جامعة الانبار

كلية العلوم

قسم الرياضيات التطبيقية

نظرية البيانات

Cut- Set Matrix / Part 1

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Cut- Set Matrix

Lecture # 5

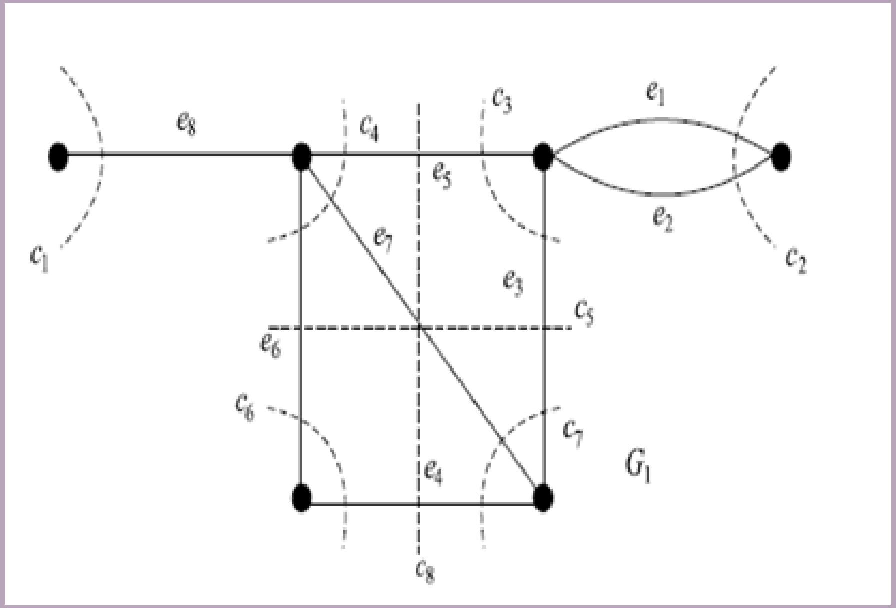
Part # 1

Cut- Set: (Edge Cut- Set)

In a connected graph G, a cut- set is a set S of edges with the following properties:

1) The removal of all the edges in S disconnects G.

2) The removal of some of edges (but not all) in S does not disconnect G.



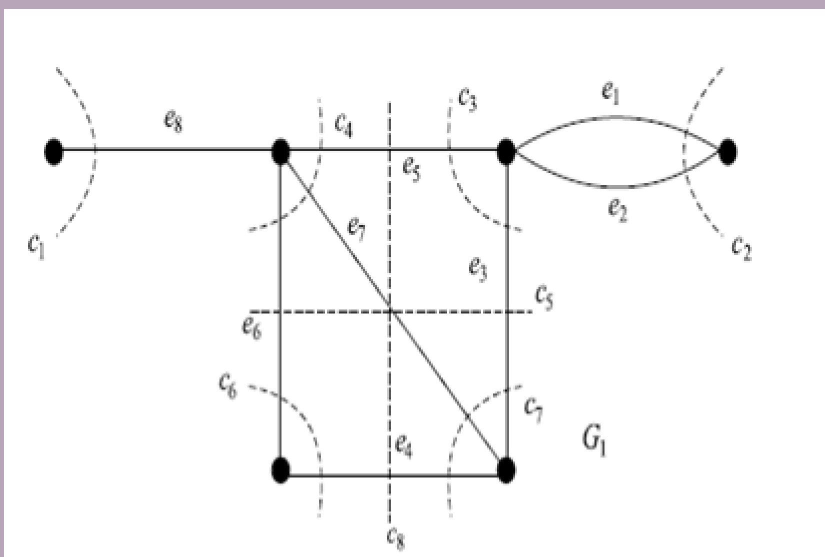
- $C_1 = \{e_8\}$
  $C_2 = \{e_1, e_2\}$
  $C_3 = \{e_3, e_5\}$
  $C_4 = \{e_5, e_6, e_7\}$
  $C_5 = \{e_3, e_6, e_7\}$
  $C_6 = \{e_4, e_6\}$
  $C_7 = \{e_3, e_4, e_7\}$
  $C_8 = \{e_4, e_5, e_7\}$

Cut- Set Matrix:

Definition: Let G be a graph with e edges and q cut-sets. The cut-set matrix of G is a matrix, denoted by $C(G)$ and is defined to be $C(G) = [c_{ij}]_{q\times e}$, is a (0,1) −matrix such that

$$c_{ij} = \begin{cases} 1 & if\ i - th\ cutset\ contains\ j - th\ edge; \\ 0 & Otherwise. \end{cases}$$

Example 1:



$C_1 = \{e_8\}, C_2 = \{e_1, e_2\}, C_3 = \{e_3, e_5\}$

$C_4 = \{e_5, e_6, e_7\}$, $C_5 = \{e_3, e_6, e_7\}$,

$C_6 = \{e_4, e_6\}$, $C_7 = \{e_3, e_4, e_7\}$,

$C_8 = \{e_4, e_5, e_7\}$.

Hence the cut-set matrix of G_1 is

$$C(G_1) = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Example 2:

$C_1 = \{e_1, e_2\}, C_2 = \{e_3, e_4\},$

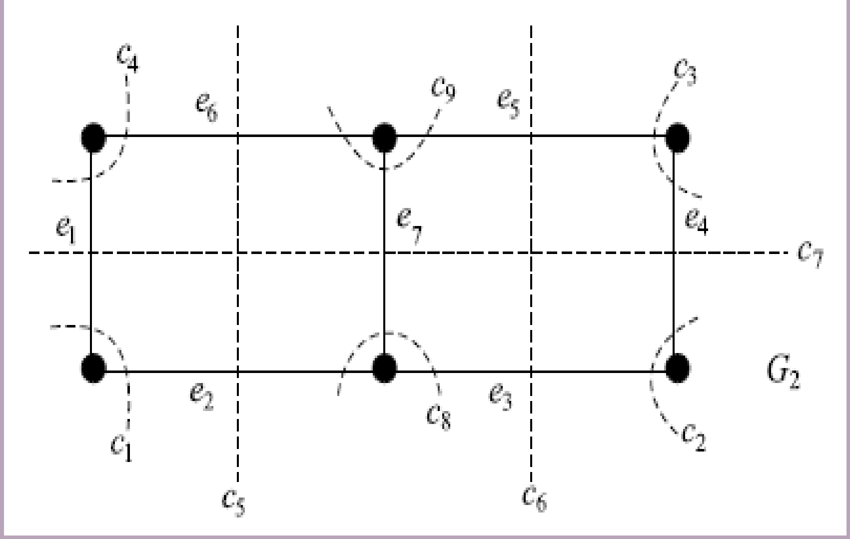
$C_3 = \{e_4, e_5\}, C_4 = \{e_1, e_6\},$

$C_5 = \{e_2, e_6\}, C_6 = \{e_3, e_5\},$

$C_7 = \{e_1, e_4, e_7\}, C_8 = \{e_2, e_3, e_7\},$

$C_9 = \{e_5, e_6, e_7\}$

Hence the cut-set matrix of G_1 is



$$C(G_2) = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \end{array} \begin{array}{c} \begin{array}{ccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

We have the following observations about the cut-set matrix $C(G)$ of a graph G.

1. The permutation of rows or columns in a cut-set matrix corresponds simply to renaming of the cut-sets and edges respectively.

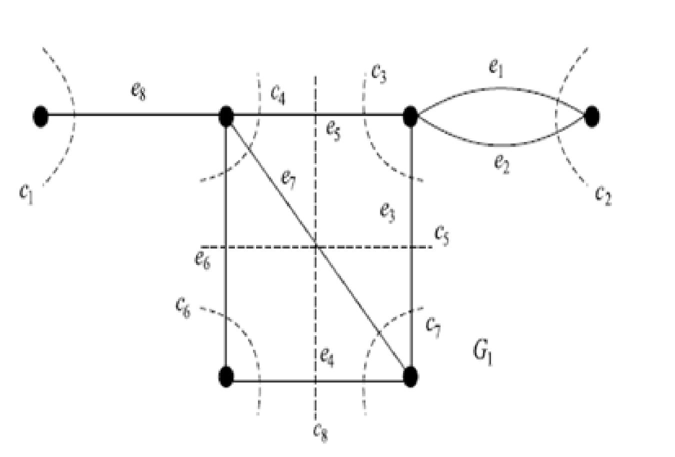
2. Each row in $C(G)$ is a cut-set vector.

3. A column with all zeros corresponds to an edge forming a self-loop.

4. Parallel edges form identical columns in the cut-set matrix.

5. In a non-separable graph, since every set of edges incident on a vertex is a cut-set, therefore every row of incidence matrix $A(G)$ is included as a row in the cut-set matrix $C(G)$. That is, for a non-separable graph G, $C(G)$ contains $A(G)$. For a separable graph, the incidence matrix of each block is contained in the cut-set matrix. For example, in the graph G_1 (Example 1), the incidence matrix of the block $\{e_3, e_4, e_5, e_6, e_7\}$ is the 4×5 submatrix of $C(G_1)$, left after deleting rows C_1, C_2, C_5, C_8 and columns e_1, e_2, e_8.

$$C(G_1) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$



6. It follows from observation 5 , that $rank\ C(G) \geq rank\ A(G)$. Hence, for a connected graph with n vertices, $rank\ C(G) \geq n - 1$.

The following result for connected graphs shows that cut-set matrix, incidence matrix and the corresponding graph matrix have the same rank.

Theorem : The rank of a cut-set matrix $C(G)$ of a connected graph G is equal to the rank of incidence matrix $A(G)$, which equals the rank of graph G.

THANK YOU

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