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College: College of Science

Department: Physics

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Stage: Third stage

Subject: Optics 1

Lecture title: Lenses

2.1 Lenses

The lens is the most widely used optical device. **A lens is a refracting device that reconfigures a transmitted energy distribution**. That much is true whether we are dealing with UV, lightwaves, IR, microwaves, radiowaves, or even sound waves. In just the reverse, it's frequently necessary to collect incoming parallel rays and bring them together at a point, thereby focusing the energy, as is done with a burning-glass or a telescope lens.

2.1.1 Aspherical Surfaces

To see how a lens works, imagine that we interpose in the path of a wave a transparent substance in which the wave's speed is different than it was initially. Figure 2.1a presents a crosssectional view of a diverging spherical wave traveling in an incident medium of index n_i impinging on the curved interface of a transmitting medium of index n_t. When n_t is greater than n_i, the wave slows upon entering the new substance. The central area of the wavefront travels more slowly than its outer extremities, which are still moving quickly through the incident medium. These extremities overtake the midregion, continuously flattening the wavefront. If the interface is properly configured, the spherical wavefront bends into a plane wave. The alternative ray representation is shown in Fig. 2.1b; the rays simply bend toward the local normal upon entering the more dense medium, and if the surface configuration is just right, the rays emerge parallel. To find the required shape of the interface, refer to Fig. 2.1c, wherein point-A can lie anywhere on the boundary. A little spherical surface of constant phase emitted from S must evolve into a flat surface of constant phase at $\overline{D\acute{D}}$. Whatever path the light takes from S to $\overline{D\acute{D}}$, it must always be the same number of wavelengths long, so that the disturbance begins and ends in-phase.

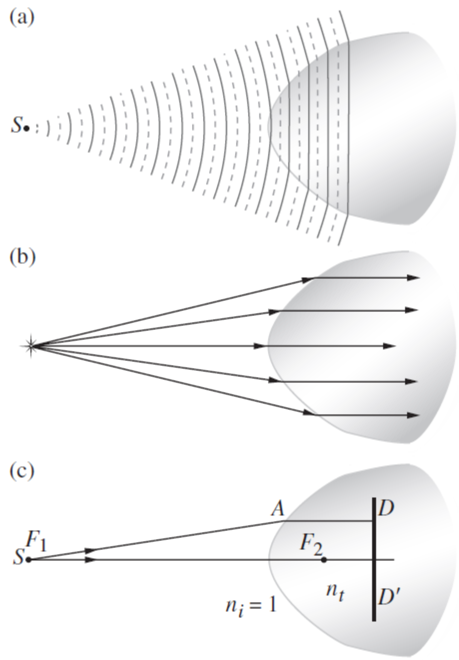


Figure 2. 1. A hyperbolic interface between air and glass. (a) The wavefronts bend and straighten out. (b) The rays become parallel. (c) The hyperbola is such that the optical path from S to A to D is the same no matter where A is.

Radiant energy leaving S as a single wavefront must arrive at the plane $\overline{D\acute{D}}$, having traveled for the same amount of time to get there, no matter what the actual route taken by any particular ray. In other words, $\overline{F_1A}/\lambda_i$ (the number of wavelengths along the arbitrary ray from F_1 to A plus $\overline{AD}/\lambda_t$ (the number of wavelengths along the ray from A to D) must be constant regardless of where on then interface A happens to be. Now, adding these and multiplying by λ_0, yields

$$n_i(\overline{F_1A}) + n_t(\overline{AD}) = \text{constant} \qquad (2.1)$$

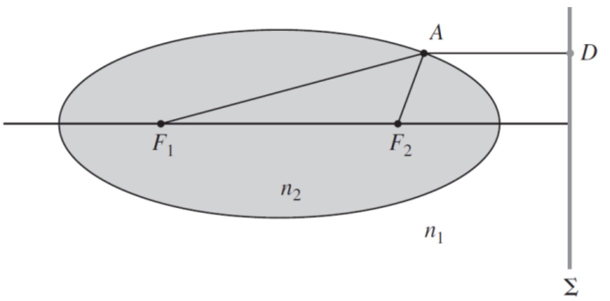
Each term on the left is the length traveled in a medium multiplied by the index of that medium, and, of course, each represents the optical path length, OPL, traversed. The optical path lengths from S to $\overline{D\acute{D}}$, are all equal. If Eq. (2.1) is divided by c, the first term becomes the time it takes light to travel from S to A and the second term, the time from A to D; the right side remains constant (not the same constant, but constant). Equation (2.1) is equivalent to saying that all paths from S to $\overline{D\acute{D}}$ must take the same amount of time to traverse. Let's return to finding the shape of the interface. Divide Eq. (2.1) by n_i, and it becomes

$$\overline{F_1A} + \left(\frac{n_t}{n_i}\right)\overline{AD} = \text{constant} \qquad (2.2)$$

This is the equation of a hyperbola in which the eccentricity (e), which measures the bending of the curve, is given by $(n_t/n_i > 1)$; that is, $e = n_{ti} > 1$. The greater the eccentricity, the flatter the hyperbola (the larger the difference in the indices, the less the surface need be curved).

Example 2.1

Use the figure to show that if a point source is placed at the focus F1 of the ellipsoid, plane waves will emerge from the far side. Remember that the defining requirement for an ellipse is that the net distance from one focus to the curve and back to the other focus is constant.



The OPL from F1 to D on Σ must be constant:

$$n_2(\overline{F_1A}) + n_1(\overline{AD}) = C \text{ and } (\overline{F_1A}) + (\overline{AD})n_{12} = C/n_2 = \acute{C}$$

if Σ corresponding to the directrix of ellipse,

$$(\overline{F_2A}) = e(\overline{AD}) \text{ where } e \text{ is the eccentricity;}$$

if $n_{12} = e$

we get $(\overline{F_1A}) + (\overline{F_2A}) = \acute{C}$

One of the first people to suggest using conic sections as surfaces for lenses and mirrors was Johann Kepler (1611), but he wasn't able to go very far with the idea without Snell's Law. Once that relationship was discovered, Descartes (1637), using his invention of Analytic Geometry, could develop the theoretical foundations of the optics of aspherical surfaces.

In Fig. 2.2a diverging incident spherical waves are made into plane waves at the first interface. These plane waves within the lens strike the back face perpendicularly and emerge unaltered: $\theta_i = 0$ and $\theta_t = 0$. Because the rays are reversible, plane waves incoming from the right will converge to point-F1, which is known as the focal point of the lens. Exposed on its flat face to the parallel rays from the Sun, our rather sophisticated lens would serve nicely as a burning-glass. In Fig. 2.2b, the plane waves within the lens are made to converge toward the axis by bending at the second interface. Both of these lenses are thicker at their midpoints than at their edges and are therefore said to be **convex** (from the Latin convexus, meaning arched). Each lens causes the incoming beam to converge somewhat, to bend a bit more toward the central axis; therefore, they are referred to as **converging lenses**. In contrast, a **concave** lens (from the Latin concavus, meaning hollow—and most easily remembered because it contains the word cave) is thinner in the middle than at the edges, as is evident in Fig. 2.2c. It causes the rays that enter as a parallel bundle to diverge. All such devices that turn rays outward away from the central axis (and in so doing add divergence to the beam) are called **diverging lenses**. In Fig. 2.2c, parallel rays enter from the left and, on emerging, seem to diverge from F_2; still, that point is taken as a focal point. ***When a parallel bundle of rays passes through a converging lens, the point to which it converges (or when passing through a diverging lens, the point from which it diverges) is a focal point of the lens***.

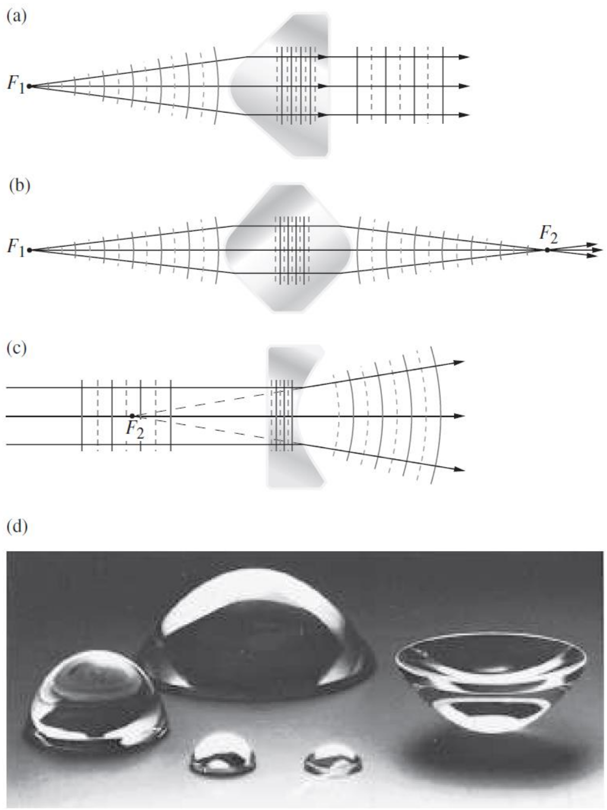


Figure 2. 2(a), (b), and (c) Several hyperbolic lenses seen in cross section. (d) A selection of aspherical lenses.

If a point source is positioned on the central or optical axis at the point-F_1 in front of the lens in Fig. 2.2b, rays will converge to the conjugate point-F_2. the conjugate point-F_2. A luminous image of the source would appear on a screen placed at F_2, an image that is therefore said to be **real**. On the other hand, in Fig. 2.2c the point source is at infinity, and the rays emerging from the system this time are diverging. They appear to come from a point-F_2, but no actual luminous image would appear on a screen at that location. The image here is spoken of as **virtual**, as is the familiar image generated by a plane mirror.

2.1.2 Refraction at Spherical Surfaces

The vast majority of quality lenses in use today have surfaces that are segments of spheres. Our intent here is to establish techniques for using such surfaces to simultaneously image a great many object points in light composed of a broad range of frequencies. Image errors, known as **aberrations**, will occur, but it is possible with the present technology to construct high-quality spherical lens systems whose aberrations are so well controlled that image fidelity is limited only by diffraction.

Figure 2.3 depicts a wave from the point source S impinging on a spherical interface of radius R Rcentered at C. The point- V is called the **vertex** of the surface. The length $s_0 = \overline{SV}$ is known as the **object distance**. The ray $\overline{SA}$ will be refracted at the interface toward the local normal ($n_2 > n_1$) and therefore toward the central or **optical axis**. Assume that at some point- P the ray will cross the axis, as will all other rays incident at the same angle θ_i (Fig. 2.4). The length $s_i = \overline{VP}$ is the image distance. Fermat's Principle maintains that the optical path length OPL will be stationary; that is, its derivative with respect to the position variable will be zero. For the ray in question,

$$OPL = n_1 l_0 + n_2 l_i \tag{2.3}$$

Using the law of cosines in triangles SAC and ACP along with the fact that

$cos\varphi = -\cos(180 - \varphi)$, we get

$$l_0 = [R^2 + (s_0 + R)^2 - 2R(s_0 + R)cos\varphi]^{1/2}$$

and
$$l_i = [R^2 + (s_i + R)^2 - 2R(s_i + R)cos\varphi]^{1/2}$$

The OPL can be rewritten as

$$OPL = n_1[R^2 + (s_0 + R)^2 - 2R(s_0 + R)cos\varphi]^{1/2} + n_2[R^2 + (s_i + R)^2 - 2R(s_i + R)cos\varphi]^{1/2}$$

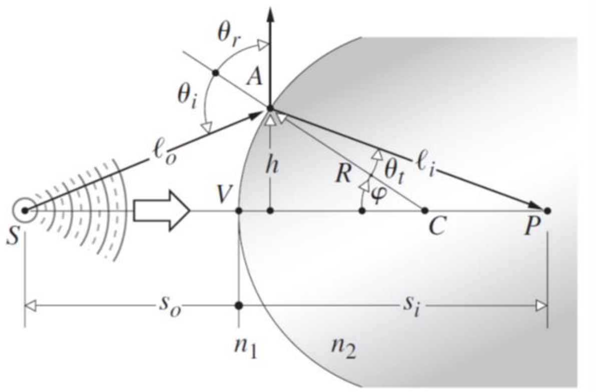


Figure 2. 3. Refraction at a spherical interface.

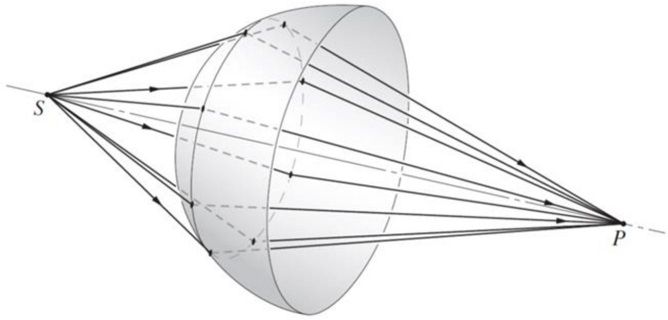


Figure 2. 4. Rays incident at the same angle.

All the quantities in the diagram (s_i, s_0, R, etc.) are positive numbers, and these form the basis of a *sign convention* that is gradually unfolding and to which we shall return time and again. Inasmuch as the point-A moves at the end of a fixed radius (i.e., R= constant), φ is the position variable, and thus setting $d(OPL)\ /d\varphi = 0$, via Fermat's Principle we have

$$\frac{n_1 R(s_0+R)sin\varphi}{2l_0} - \frac{n_2 R(s_i+R)sin\varphi}{2l_i} = 0 \qquad (2.4)$$

From which it follow that

$$\frac{n_1}{l_0} + \frac{n_2}{l_i} = \frac{1}{R}\left(\frac{n_2 s_i}{l_i} - \frac{n_1 s_0}{l_{i0}}\right) \qquad (2.5)$$

Although this expression is exact, it is rather complicated. If A is moved to a new location by changing φ, the new ray will not intercept the optical axis at P. The approximations that are used to represent l_0 and l_i and thereby simplify Eq. (2.5), are crucial in all that is to follow. Recall that

$$cos\varphi = 1 - \frac{\varphi^2}{2!} + \frac{\varphi^4}{4!} + \frac{\varphi^6}{6!} + \cdots \qquad (2.6)$$

and
$$sin\varphi = \varphi - \frac{\varphi^3}{3!} + \frac{\varphi^5}{5!} + \frac{\varphi^7}{7!} + \cdots \qquad (2.7)$$

If we assume small values of φ (i.e., A close to V), $cos\varphi \approx 1$. Consequently, the expressions for l_0 and l_i yield $l_0 \approx s_0$ and $l_i \approx s_i$ and to that approximation.

$$\frac{n_1}{s_0} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R} \qquad (2.8)$$

This approximation delineates the domain of what is called first-order theory; we'll examine *third-order theory*. Rays that arrive at shallow angles with respect to the optical axis (such that φ and h are appropriately small) are known as **paraxial rays**. *The emerging wavefront segment corresponding to these paraxial rays is essentially spherical and will form a "perfect" image at its centre P located at s_i.* Notice that Eq. (2.8) is independent of the location of A over a small area about the symmetry axis, namely, the *paraxial region*.

Gauss, in 1841, was the first to give a systematic exposition of the formation of images under the above approximation, and the result is variously known as *first-order, paraxial*, or **Gaussian Optics**. It soon became the basic theoretical tool by which lenses would be designed for several decades to come. If the optical system is well corrected, an incident spherical wave will emerge in a form very closely resembling a spherical wave. Consequently, as the perfection of the system increases, it more closely approaches first-order theory. Deviations from that of paraxial analysis will provide a convenient measure of the quality of an actual optical device.

If point-F_o in Fig. 2.5 is imaged at infinity ($s_i = \infty$), we have

$$\frac{n_1}{s_0} + \frac{n_2}{\infty} = \frac{n_2 - n_1}{R}$$

That special object distance is defined as the **first focal length** or the **object focal length**, $s_0 = f_0$, so that

$$f_0 = \frac{n_1}{n_2 - n_1} R \qquad (2.9)$$

Point-F_o is known as the **first** or **object focus**. Similarly, the second or image focus is the axial point-F_i, where the image is formed when $s_0 = \infty$; that is,

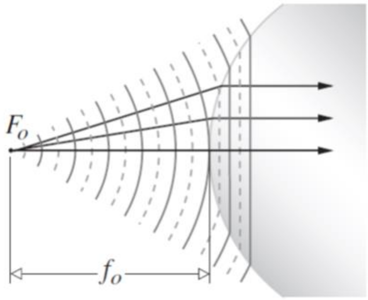


Figure 2. 5. Plane waves propagating beyond a spherical interface—the object focus.

$$\frac{n_1}{\infty} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$$

Defining the **second** or **image focal length** F_i as equal to s_i in this special case (Fig.2.5), we have

$$f_i = \frac{n_2}{n_2 - n_1} R \qquad (2.10)$$

Recall that an image is virtual when the rays diverge from it (Fig. 2.6). Analogously, ***an object is virtual when the rays converge toward it*** (Fig. 2.7). Observe that the virtual object is now on the right-hand side of the vertex, and therefore s_0 will be a negative quantity. Moreover, the surface is concave, and its radius will also be negative, as required by Eq. (2.9), since f_0 would be negative. In the same way, the virtual image distance appearing to the left of V is negative.

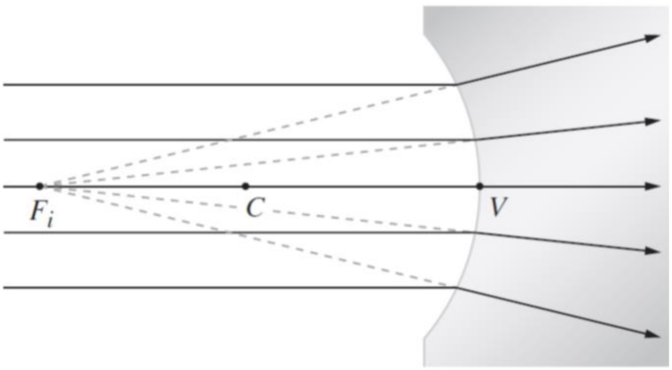


Figure 2. 6. A virtual image point.

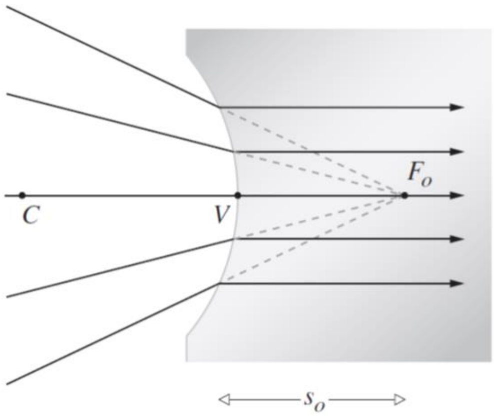
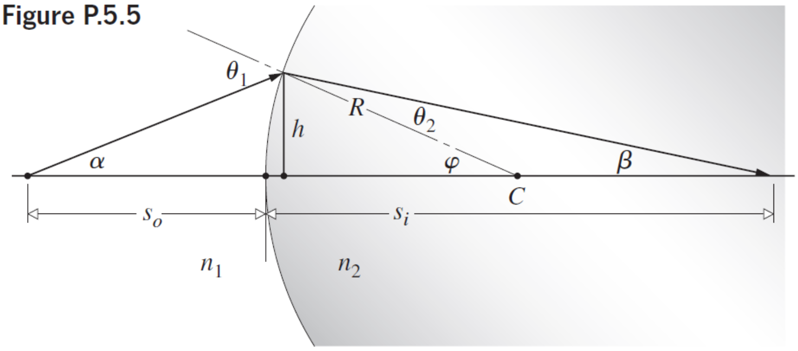


Figure 2. 7. A virtual object point.

Example

Making use of Fig. P.5.5, Snell's Law, and the fact that in the paraxial region $\alpha = h/s_0$, $\varphi \approx h/R$, and $\beta \approx h/s_i$, derive Eq. (2.8).

Figure P.5.5



Solution

$$\theta_2 + (180° - \varphi) + \beta = 180°$$

$$\theta_2 = \varphi - \beta$$

$$sin\theta_2 = sin(\varphi - \beta)$$

$$= sin\varphi \cos(-\beta) + cos\varphi sin(-\beta)$$

$$\approx sin\varphi - sin\beta$$

$$h/R - h/s_i$$

$$(180° - \theta_1) + \varphi - \alpha = 180°$$

$$\theta_1 = \varphi + \alpha$$

$$sin\theta_1 = \sin(\varphi + \alpha)$$

$$= \sin\varphi cos\alpha + \cos\varphi sin\alpha$$

$$\approx h/R + h/s$$

$n_1 sin\theta_1 = n_2 sin\theta_2;\ n_1(h/R + h/s_0) = n_2\,(h/R + h/s_i)$

$$n_1/s_0 + n_2/s_i = (n_2 - n_1)/R$$

Example

A long horizontal flint-glass ($n_g = 1.800$) cylinder is 20.0 cm in diameter and has a convex hemispherical left end ground and polished onto it. The device is immersed in ethyl alcohol ($n_a = 1.361$) and a tiny LED is located on the central axis in the liquid 80.0 cm to the left of the vertex of the hemisphere. Locate the image of the LED. What would happen if the alcohol was replaced by air?

$$\frac{n_1}{s_0} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$$

Here $n_1 = 1.361, n_2 = 1.800,\ s_0 = +80\ cm, and\ R = 10\ cm$ We can work the problem in centimetres, whereupon the equation becomes

$$\frac{1.361}{80} + \frac{1.8}{s_i} = \frac{1.8 - 1.361}{10}$$

$$\frac{1.8}{s_i} = \frac{0.439}{10} - \frac{1.361}{80}$$

$$s_i = 66.9\ cm$$

With the alcohol in place the image is within the glass, 66.9 cm to the right of the vertex ($s_i > 0$). Removing the liquid,

$$\frac{1}{80} + \frac{1.8}{s_i} = \frac{0.8}{10}$$

$$s_i = 26.7\ cm$$

References:

1- Principles of optics- Max Born

2- Optics,-Eugene-Hecht

3- Optics and photonics an introduction