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College: College of Science

Department: Physics

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Subject: Optics 1

Lecture title: Reflectance and Transmittance

Reflectance and Transmittance

Consider a circular beam of light incident on a surface, as shown in Fig. 1.20, such that there is an illuminated spot of area *A*. The radiant flux density (W/m^2) or irradiance is

$$I = \langle S \rangle_T = \frac{c\epsilon_0}{2} E_0^2 \qquad (1.44)$$

In the case at hand (Fig. 1.20), let I_i, I_r, and I_t be the incident, reflected, and transmitted flux densities, respectively. The cross-sectional areas of the incident, reflected, and transmitted beams are, respectively, $Acos\theta_i$, $Acos\theta_r$, and $Acos\theta_t$. Accordingly, the incident power is $I_i Acos\theta_i$; this is the energy per unit time flowing in the incident beam, and it's therefore the power arriving on the surface over *A*. Similarly, $I_r Acos\theta_r$is the power in the reflected beam, and $I_t Acos\theta_t$ is the power being transmitted through A. We define the **reflectance** *R* to be the ratio of the reflected power (or flux) to the incident power:

$$R = \frac{I_r Acos\theta_r}{I_i Acos\theta_i} = \frac{I_r}{I_i} \qquad (1.45)$$

In the same way, the **transmittance** T is defined as the ratio of the transmitted to the incident flux and is given by

$$T = \frac{I_t cos\theta_t}{I_i cos\theta_i} \qquad (1.46)$$

The quotient I_r/I_i equals $(\upsilon_r \epsilon_r E_{0r}^2/2)$ / $(\upsilon_i \epsilon_i E_{0i}^2/2)$, and since the incident and reflected waves are in the same medium, $\upsilon_r = \upsilon_i$, $\epsilon_r = \epsilon_i$, and

$$R = \left(\frac{E_{0r}}{E_{0i}}\right)^2 = r^2 \qquad (1.47)$$

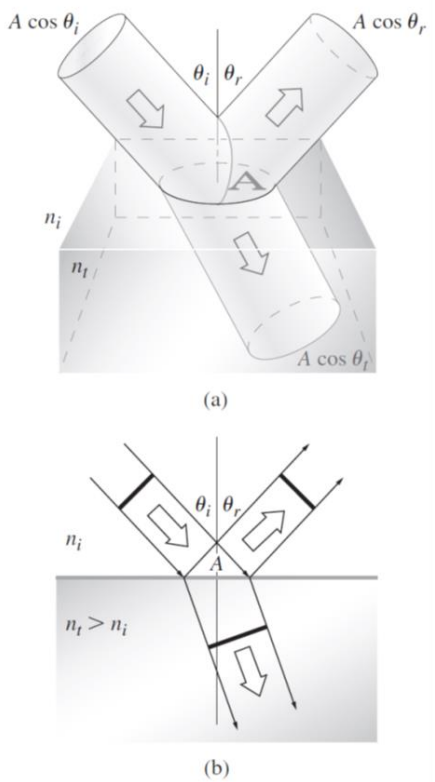


Figure 1. 1. Reflection and transmission of an incident beam.

In like fashion (assuming $\mu_i = \mu_t = \mu_0$),

$$T = \frac{n_t cos\theta_t}{n_i cos\theta_i}\left(\frac{E_{0t}}{E_{0i}}\right)^2 = \left(\frac{n_t cos\theta_t}{n_i cos\theta_i}\right)^2 t^2 \tag{1.48}$$

where use was made of the fact that $\mu_0\epsilon_t = 1/\upsilon_t^2$ and $\mu_0\upsilon_t\epsilon_t = n_t/c$. Notice that at normal incidence, which is a situation of great practical interest, $\theta_t = \theta_i = 0$, and the transmittance [Eq. (1.46)], like the reflectance [Eq. (1.45)], is then simply the ratio of the appropriate irradiances. Observe that in Eq. (1.48) T is not simply equal to t^2, for two reasons. First, the ratio of the indices of refraction must be there, since the speeds at which energy is transported into and out of the interface are different, in other words, $I \propto \upsilon$. Second, the cross-sectional areas of the incident and refracted beams are different. The energy flow per unit area is affected accordingly, and that manifests itself in the presence of the ratio of the cosine terms.

Let's now write an expression representing the conservation of energy for the configuration depicted in Fig. 1.20. In other words, the total energy flowing into area A per unit time must equal the energy flowing outward from it per unit time:

$$I_i A cos\theta_i = I_r A cos\theta_r + I_t A cos\theta_t \tag{1.49}$$

When both sides are multiplied by *c,* this expression becomes

$$n_i E_{0i}^2 cos\theta_i = n_i E_{0r}^2 cos\theta_i + n_i E_{0t}^2 cos\theta_t$$

Or

$$1 = \left(\frac{E_{0r}}{E_{0i}}\right)^2 + \left(\frac{n_t cos\theta_t}{n_i cos\theta_i}\right)\left(\frac{E_{0t}}{E_{0i}}\right)^2 \tag{1.50}$$

But this is simply

$$R + T = 1 \tag{1.51}$$

where there was no absorption.

The electric field is a vector field and, as in the Fresnel analysis, we can again think of light as being composed of two orthogonal components whose E-fields are either parallel or perpendicular to the plane-of-incidence. In fact, for ordinary "unpolarized" light, half oscillates parallel to that plane and half oscillates perpendicular to it. Thus, if the incoming net irradiance is, say, 500 W/m^2 the amount of light oscillating

perpendicular to the incident plane is 250 W/m^2. It follows from Eqs. (1.47) and (1.48) that

$$R_{\perp} = r_{\perp}^2 \tag{1.52}$$

$$R_{\parallel} = r_{\parallel}^2 \tag{1.53}$$

$$T_{\perp} = \left(\frac{n_t cos\theta_t}{n_i cos\theta_i}\right) t_{\perp}^2 \tag{1.54}$$

$$T_{\parallel} = \left(\frac{n_t cos\theta_t}{n_i cos\theta_i}\right) t_{\parallel}^2 \tag{1.55}$$

$$R_{\parallel} + T_{\parallel} = 1 \tag{1.56a}$$

$$R_{\perp} + T_{\perp} = 1 \tag{1.56b}$$

Notice that $R_{\perp}$is the fraction of $l_{i\perp}$ that is reflected, and not the fraction of l_i reflected. Accordingly, both $R_{\perp}$ and $R_{\parallel}$ can equal 1, and so the total reflectance for natural light is given by

$$R = \frac{1}{2}(R_{\parallel} + R_{\perp}) \tag{1.57}$$

When $\theta_i = 0$, the incident plane becomes undefined, and any distinction between the parallel and perpendicular components R and T vanishes. In this case Eqs. (1.52) through (1.55),

along with (1.38) and (1.39), lead to

$$R = R_{\parallel} = R_{\perp} = \left(\frac{n_t - n_i}{n_t + n_i}\right)^2 \tag{1.58}$$

and

$$T = T_{\parallel} = T_{\perp} = \frac{4n_t n_i}{(n_t + n_i)^2} \tag{1.59}$$

Thus 4% of the light incident normally on an air–glass ($n_g = 1.5$) interface will be reflected back, whether internally, $n_i > n_t$, or externally, , $n_i < n_t$. This will be of concern to anyone who is working with a complicated lens system, which might have 10 or 20 such air–glass boundaries.

Problems 1.1

We can define the deviation angle θ_d for refraction as the angle between the direction of the incident ray and the direction of the transmitted ray. What is the deviation angle for a light beam incident from air on a sheet of glass (n = 1.6) at an angle of 50°?

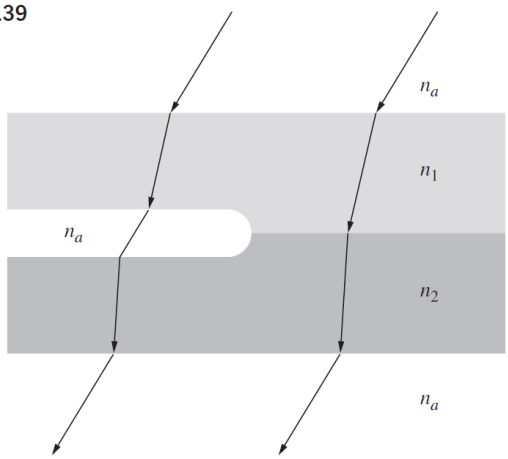
Problems 1.2

A beam of light is directed toward the water surface (n = 1.33) from below. The beam is incident at the water- air interface at 20°. At what angle will it emerge into the air?

Problems 1.3

Show that the two rays that enter the system in Fig. P.4.39 parallel to each other emerge from it being parallel. (start from Eq.1.31)

Figure P.4.39



Problems 1.4

A linearly polarized lightwave moving through air impinges at 20° on a plate of glass (n = 1.62) such that its electric vector is perpendicular to the plane of incidence. Compute the amplitude reflection and transmission coefficients at this interface.

References:

1- Principles of optics- Max Born

2- Optics,-Eugene-Hecht

3- Optics and photonics an introduction