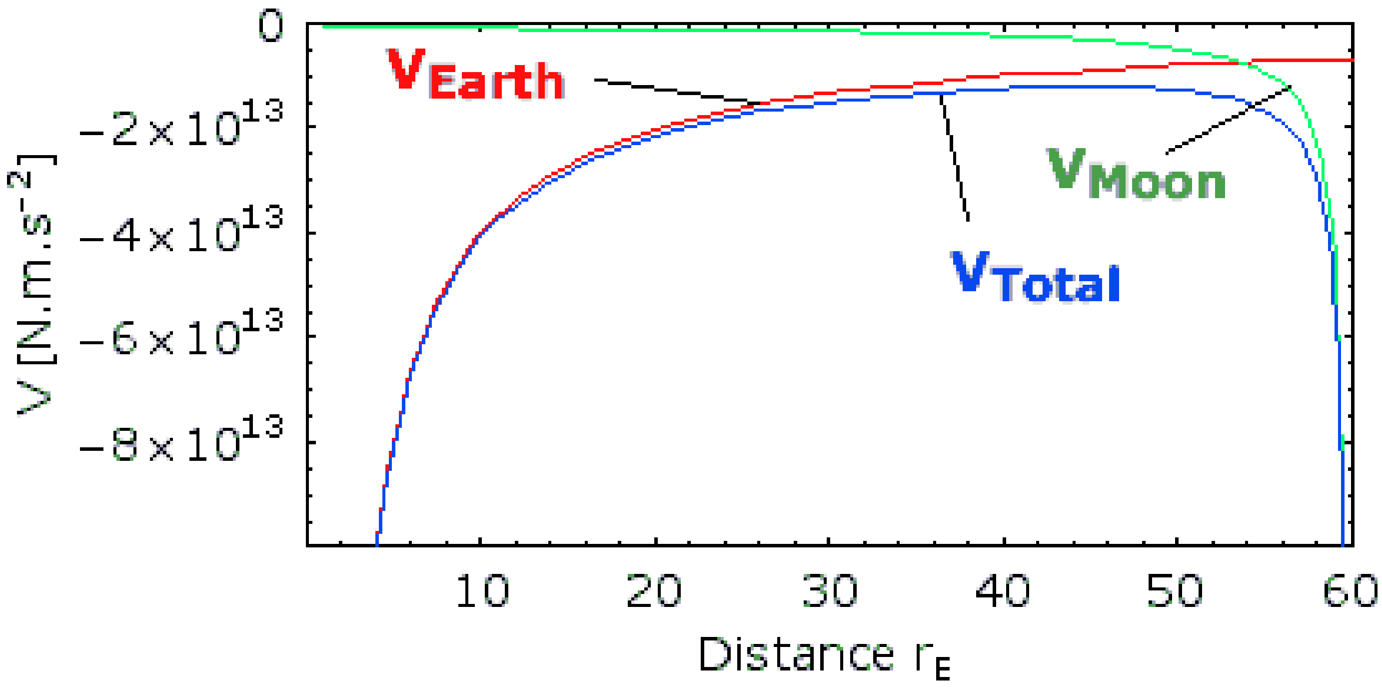
- The potential is defined as the work done in bringing a body of unit mass from infinity to a point. Since work is done by gravity on the body the work done is negative.
- In mathematical terms,

$$V = \int F(r)dr = \int \frac{GMm}{r^2}dr = \left[-\frac{GMm}{r} + c\right] = -\frac{GMm}{r}$$

Gravitational Potential

- The zero for gravitational potential is taken at infinity because it is a convenient reference point for any object in the universe.
- Gravitational potential is always negative indicating that the field does work in moving an object closer.
- The figure below shows the gravitational potential for the Earth and moon. The potential fields add together algebraically.

Gravitational Potential



The gravitational potential of the Earth, Moon and their sum.

Gravitational Potential

- We can see that to send a rocket to the moon we need only provide the force for the rocket to escape the Earths gravitational potential field and enter the moon's gravitational potential field. After this the rocket will accelerate towards the moon by the pull of the moon's potential.
- The force on an object at a given potential is given by the slope of the potential at that point.

$$F = \frac{dV}{dr}$$

Gravity and the Figure of the Earth

- The figure of the earth means the set of parameters that define the size and shape of the earth.
- The size of Earth, like the size of all of the celestial bodies, is measured in a number of parameters including mass, volume, density, surface area, and equatorial/polar/mean diameter.

Gravity and the Figure of the Earth

- Below is a table with the parameters used to measure the size of the Earth.

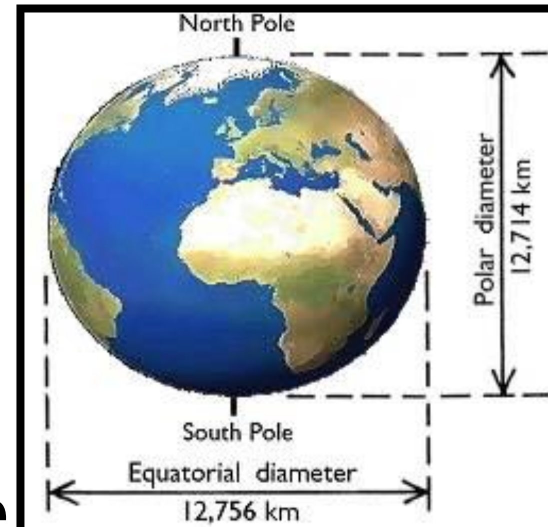
Mass	5.9736×10^{24}kg
Volume	1.083×10^{12} km^3
Mean diameter	12,742 km
Surface area	510,072,000 km^2
Density	5.515 g/cm^3
Circumference	40,041 km

Gravity and the Figure of the Earth :Shape of the Earth

- The shape of the Earth has intrigued scientists throughout history.
- The general acceptance of the fact that the Earth is round came about in the first century A.D., although Pythagoras had already postulated a spherical Earth 600 years earlier.
- The flat Earth concept resurfaced now and again in the Middle Ages, sometimes on religious grounds, but it is safe to say that mankind has known for 2000 years that we live on a sphere.

Gravity and the Figure of the Earth: Shape of the Earth

- We know also that it is not a perfect sphere: the diameter from pole to pole is shorter than the diameter at the equator.
- The difference is small:
- the equatorial diameter is
- about 12,756 kilometers,
- and the polar diameter
- is about 40 km shorter.



- The reason for this difference rotation, which creates a centrifugal force perpendicular to the rotation axis.

Gravity and the Figure of the Earth : Shape of the Earth

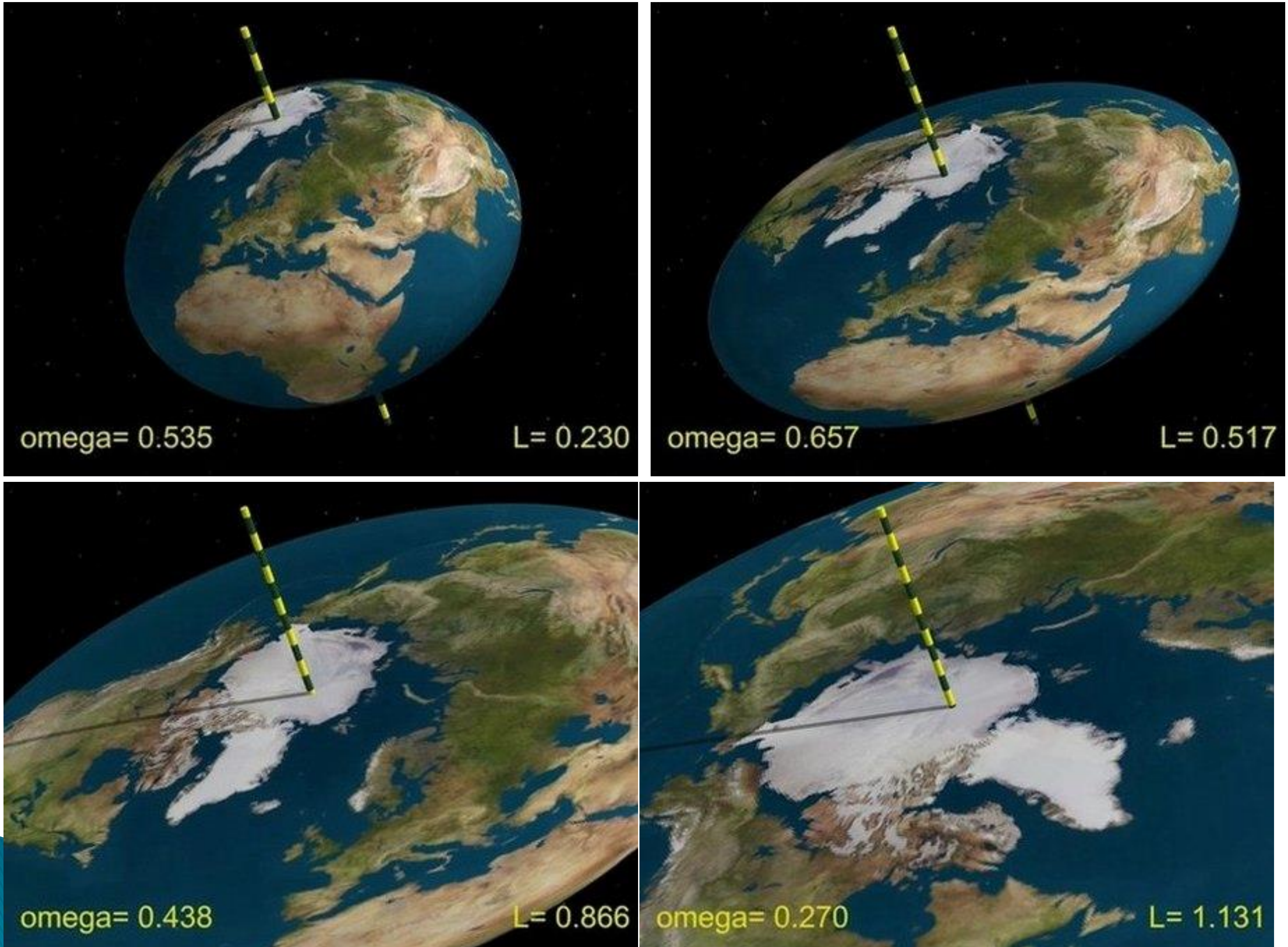
- The Earth is "viscous" and this accounts for the slight flattening at the poles.
- Because of the rotation, the Earth has an ***angular momentum***, *L*.
- The energy stored in a rotating mass is $L*\omega/2$, where ω is the rotation speed of 2π radians per day. The value for *L* is 5.86×10^{33} kJoule.sec.

Gravity and the Figure of the Earth : Shape of the Earth

- The theoretical shape of the Earth has been studied by mathematicians over the past 4 centuries .
- It was Isaac Newton who first claimed that the Earth is not spherical, but "oval".
- Setting up a mathematical model of the Earth, taking into account all its properties, is extremely complicated: the Earth has a non-homogeneous interior, there are internal flows of the molten material, there is a relatively thin crust, etc.

Flat (Oblate) Earth

- In 1742, the Scotsman Colin MacLaurin showed that , as the angular momentum increases, the Earth will get ever more flat.
- The shape is an ellipsoid with two equal axes, rotating around the short axis.
- The ellipsoid becomes a disc with an ever increasing radius.
- The images below show consecutive stages of the shape of the Earth as the angular momentum increases.


omega= 0.535
L= 0.230
omega= 0.657
L= 0.517
omega= 0.438
L= 0.866
omega= 0.270
L= 1.131

Flat (Oblate) Earth

- the **flattening** (also called **oblateness**) *f*, is the ratio of the equatorial–polar length difference to the equatorial length:

$$f = \frac{a - b}{a} = 1 - b : a.$$

Earth Ellipsoid

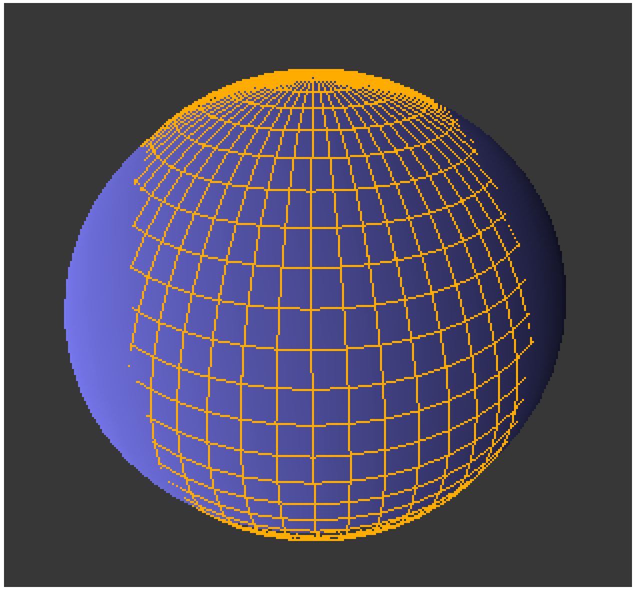
- An **Earth ellipsoid** is a mathematical figure approximating the shape of the earth, used as a reference frame for computations in geodesy, astronomy and the geosciences.
- The bad news is that the Earth is not an exact ellipsoid. In fact, because the Earth is such a "lumpy" ellipsoid no single smooth ellipsoid will provide a perfect reference surface for the entire Earth.

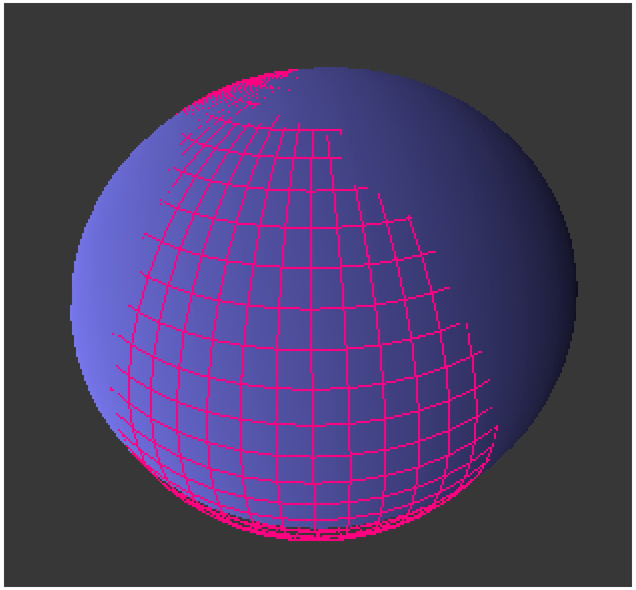
-The practical solution to this is to measure the Earth's shape in different areas and to then create different reference ellipsoids used for mapping different regions on Earth.

Earth Ellipsoid

- For example, the ellipsoid shown in yellow below is a fair match to the Earth's surface (shown in blue) in some areas but not in others. In some areas the Earth's surface protrudes above the even ellipsoid shape and in other areas the Earth's surface is lower than the ellipsoid's surface. We can use the yellow ellipsoid for precision mapping in areas where the Earth's surface is a close match.

Earth Ellipsoid





Earth Ellipsoid

–We can specify many different standard ellipsoids to map different areas of the Earth (Table below).

Name	equatorial radius	polar radius	1/f=A/(A-C)
WGS84	6378137.00	6356752.31	298.25722
MERIT,GEM-T1,T3	6378137.00	6356752.30	298.25700
GRS-80	6378137.00	6356752.31	298.25722
IAU-1976	6378140.00	6356755.29	298.25700
GEM-8	6378145.00	6356760.13	298.25500
GEM-9, GEM-10	6378140.00	6356755.14	298.25500
GEM-10B	6378138.00	6356753.29	298.25700
WGS-72	6378135.00	6356750.52	298.26000
WGS-66	6378145.00	6356759.77	298.25000
GRS-67 (IUGG-67)	6378160.00	6356774.52	298.24717
IERS-1989	6378136.00	6356751.3	298.25700
Clarke-1866	6378206.40	6356583.8	294.97870
Intl_1924/Hayford	6378388.00	6356911.95	297.00000
Australian_Natl	6378160.00	6356774.72	298.25000
Krassovski_1942	6378245.00	6356863.02	298.30000
Clarke_1880-Modified	6378249.15	6356514.97	293.46630
CERES-TOA	6408137.00	6386651.73	298.25722

References

-Stacey F. and Davis P., Physics of the Earth, Cambridge University Press, 2008.

- Lowrie, W., Fundamentals of Geophysics, Cambridge University Press, 2007.