جامعة الانبار

كلية العلوم

قسم الرياضيات التطبيقية

نظرية البيانات

Relation between A_f , B_f,C_f

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Relation between A_f , B_f and C_f

Lecture # 6

Review:

Let G be a connected graph and A_f , B_f and C_f be respectively the reduced incidence matrix, the fundamental cycle matrix, and the fundamental cut-set matrix of G. We have shown that

$$B_f = [I_\mu \vdots B_t]$$

Fundamental cycle matrix of order $(e-n+1) \times e$

Identity matrix corresponding to the chord of T of order $\mu = e-n+1$

Submatrix of order $(e-n+1) \times (n-1)$ corresponding to the branches of T

$$C_f = [C_c \vdots I_{n-1}]$$

Fundamental Cut-Set matrix of order $(n-1) \times e$

Matrix of order $(n-1) \times (e-n+1)$ whose columns correspond to the chords of T

Identity matrix of order $n-1$ whose columns correspond to the branches of T.

Reduced incidence matrix of order $(n-1) \times e$ formed by remove an arbitrary row

$$A_f = [A_c \vdots A_t]$$

An $(n-1) \times (e-n+1)$ submatrix whose columns correspond to the chords of T

An $(n-1) \times (n-1)$ submatrix whose columns correspond to the branches of T

Relation between A_f, B_f and C_f:

Let the edges (i.e., the columns) be arranged in the same order as in A_f, B_f and C_f.

- Since the columns in A_f and B_f are arranged in the same order, the equation $AB^T = BA^T \equiv 0\ (mod\ 2)$ gives:

Since $AB^T = 0$, therefore $A_f\, B_f^T = 0$.

Thus, $[A_c : A_t]\begin{bmatrix} I_\mu \\ .. \\ B_t^T \end{bmatrix} = 0$, so that $A_c + A_t B_t^T = 0$.

Since A_t is non-singular, we have $A_t^{-1}[A_c + A_t B_t^T] = 0$.

Therefore, $A_t^{-1}A_c + B_t^T = 0$ and so $A_t^{-1}A_c = -B_t^T$.

Since in $mod\,2$ arithmetic $-1 = 1$, we have

$$B_t^T = A_t^{-1}A_c.$$

- Since the columns in B_f and C_f are arranged in the same order, then

$C_f B_f^T = 0.$

Therefore, $[C_c : I_{n-1}]\begin{bmatrix} I_\mu \\ .. \\ B_t^T \end{bmatrix} = 0$, and so $C_c + B_t^T = 0$.

Thus, $C_c = -B_t^T$ and so $C_c = A_t^{-1}A_c$.

$$C_c = B_t^T = A_t^{-1} A_c.$$

$$A_f = [A_c \vdots A_t]$$

$$B_f = [I_\mu \vdots B_t]$$

$$C_f = [C_c \vdots I_{n-1}]$$

We make the following observations from the above relations.

1. If A or A_f is given, we can construct B_f and C_f starting from an arbitrary spanning tree and its submatrix A_t in A_f .

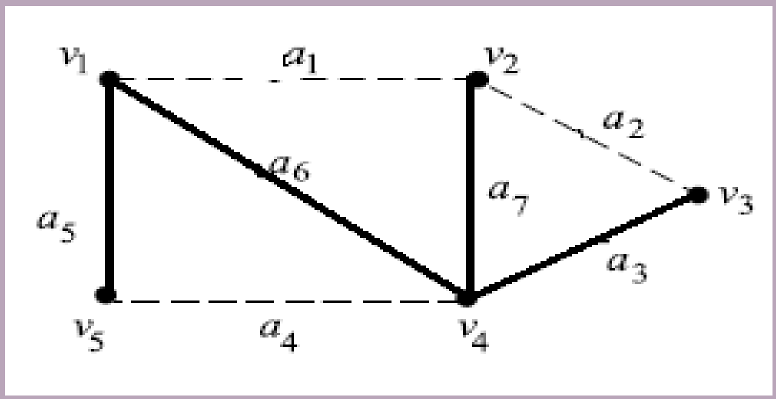
2. If either B_f or C_f is given, we can construct the other. Therefore, since B_f determines a graph within 2-isomorphism, so does C_f.

3. If either B_f and C_f is given, then A_f, in general, cannot be determined completely.

Example:

$$C_c = B_t^T = A_t^{-1} A_c$$

$A_c \rightarrow Chord$
$A_t \rightarrow Branch$
$B_t \rightarrow Branch$
$C_c \rightarrow Chord$



$$B_f(G) = \begin{array}{c} \begin{array}{cccccccc} a_2 & a_1 & a_4 & \vdots & a_5 & a_3 & a_7 & a_6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & \vdots & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & \vdots & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & \vdots & 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$B_f = [I_\mu \vdots B_t]$$

$$A_f(G) = \begin{array}{c} \begin{array}{cccccccc} a_2 & a_1 & a_4 & \vdots & a_5 & a_3 & a_7 & a_6 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & \vdots & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & \vdots & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \vdots & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & \vdots & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$A_f = [A_c \vdots A_t]$$

$$C_f(G) = \begin{array}{c} \begin{array}{cccccccc} a_2 & a_1 & a_4 & \vdots & a_5 & a_3 & a_7 & a_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & \vdots & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & \vdots & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & \vdots & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & \vdots & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$C_f = [C_c \vdots I_{n-1}]$$

THANK YOU

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