جامعة الانبار

كلية العلوم

قسم الرياضيات التطبيقية

المادة: التفاضل والتكامل2

للطلبة المرحلة الاولى

الفصل الخامس: المحاضرة الاولى

(حساب المساحة التقريبية باستخدام المستطيلات)

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CH. 5: Integration

5.1 Area and Estimation with Finite Sums

The definite integral is the key tool in calculus for defining and calculating quantities important to mathematics and science, such as areas, volumes, lengths of curved paths, probabilities, and the weights of various objects, just to mention a few. The idea behind the integral is that we can effectively compute such quantities by breaking them into small pieces and then summing the contributions from each piece. We then consider what happens when more and more, smaller and smaller pieces are taken in the summation process. Finally, if the number of terms contributing to the sum approaches infinity and we take the limit of these sums in the way described in Section 5.3, the result is a definite integral.

Area

Suppose we want to find the area of the shaded region R that lies above the x-axis, below the graph of $f(x) = 4 - x^2$, and between the vertical lines $x = 0$ and $x = 2$ (Figure 5.1). Unfortunately, there is no simple geometric formula for calculating the areas of general shapes having curved boundaries like the region R. How, then, can we find the area of R?

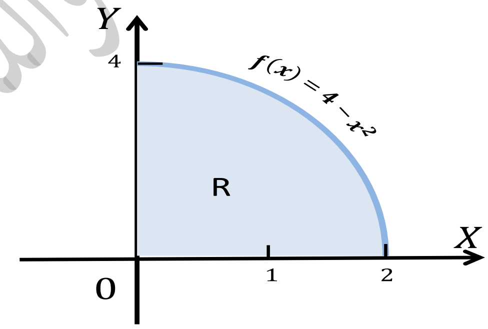

Figure 5.1 The area of the region R cannot be found by a simple formula.

Ex. Use finite approximation to estimate the area under the graph of the function $f(x) = 4 - x^2$ *between* $x = 0$ *and* $x = 2$ using:

a. a lower sum with four rectangles of equal width.

b. an upper sum with four rectangles of equal width.

c. the midpoint rule with four rectangles of equal width.

Sol. (a)(*See Figure 5.2*) $f\left(\frac{1}{2}\right) = 4 - \left(\frac{1}{2}\right)^2 = 4 - \frac{1}{4} = \frac{15}{4}$

$$f(1) = 4 - (1)^2 = 4 - 1 = 3$$

$$f\left(\frac{3}{2}\right) = 4 - \left(\frac{3}{2}\right)^2 = 4 - \frac{9}{4} = \frac{7}{4}$$

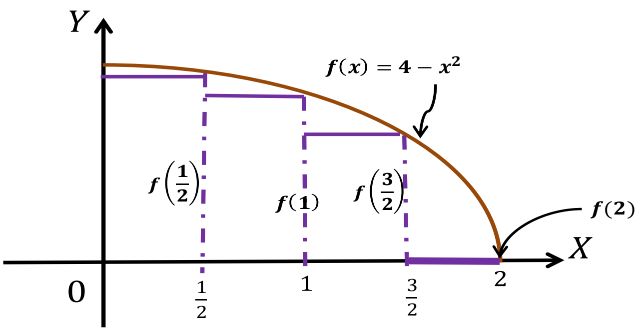

Figure 5.2 The area by using a lower sum with four rectangles of equal width.

The total area of the rectangles

$$A \approx \frac{15}{4}\cdot\frac{1}{2} + 3\cdot\frac{1}{2} + \frac{7}{4}\cdot\frac{1}{2} + 0\cdot\frac{1}{2} = \frac{1}{2}\left(\frac{15}{4} + 3 + \frac{7}{4}\right)$$

$$= \frac{1}{2}\left(3 + \frac{22}{4}\right) = \frac{1}{2}\left(3 + \frac{11}{2}\right) = \frac{1}{2}\left(\frac{17}{2}\right) = \frac{17}{4} = 4.25$$

(b) (*See Figure 5.3*) $f(0) = 4,\ f\left(\frac{1}{2}\right) = \frac{15}{4},\ f(1) = 3,\ f\left(\frac{3}{2}\right) = \frac{7}{4}$

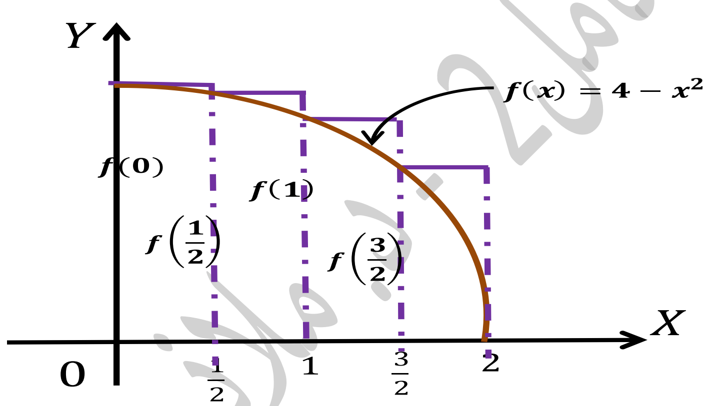

Figure 5.3 The area by using an upper sum with four rectangles of equal width.

The upper sum is

$$A \approx 4\cdot\frac{1}{2} + \frac{15}{4}\cdot\frac{1}{2} + 3\cdot\frac{1}{2} + \frac{7}{4}\cdot\frac{1}{2} = \frac{1}{2}\left(4 + \frac{15}{4} + 3 + \frac{7}{4}\right)$$

$$= \frac{1}{2}\left(7 + \frac{22}{4}\right) = \frac{1}{2}\left(7 + \frac{11}{2}\right) = \frac{1}{2}\left(\frac{25}{2}\right) = \frac{25}{4} = 6.25$$

So, the area of the region R is A where $4.25 < A < 6.25$

(c)(*See Figure 5.3*) $f\left(\frac{1}{4}\right) = 4 - \left(\frac{1}{4}\right)^2 = 4 - \frac{1}{16} = \frac{63}{16}$

$$f\left(\frac{3}{4}\right) = 4 - \left(\frac{3}{4}\right)^2 = 4 - \frac{9}{16} = \frac{55}{16}$$

$$f\left(\frac{5}{4}\right) = 4 - \left(\frac{5}{4}\right)^2 = 4 - \frac{25}{16} = \frac{39}{16}$$

$$f\left(\frac{7}{4}\right) = 4 - \left(\frac{7}{4}\right)^2 = 4 - \frac{49}{16} = \frac{15}{16}$$

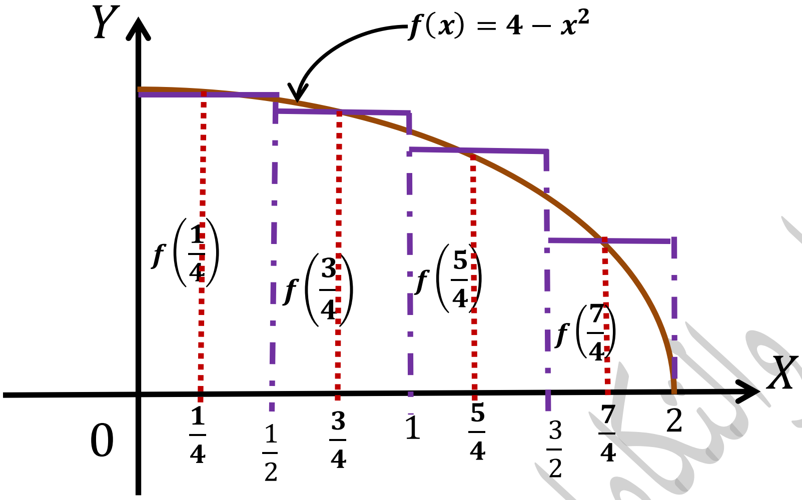

Figure 5.4 The area by using the midpoint rule sum with four rectangles of equal width.

The estimating sum of the area is

$$A \approx \frac{63}{16}\cdot\frac{1}{2} + \frac{55}{16}\cdot\frac{1}{2} + \frac{39}{16}\cdot\frac{1}{2} + \frac{15}{16}\cdot\frac{1}{2} = \frac{1}{2}\cdot\frac{1}{16}(63 + 55 + 39 + 15)$$

$$= \frac{172}{2(16)} = \frac{43}{8} = 5\cdot\frac{3}{8} = 5.375$$

Exercises 5.1:

Use finite approximations to estimate the area under the graph of the function using:

a. a lower sum with four rectangles of equal width.

b. an upper sum with four rectangles of equal width.

c. the midpoint rule with four rectangles of equal width.

1. $f(x) = x^2 \;\; between \;\; x = 0 \;\; and \;\; x = 1.$

Answer: $a)\frac{7}{32}\,, \quad b)\frac{15}{32}\,, \quad c)\frac{21}{64}$

2. $f(x) = \frac{1}{x} \;\; between \;\; x = 1 \;\; and \;\; x = 5.$

Answer: $a)\frac{77}{60}\,, \quad b)\frac{25}{12}\,, \quad c)\frac{496}{315}$

3. $f(x) = 4 - x^2 \;\; between \;\; x = -2 \;\; and \;\; x = 2.$

Answer: $a)6\,, \quad b)14\,, \quad c)11$

المصادر:

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