جامعة الانبار

كلية العلوم

قسم الرياضيات التطبيقية

نظرية البيانات

Path Matrix

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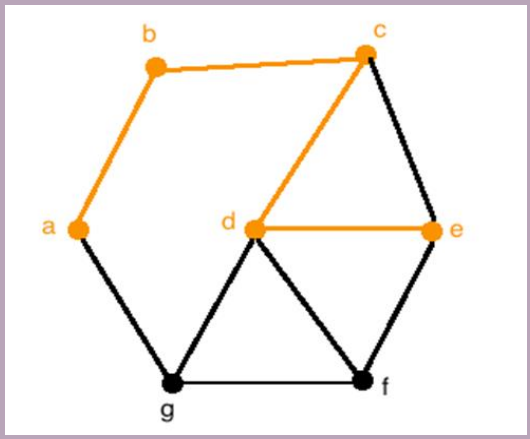
Path Matrix

Lecture # 8

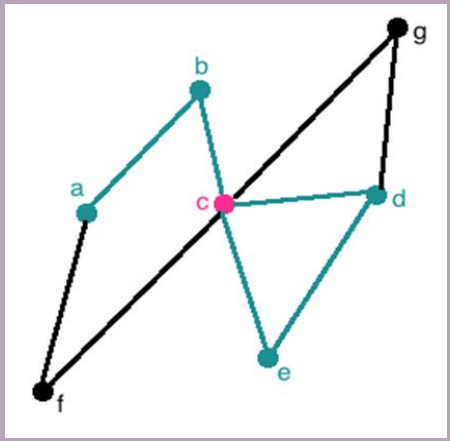
All of these are sequences of vertices and edges. They have the following properties :

1. **Walk** : Vertices may repeat. Edges may repeat (Closed or Open)
2. **Trail** : Vertices may repeat. Edges cannot repeat (Open)
3. **Circuit** : Vertices may repeat. Edges cannot repeat (Closed)
4. **Path** : Vertices cannot repeat. Edges cannot repeat (Open)
5. **Cycle** : Vertices cannot repeat. Edges cannot repeat (Closed)

NOTE : For closed sequences start and end vertices are the only ones that can repeat.



path



Not path

Vertex *c* repeat twice

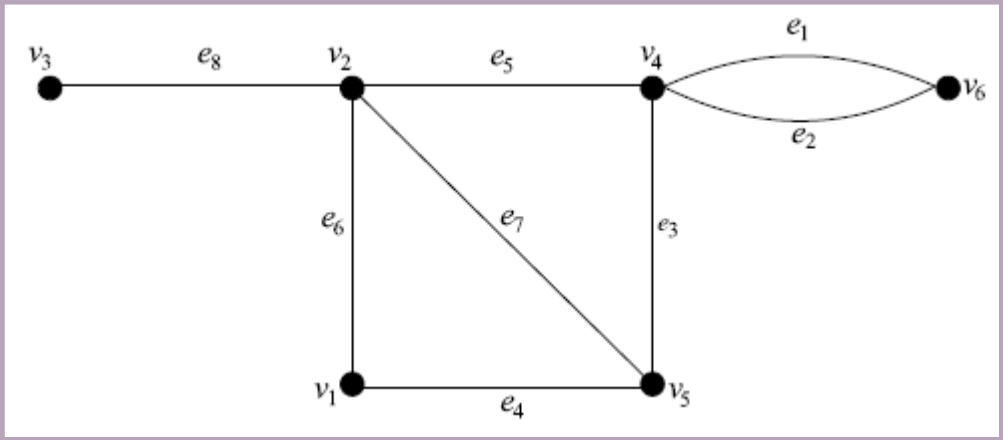
Path Matrix:

Let G be a graph with m edges, and u and v be any two vertices in G. The path matrix for vertices u and v denoted by $P(u, v) = [p_{ij}]_{q \times m}$, where q is the number of different paths between u and v, is defined as

$$p_{ij} = \begin{cases} 1, & if\ jth\ edge\ lies\ in\ the\ ith\ path, \\ 0, & otherwise. \end{cases}$$

if $i-th$ path contains the $j-th$ edge

Clearly, a path matrix is defined for a particular pair of vertices, the rows in $P(u, v)$ correspond to different paths between u and v, and the columns correspond to different edges in G.



The different paths between the vertices v_3 and v_4 are

$$p_1 = \{e_8, e_5\},\ p_2 = \{e_8, e_7, e_3\} \text{ and } p_3 = \{e_8, e_6, e_4, e_3\}.$$

The path matrix for v_3, v_4 is given by

$$P(v_3, v_4) = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array} \begin{array}{c} \begin{array}{cccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

We have the following observations about the path matrix.

1. A column of all zeros corresponds to an edge that does not lie in any path between u and v.
2. A column of all ones corresponds to an edge that lies in every path between u and v.
3. There is no row with all zeros.
4. The ring sum of any two rows in $P(u, v)$ corresponds to a cycle or an edge-disjoint union of cycles.

The next result gives a relation between incidence and path matrix of a graph.

The next result gives a relation between incidence and path matrix:
Theorem: If the columns of the incidence matrix A and the path matrix $P(u,v)$ of a connected graph are arranged in the same order, then under the product (mod 2) $AP^T(u,v) = M$, where M is a matrix having ones in two rows u and v, and the zeros in the remaining $n-2$ rows.

$$AP^T(v_3, v_4) = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{matrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{(mod2).}$$

THANK YOU

References:

1. S. Arumugam and S. Ramachandran, (2015), Invitation to graph theory, Scitech Publ., Kolkata, India.
2. G. S. Singh, (2013). Graph theory, Prentice Hall of India, New Delhi.
3. R. Balakrishnan and K. Ranganathan, (2012). A textbook of graph theory, Springer, New York.
4. J.A. Bondy and U.S.R Murty, (2008). Graph theory, Springer.
5. G. Agnarsson and R. Greenlaw, (2007). Graph theory: Modeling, applications & algorithms, Pearson Education, New Delhi.
6. G. Chartrand and P. Zhang, (2005). Introduction to graph theory, McGraw-Hill Inc.
7. G. Sethuraman, R. Balakrishnan, and R.J. Wilson, (2004). Graph theory and its applications, Narosa Pub. House, New Delhi.
8. D.B. West, (2001). Introduction to graph theory, Pearson Education Inc., Delhi.
9. V.K. Balakrishnan, (1997). Graph theory, McGrawhill, New York.
10. G. Chartrand and L. Lesniak, (1996). Graphs and digraphs, CRC Press.
11. J.A. Bondy and U.S.R Murty, (1976). Graph theory with applications, North-Holland, New York.