# Institute: University of Anbar 

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Lecture title: Case 2: $\mathrm{E}^{\boldsymbol{7}}$ parallel to the plane-ofincidence

## Case 2: $\vec{E}$ parallel to the plane-of-incidence.

A similar pair of equations can be derived when the incoming $\vec{E}$ - field lies in the plane-of-incidence, as shown in Fig. 1.19. Continuity of the tangential components of $\vec{E}$ on either side of the boundary leads to

$$
\begin{equation*}
E_{0 i} \cos \theta_{i}-E_{0 r} \cos \theta_{r}=E_{0 t} \cos \theta_{t} \tag{1.34}
\end{equation*}
$$

In much the same way as before, continuity of the tangential components of $\vec{B} / \mu$ yields
$\frac{1}{\mu_{i} v_{i}} E_{0 i}-\frac{1}{\mu_{r} v_{r}} E_{0 r}=\frac{1}{\mu_{t} v_{t}} E_{0 t}$ (1.35)

Using the fact that $\mu_{i}=\mu_{r}$ and $\theta_{i}=\theta_{r}$, we can combine these formulas to obtain two more of the Fresnel Equations:
$r_{\|}=\left(\frac{E_{0 r}}{E_{0 i}}\right)_{\perp}=\frac{\frac{n_{t}}{\mu_{t}} \cos \theta_{i}-\frac{n_{i}}{\mu_{i}} \cos \theta_{t}}{\frac{n_{i}}{\mu_{i}} \cos \theta_{t}+\frac{n_{t}}{\mu_{t}} \cos \theta_{i}}$
(1.36)
and
$r_{\|}=\left(\frac{E_{0 t}}{E_{0 i}}\right)_{\perp}=\frac{2 \frac{n_{i}}{\mu_{i}} \cos \theta_{i}}{\frac{n_{i}}{\mu_{i}} \cos \theta_{t}+\frac{n_{t}}{\mu_{t}} \cos \theta_{i}}$


Figure 1. 1. An incoming wave whose $\vec{E}$-field is in the plane-of incidence.

When both media forming the interface are dielectrics that are essentially the amplitude coefficients become

$$
\begin{align*}
& r_{\|}=\frac{n_{t} \cos \theta_{i}-n_{i} \cos \theta_{t}}{n_{i} \cos \theta_{t}+n_{t} \cos \theta_{i}}  \tag{1.38}\\
& r_{\|}=\frac{2 n_{i} \cos \theta_{i}}{n_{i} \cos \theta_{t}+n_{t} \cos \theta_{i}} \tag{1.39}
\end{align*}
$$

and

One further notational simplification can be made using Snell's Law, whereupon the Fresnel Equations for dielectric media become

$$
\begin{gather*}
r_{\perp}=-\frac{\sin \left(\theta_{i}-\theta_{t}\right)}{\sin \left(\theta_{i}+\theta_{t}\right)}  \tag{1.40}\\
r_{\|}=+\frac{\tan \left(\theta_{i}-\theta_{t}\right)}{\tan \left(\theta_{i}+\theta_{t}\right)}  \tag{1.41}\\
t_{\perp}=+\frac{2 \sin \theta_{t} \cos \theta_{i}}{\sin \left(\theta_{i}+\theta_{t}\right)}  \tag{1.42}\\
t_{\|}=+\frac{2 \sin \theta_{t} \cos \theta_{i}}{\sin \left(\theta_{i}+\theta_{t}\right) \cos \left(\theta_{i}-\theta_{t}\right)} \tag{1.43}
\end{gather*}
$$

Bear in mind that the directions (or more precisely, the phases) of the fields in Figs. 1.18 and 1.19 were selected rather arbitrarily. For example, in Fig. 1.18 we could have assumed that $\vec{E}$ pointed inward, whereupon $\vec{B}$ would have had to be reversed as well. Had we done that, the sign of $r_{\perp}$ would have turned out to be positive, leaving the other amplitude coefficients unchanged. The signs appearing in Eqs. (1.40) through (1.43), which are positive except for the first, correspond to the particular set of field directions selected. The minus sign in Eq. (1.40), as we will see, just means that we didn't guess correctly concerning $\vec{E}$ in Fig. 1.18. Nonetheless, be aware that the literature is not standardized, and all possible sign variations have been labelled the Fresnel Equations. To avoid confusion they must be related to the specific field directions from which they were derived.

## Example

An electromagnetic wave having an amplitude of $1.0 \mathrm{~V} / \mathrm{m}$ arrives at an angle of $30.0^{\circ}$ to the normal in air on a glass plate of index 1.60. The wave's electric field is entirely perpendicular to the plane-ofincidence. Determine the amplitude of the reflected wave.

## SOLUTION

$$
\begin{aligned}
& \text { Since }\left(E_{0 r}\right)_{\perp}=r_{\perp}\left(E_{0 i}\right)_{\perp}=r_{\perp}(1 . \mathrm{V} / \mathrm{m}) \text { we have to find } \\
& \qquad r_{\perp}=-\frac{\sin \left(\theta_{i}-\theta_{t}\right)}{\sin \left(\theta_{i}+\theta_{t}\right)}
\end{aligned}
$$

But first we'll need $\theta_{t}$, and so from Snell's Law

$$
\begin{gathered}
n_{i} \sin \theta_{i}=n_{t} \sin \theta_{t} \\
\sin \theta_{t}=\frac{n_{i}}{n_{t}} \sin \theta_{i} \\
\sin \theta_{t}=\frac{1}{1.6} \sin 30 \\
\theta_{t}=18.21^{\circ}
\end{gathered}
$$

Hence

$$
\begin{gathered}
r_{\perp}=-\frac{\sin (30-18.2)}{\sin (30+18.2)}=-\frac{\sin 11.8}{\sin 48.2} \\
r_{\perp}=-\frac{0.2045}{0.7455}=-0.274
\end{gathered}
$$

And so

$$
\begin{aligned}
& \left(E_{0 r}\right)_{\perp}=r_{\perp}\left(E_{0 i}\right)_{\perp}=r_{\perp}(1.0 \mathrm{~V} / \mathrm{m}) \\
& \left(E_{0 r}\right)_{\perp}=-0.27 \mathrm{~V} / \mathrm{m}
\end{aligned}
$$

## References:

1- Principles of optics- Max Born
2- Optics,-Eugene-Hecht
3- Optics and photonics an introduction

