جامعة الانبار

كلية العلوم

قسم الرياضيات التطبيقية

المادة: التفاضل والتكامل2

للطلبة المرحلة الاولى

الفصل السابع: المحاضرة الثامنة

(اللوغاريتم الطبيعي)

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CH. 7: Transcendental Functions

7.2 Natural Logarithms

Def. The natural logarithms function $\ln x = \int_1^x \frac{1}{t}\, dt, \quad x > 0$.

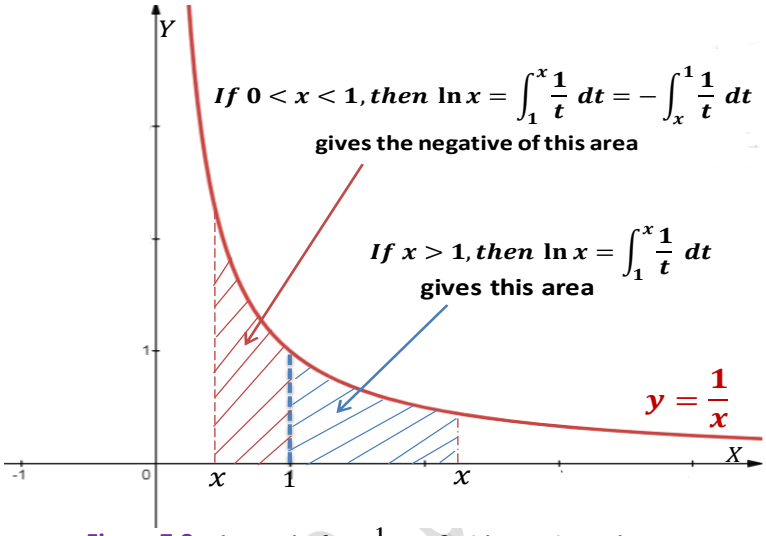

Figure 7.9 The graph of $y = \frac{1}{x}, x > 0$ with negative and positive area for $\ln x$

Some Properties of the function $y = \ln x$

1)If $x > 0$ then $\ln x = \int_1^x \frac{1}{t} dt$ =positive area between the graph of $y = \frac{1}{t}$ from $t = 1$ to $t = x$.

2)If $0 < x < 1$, then

$\ln x = \int_1^x \frac{1}{t} dt = -\int_x^1 \frac{1}{t} dt$ =the negative area between the graph of $y = \frac{1}{t}$ from $t = x$ to $t = 1$.

3)If $x = 1$, then $\ln 1 = \int_1^1 \frac{1}{t} dt = 0$.

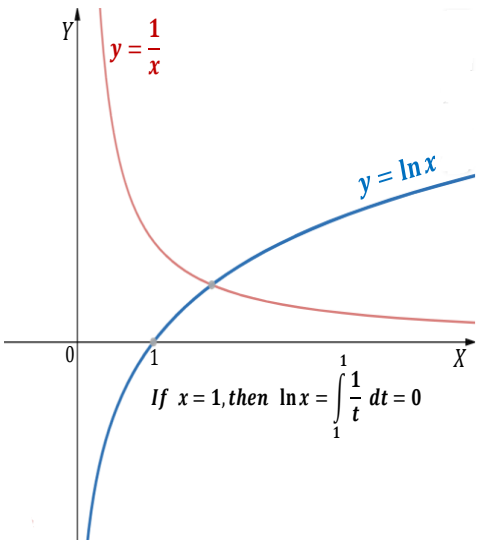

Figure 7.10 The graph of $y = \ln x$ and its relation to the function $y = \frac{1}{x}, x > 0$.

4)$\frac{d}{dx}(\ln x) = \frac{d}{dx}\left(\int_1^x \frac{1}{t} dt\right) = \frac{1}{x} > 0,$

$\forall x > 0 \rightarrow \ln x$ is an increasing function.

Def. The number e is a number in the domain of $\ln x$ that satisfy $\ln e = 1$.

Ex.(1) **Simplify the following expressions.**

a) $\frac{\ln 49}{\ln 21+\ln\frac{1}{3}} = \frac{\ln 7^2}{\ln(3\cdot 7)-\ln 3} = \frac{2\ln 7}{\ln 3+\ln 7-\ln 3} = \frac{2\ln 7}{\ln 7} = 2$

b) $3\ln\sqrt[3]{2}+\frac{1}{2}\ln\left(\frac{9}{4}\right) = 3\ln(2)^{\frac{1}{3}}+\frac{1}{2}\ln\left(\frac{3}{2}\right)^2 = 3\cdot\frac{1}{3}\ln 2+\frac{1}{2}\cdot 2\ln\frac{3}{2}$

$= \ln 2+\ln\frac{3}{2} = \ln\left(2\cdot\frac{3}{2}\right) = \ln 3$

Ex.(2) **Solve for x: $\ln[(2x+1)(x+2)] = 2\ln(x+2)$**

Sol. $\ln(2x+1)+\ln(x+2) = 2\ln(x+2)$

$\ln(2x+1) = 2\ln(x+2)-\ln(x+2)$

$\ln(2x+1) = \ln(x+2) \rightarrow 2x+1 = x+2 \rightarrow x = 1$

Ex.(3) **Find y' for each the following.**

1) $y = x\sqrt{\ln x}$

Sol. $y' = x\frac{1}{2\sqrt{\ln x}}\cdot\frac{1}{x}+\sqrt{\ln x}\cdot 1 = \frac{1}{2\sqrt{\ln x}}+\sqrt{\ln x}$

2) $y = (x^2\ln x)^4$

Sol.$y' = 4(x^2\ln x)^3\cdot\left[x^2\cdot\frac{1}{x}+\ln x\cdot(2x)\right] = 4(x^2\ln x)^3[x+2x\ln x]$

3) $y = \ln(\ln(3x+4)) \overset{Sol}{\Longrightarrow} y' = \frac{1}{\ln(3x+4)}\cdot\frac{1}{(3x+4)}\cdot 3$

Ex.(4) **Evaluate the following integrals.**

1)i. $\int \cot x\,dx = \int\frac{\cos x}{\sin x}dx = \ln|\sin x|+C$

ii. $\int \tan x\,dx = \int\frac{\sin x}{\cos x}dx = -\int\frac{-\sin x}{\cos x}dx = -\ln|\cos x|+C$

or $= \ln|\cos x|^{-1}+C = \ln\frac{1}{|\cos x|}+C = \ln|\sec x|+C$

iii. $\int \sec x\,dx = \int\frac{\sec x(\sec x+\tan x)}{\sec x+\tan x}dx = \int\frac{\sec^2 x+\sec x\tan x}{\sec x+\tan x}dx$

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$$= \ln|\sec x + \tan x| + C$$

iv. $\int \csc x\, dx = \int \frac{\csc x(\csc x - \cot x)}{\csc x - \cot x} dx = \int \frac{\csc^2 x - \csc x \cot x}{\csc x - \cot x} dx$

$$= \ln|\csc x - \cot x| + C$$

or $\int \frac{\csc x(\csc x + \cot x)}{\csc x + \cot x} dx = -\ln|\csc x + \cot x| + C$

2) $\int \frac{x^3}{2x^4+3} dx \overset{Sol.}{\Longrightarrow} \frac{1}{8}\int \frac{8x^3}{2x^4+3} dx = \frac{1}{8}\ln|2x^4 + 3| + C$

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