

جامعة الانبار

كلية العلوم

قسم الرياضيات التطبيقية

نظرية البيانات

Adjacency Matrix / Part 2

Power of Adjacency Matrix

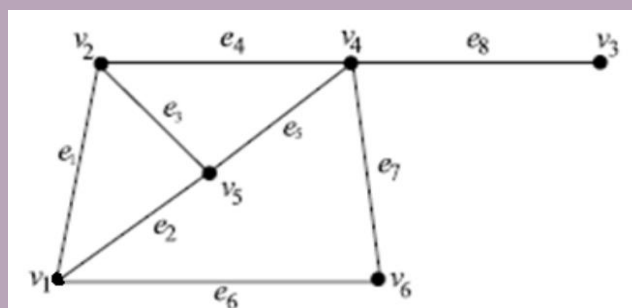
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Power of Adjacency Matrix

Lecture # 7 Part 2

Power of $X(G)$:

1) $X^2(G)$



$$X^2(G) = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 0 & 3 & 1 & 0 \\ 1 & 3 & 1 & 1 & 2 & 2 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 4 & 1 & 0 \\ 1 & 2 & 1 & 1 & 3 & 2 \\ 0 & 2 & 1 & 0 & 2 & 2 \end{bmatrix}$$

For $k = 2, i \neq j$, we have

- $[X^2]_{ij}$ = number of ones in the product of i th row and j th column (or j th row) of X
- = number of positions in which both i th and j th rows of X have ones
- = number of vertices that are adjacent to both i th and j th vertices
- = number of different paths of length two between i th and j th vertices

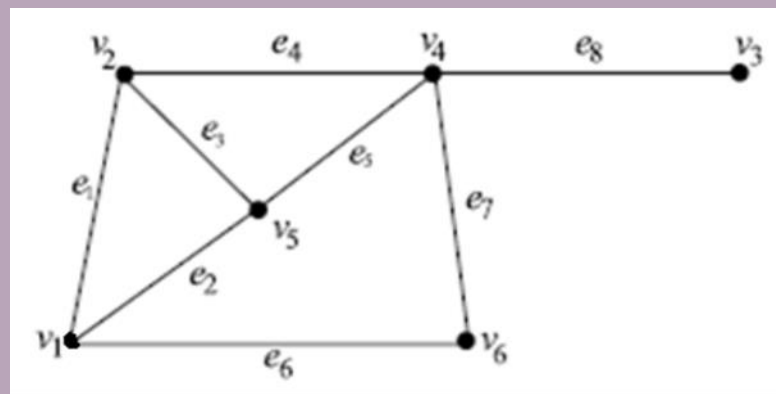
Also, $[X^2]_{ii}$ = number of ones in the i th row (or column) of X

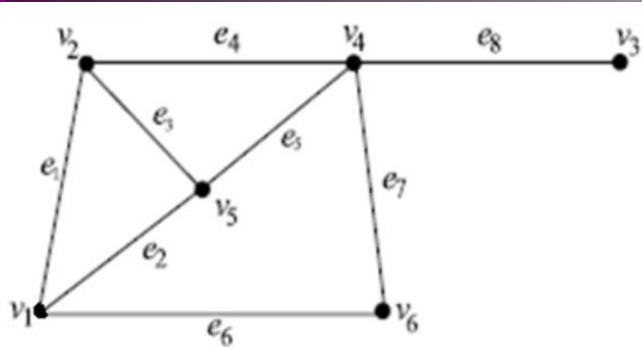
= degree of the corresponding vertex.

2) $X^3(G)$

Since a matrix commutes with matrices that are its own power, $XX^2 = X^2X = X^3$. Since the product of two square symmetric matrices that commute is also a symmetric matrix, X^3 is a symmetric matrix.

$$X^3(G) = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 3 & 1 & 0 & 3 & 1 & 0 \\ 1 & 3 & 1 & 1 & 2 & 2 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 4 & 1 & 0 \\ 1 & 2 & 1 & 1 & 3 & 2 \\ 0 & 2 & 1 & 0 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 7 & 3 & 2 & 7 & 6 \\ 7 & 4 & 1 & 8 & 5 & 2 \\ 3 & 1 & 0 & 4 & 1 & 0 \\ 2 & 8 & 4 & 2 & 8 & 7 \\ 7 & 5 & 1 & 8 & 4 & 2 \\ 6 & 2 & 0 & 7 & 2 & 0 \end{bmatrix}$$





An edge sequence is a sequence of edges in which each edge, except the first and the last, has one vertex in common with the edge preceding it and one vertex in common with the edge following it. A walk and a path are the examples of an edge sequence. An edge can appear more than once in an edge sequence.

$$X^3(G) = \begin{bmatrix} 3 & 1 & 0 & 3 & 1 & 0 \\ 1 & 3 & 1 & 1 & 2 & 2 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 4 & 1 & 0 \\ 1 & 2 & 1 & 1 & 3 & 2 \\ 0 & 2 & 1 & 0 & 2 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 7 & 3 & 2 & 7 & 6 \\ 7 & 4 & 1 & 8 & 5 & 2 \\ 3 & 1 & 0 & 4 & 1 & 0 \\ 2 & 8 & 4 & 2 & 8 & 7 \\ 7 & 5 & 1 & 8 & 4 & 2 \\ 6 & 2 & 0 & 7 & 2 & 0 \end{bmatrix}$$

Let us now observe that the ij -th entry of X^3 is equal to the following:

- (i) the dot product (inner product) of i -th row of X^2 and j -th column (or j -th row) of X .
- (ii) $\sum_{k=1}^n$ (ik -th entry of X^2 \cdot kj -th entry of X).
- (iii) $\sum_{k=1}^n$ (the number of all different edge sequences of three edges from i -th vertex to j -th vertex via k -th vertex).
- (iv) the number of different edge sequences of three edges between i -th and j -th vertices.

Theorem: Let X be the adjacency matrix of a simple graph G . Then the i, j th entry in X^r is the number of different edge sequences of r edges between vertices v_i & v_j .

Corollary 1: In a connected graph, the distance between two vertices v_i & v_j (for $i \neq j$) is k , iff k is the smallest integer for which the i, j th entry in X^k is non-zero.

Corollary 2: If X is the adjacency matrix of a graph G with n vertices, and $Y = X + X^2 + X^3 + \dots + X^{n-1}$, (in the ring of integers), then G is disconnected iff there exists at least one entry in matrix Y that is zero.



THANK YOU

References:

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