جامعة الانبار

كلية العلوم

قسم الرياضيات التطبيقية

نظرية البيانات

Cycle Matrix / Part 2

Fundamental Cycle Matrix

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Fundamental Cycle Matrix

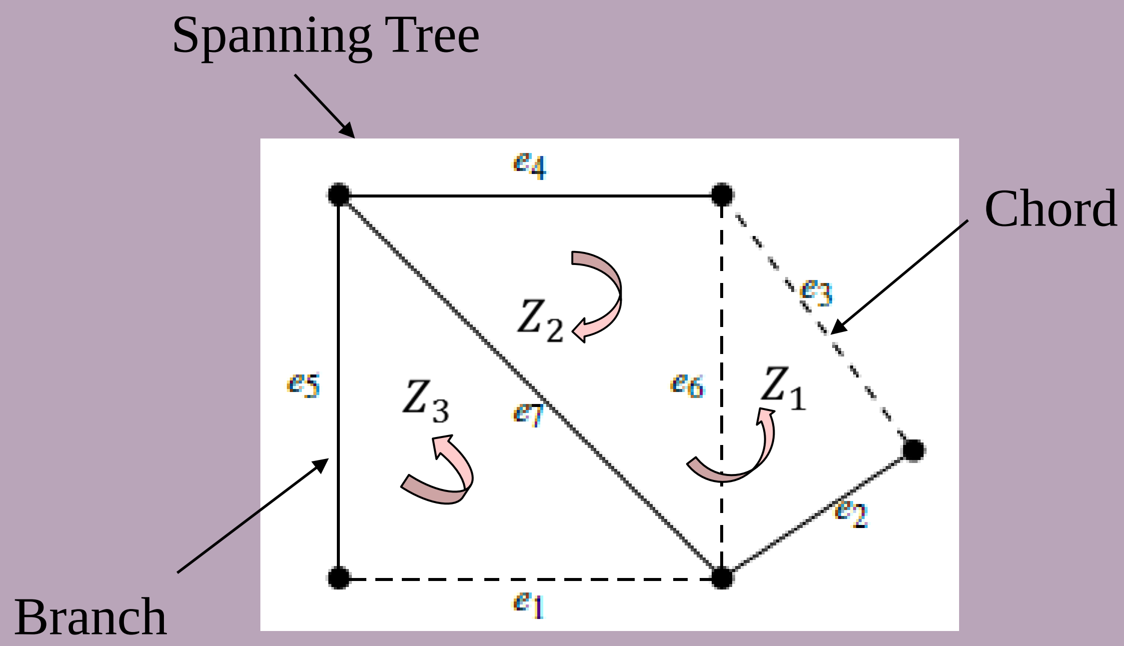
Lecture # 4

Part # 2

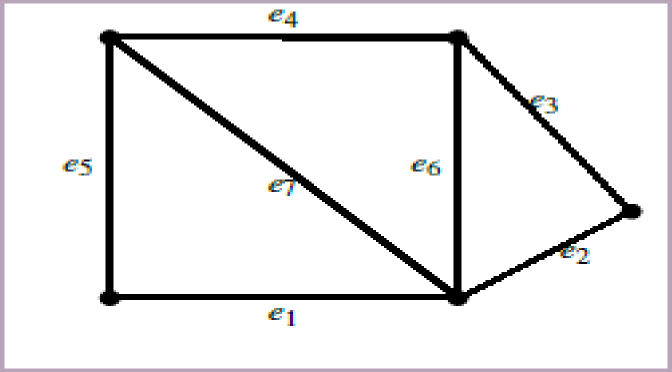
Fundamental Cycle :

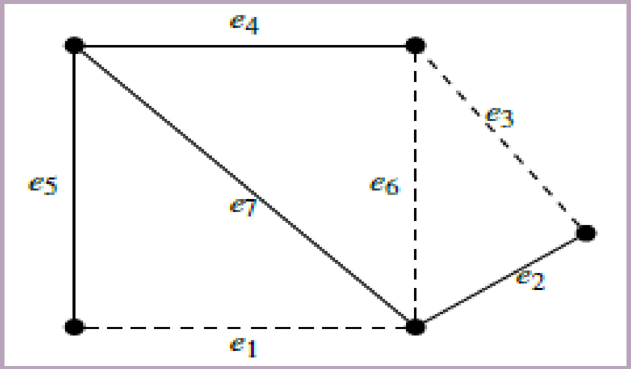
Definition : A cycle formed in a graph G by adding a chord of a spanning tree T of G is called a fundamental circuit or fundamental cycle in G.

- $Z_1 = \{e_2, e_3, e_4, e_7\}$
  $Z_2 = \{e_4, e_6, e_7\}$
  $Z_3 = \{e_1, e_5, e_7\}$



The set of fundamental cycles with respect to any spanning tree in a connected graph are the only independent cycles in a graph. The remaining cycles can be obtained as ring sums of these cycles.

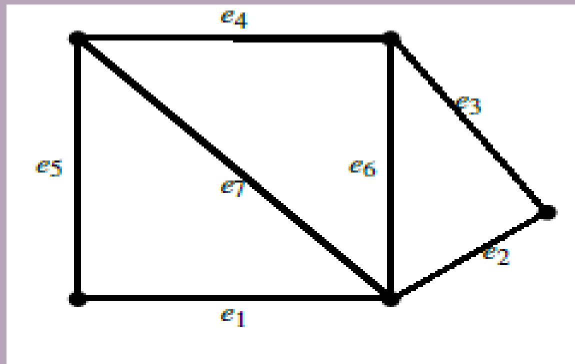


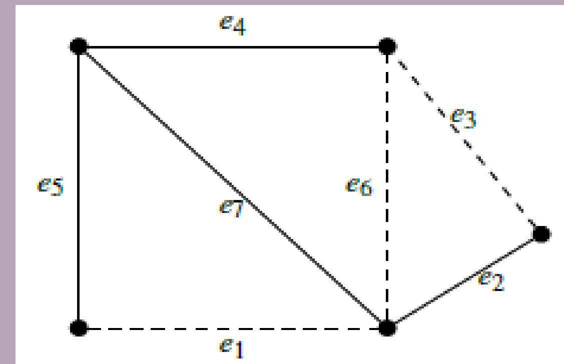


- $C_1 = \{e_2, e_3, e_4, e_7\}$
  $C_2 = \{e_4, e_6, e_7\}$
  $C_3 = \{e_1, e_5, e_7\}$
  $C_4 = \{e_2, e_3, e_6\}\,; C_4 = C_1 \oplus C_2$
  $C_5 = \{e_1, e_4, e_5, e_6\}\,; C_5 = C_2 \oplus C_3$
  $C_6 = \{e_1, e_2, e_3, e_4, e_5\}\,; C_6 = C_1 \oplus C_3$

- $Z_1 = \{e_2, e_3, e_4, e_7\}$
  $Z_2 = \{e_4, e_6, e_7\}$
  $Z_3 = \{e_1, e_5, e_7\}$

A submatrix of a cycle matrix in which all rows correspond to a set of fundamental cycles (independent cycles) is called a fundamental cycle matrix, denoted by B_f . Note that the permutation of rows or columns or both do not affect the matrix B_f . If the order and size of a connected graph G are respectively n and e, then B_f is an $(e-n+1) \times e$ matrix because the number of fundamental cycles is $e-n+1$, each fundamental cycle being produced by one chord.



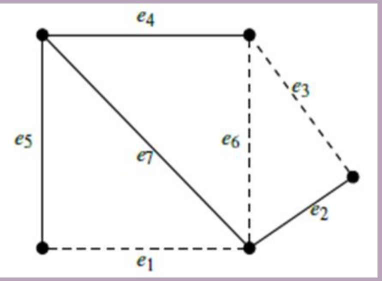


- $$B(G) = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

$$B(G) = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ \cancel{0} & \cancel{1} & \cancel{1} & \cancel{0} & \cancel{0} & \cancel{1} & \cancel{0} \\ \cancel{1} & \cancel{0} & \cancel{0} & \cancel{1} & \cancel{1} & \cancel{1} & \cancel{0} \\ \cancel{1} & \cancel{1} & \cancel{1} & \cancel{1} & \cancel{1} & \cancel{0} & \cancel{0} \end{bmatrix}$$

$$B_f(G) = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

By arranging the columns in B_f such that all the $e-n+1$ chords correspond to the first $e-n+1$ columns and rearranging the rows such that the first row corresponds to the fundamental cycle made by the chord in the first column, the second row to the fundamental cycle made by the second, and so on. A matrix B_f thus arranged has the form $B_f = [I_\mu : B_t]$, where I_μ is an identity matrix of order $\mu = e-n+1$ and B_t is the remaining $\mu \times (n-1)$ submatrix, corresponding to the branches of the spanning tree.



$$B_f = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \sim \left[\begin{array}{ccc|cccc} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{array}\right]$$

Identity matrix corresponding to the chord of T of order $\mu = e-n+1$ $\{e_1, e_3, e_6\}$

$$\text{where } I_\mu = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } B_t = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Submatrix of order $(e-n+1) \times (n-1)$ corresponding to the branches of T $\{e_2, e_4, e_5, e_7\}$

Note:

From equation $B_f = [I_\mu \vdots B_t]$, we have rank $B_f = \mu = \varepsilon - n + 1$. Since B_f is a submatrix of the cycle matrix B, then, $rankB \geq rank(B_f)$ and thus, $rank(B) \geq \varepsilon - n + 1$.

Theorem : (Sylvester's Theorem) If A and B are matrices of order $k \times n$ and $n \times p$ respectively, then $nullity(AB) \leq nullity(A) + nullity(B)$.

Theorem: If $B(G)$ is a cycle matrix of a connected graph G with n vertices and e edges, then rank of $B(G)$ is $\mathrm{e} - n + 1$.

Theorem: If $B(G)$ is a cycle matrix of a disconnected graph G with n vertices , e edges and k components , then rank of $B(G)$ is $\mathrm{e} - n + k$.

THANK YOU

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