First, we write expressions for the position of each vehicle as a function of time. It is convenient to choose the position of the billboard as the origin and to set $t_B = 0$ as the time the trooper begins moving. At that instant, the car has already traveled a distance of (45 m) from the billboard because it has traveled at a constant speed of $v_x = 45$ m/s for 1 s. Therefore, the initial position of the speeding car is $x_B = 45$ m.

Apply equation (2.3) to give the car's position at any time (t):

$$x_{car} = x_B + v_{x\,car}\, t$$

At ($t = 0$), this expression gives the car's correct initial position when the trooper begins to move: $x_{car} = x_B = 45$ **m**

The trooper starts from rest at $t_B = 0$ and accelerates at $a_x = 3$ m/s^2 away from the origin. Use equation (2.13) to give his position at any time (t):

$$x_f = x_i + v_{xi} t + \frac{1}{2} a_x t^2$$

$$x_{trooper} = 0 + 0(t) + \frac{1}{2} a_x t^2 = \frac{1}{2} a_x t^2$$

Set the positions of the car and trooper equal to represent the trooper overtaking the car at position (C): $x_{trooper} = x_{car}$

$$\frac{1}{2} a_x t^2 = x_B + v_{x\,car}\, t$$

Rearrange: $$\frac{1}{2} a_x t^2 - v_{x\,car}\, t - x_B = 0$$

$$1.5\, t^2 - 45t - 45 = 0$$

$$t = 31 \text{ s.}$$

2.5 Freely Falling Objects

- A freely falling object is (any object moving freely under the influence of gravity alone, regardless of its initial motion).
- We denote the magnitude of the *free-fall acceleration* by the symbol (g). The value of (g) decreases with increasing altitude

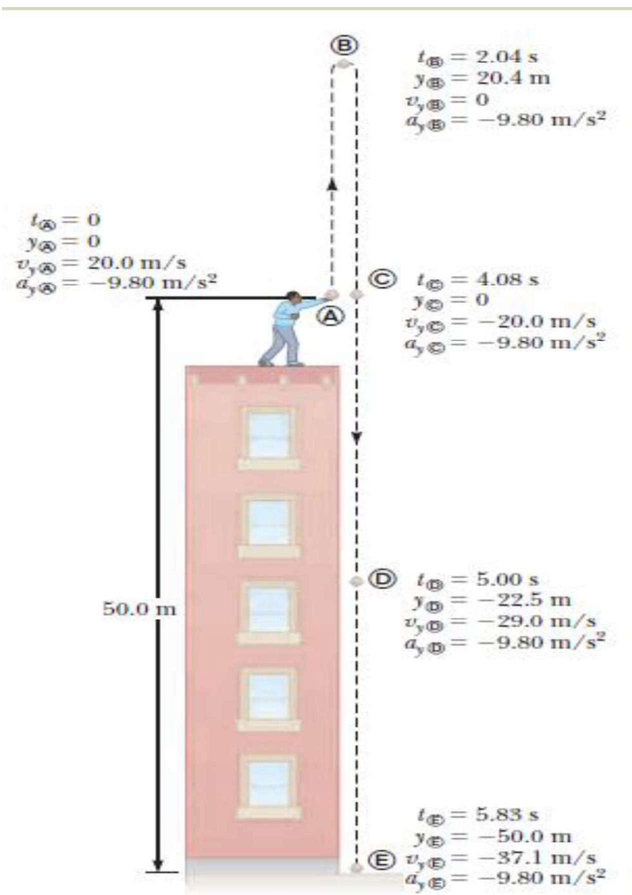
above the Earth's surface. Furthermore, slight variations in (g) occur with changes in latitude. At the Earth's surface, the value of (g) is approximately equal 9.80 m/s^2.

- Note that the motion is in the vertical direction (the *y* direction) rather than in the horizontal direction (*x*)
- We choose (g= - 9.80 m/s^2), where the negative sign means that the acceleration of a freely falling object is downward.

Example (2.4):

A stone thrown from the top of a building is given an initial velocity of 20 m/s straight upward. The stone is launched 50 m above the ground, and the stone just misses the edge of the roof on its way down as shown in the figure below.

(A) Using t_A= 0 as the time the stone leaves the thrower's hand at position A. Determine the time at which the stone reaches its maximum height?



Solution:

Use equation (2.10) to calculate the time at which the stone reaches its maximum height:

$$v_{yf} = v_{yi} + a_y t ,$$

$$t = \frac{v_{yf} - v_{yi}}{ay} = t_B$$

$$t = \frac{0-20}{-9.8} = 2.04 \text{ s}$$

(B) Find the maximum height of the stone?

As in part (A), choose the initial and final points at the beginning and the end of the upward flight. Set (y_A= 0) and substitute the time from part (A) into equation (2.13) to find the maximum height:

$$y_{max} = y_B = y_A + v_{yA}t + \frac{1}{2}a_y t^2$$

$$y_B = 0 + (20)(2.04) + \frac{1}{2}(-9.8)(2.04)^2 = 20.4 \text{ m}$$

(C) Determine the velocity of the stone when it returns to the height from which it was thrown?

Choose the initial point where the stone is launched and the final point when it passes this position coming down.

Substitute known values into equation (2.14):

$$v_{yC}^2 = v_{yA}^2 + 2a_y(y_C - y_A)$$

$$v_{yC}^2 = (20)^2 + 2(-9.8)(0 - 0) = 400 \text{ m}^2/\text{s}^2$$

$$v_{yC} = -20 \text{ m/s.}$$

When taking the square root, we could choose either a positive or a negative root. We choose the negative root because we know that the stone is moving downward at point C. The velocity of the stone when it arrives back at its original height is equal in magnitude to its initial velocity but is opposite in direction.

(D) Find the velocity and position of the stone at $t = 5$ s?

Choose the initial point just after the throw and the final point (5 s.) later.

Calculate the velocity at (D) from equation (2.10):

$$v_{yD} = v_{yA} + a_y t = 20 + (-9.8)(5) = -29 \text{ m/s}$$

Use equation (2.13) to find the position of the stone at $t_D = 5$ s:

$$y_D = y_A + v_{yA}t + \frac{1}{2}a_y t^2$$

$$= 0 + (20)(5) + \frac{1}{2}(-9.8)(5)^2$$

$$= -22.5 \text{ m}$$

Chapter 3

Vectors

3.1 Vector and Scalar Quantities

- A **scalar quantity** is completely specified by a single value with an appropriate unit and has no direction.

Examples of scalar quantities are temperature, volume, mass, speed, and time intervals.

- A **vector quantity** is completely specified by a number with an appropriate unit plus a direction.

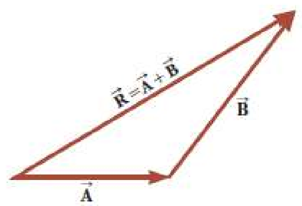
Examples of vector quantity are displacement and velocity.

3.2 Some Properties of Vectors

Adding Vectors

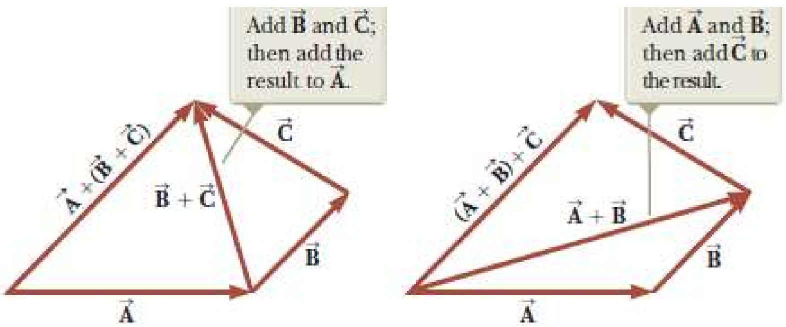
When two vectors (vector $\vec{A}$ and vector $\vec{B}$) are added, the sum is independent of the order of the addition. This property is known as the **(commutative law of addition):**

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$



Another property is called the **associative law of addition:**

$$\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$$



- A vector quantity has both magnitude and direction and also obeys the laws of vector addition.

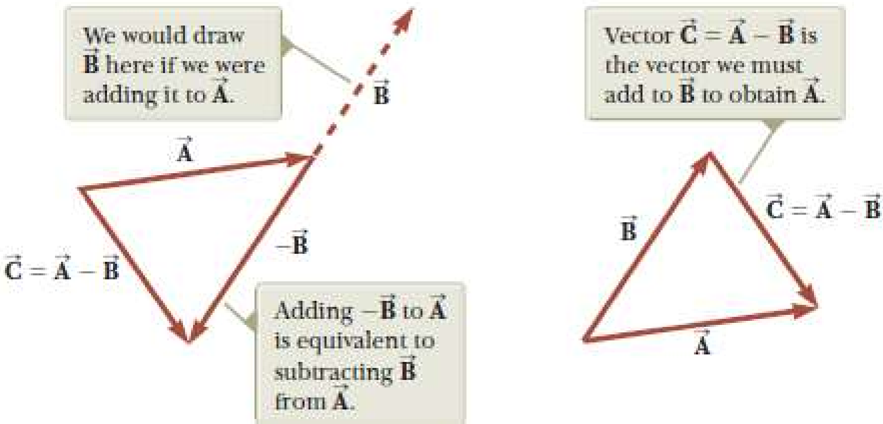
Negative of a Vector

The negative of the vector $\vec{A}$ is defined as the vector that when added to $\vec{A}$ gives zero for the vector sum. That is, $\vec{A} + (-\vec{A}) = 0$. The vectors $\vec{A}$ and (-$\vec{A}$) have the same magnitude but point in opposite directions.

Subtracting Vectors

The operation of vector subtraction makes use of the definition of the negative of a vector. We define the operation ($\vec{A} - \vec{B}$) as vector $(-\vec{B})$ added to vector ($\vec{A}$): $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

The geometric construction for subtracting two vectors in this way is illustrated in the figure below:



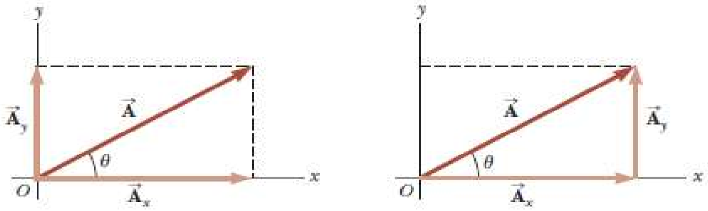
3.3 Components of a Vector and Unit Vectors

Components of a Vector

Any vector can be completely described by its components.

Consider a vector $(\vec{A})$ lying in the (xy plane) and making an angle (θ) with the positive (x-axis) as shown in the figure below. This vector can be expressed as the sum of two other *component* vectors ($\vec{A}_x$), which is parallel to the (x-axis), and ($\vec{A}_y$), which is parallel to the (y-axis).

$$\vec{A} = \vec{A}_x + \vec{A}_y$$



From the figure and the definition of sine and cosine, we see that

($\cos\theta = A_x / A$) and that ($\sin\theta = A_y / A$). Hence, the components of $\vec{A}$

$$A_x = A\cos\theta \text{ and } A_y = A\sin\theta$$

The magnitude and direction of ($\vec{A}$) are related to its components through the expressions:

$$A = \sqrt{A_x^2 + A_y^2} \qquad \text{(magnitude of } \vec{A}\text{)}$$

$$\theta = \tan^{-1}\left(\frac{Ay}{Ax}\right) \qquad \text{(direction of } \vec{A}\text{)}$$

Unit Vectors

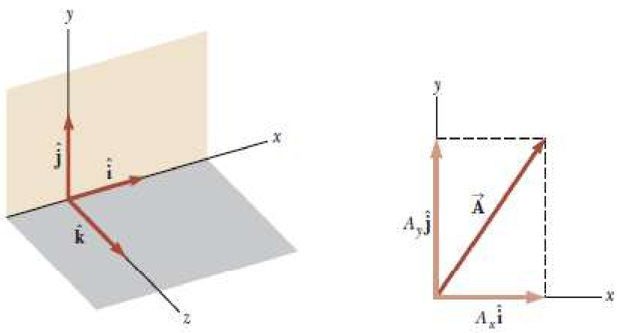
A **unit vector** is (a dimensionless vector having a magnitude of exactly one, and are used to specify a given direction).

We shall use the symbols ($\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$) to represent unit vectors pointing in the positive (*x, y,* and *z*) directions, respectively.

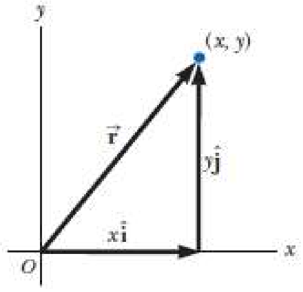
The magnitude of each unit vector equals 1; that is, $|\hat{\imath}| = |\hat{\jmath}| = |\hat{k}| = 1$

$\vec{A}_x = \hat{\imath}\, A_x$, $\vec{A}_y = \hat{\jmath}\, A_y$. Therefore, the unit-vector notation for the vector $\vec{A}$ is:

$$\vec{A} = \hat{\imath}\, A_x + \hat{\jmath}\, A_y$$



- Consider a point lying in the xy plane and having Cartesian coordinates (x, y) as in the figure below. The point can be specified by the **position vector** ($\vec{r}$) which in unit-vector form is given by:



$$\vec{r} = \hat{\imath}\, x + \hat{\jmath}\, y$$

- The resultant vector ($\vec{R} = \vec{A} + \vec{B}$) is:

$$\vec{R} = (\hat{\imath}\, A_x + \hat{\jmath}\, A_y) + (\hat{\imath}\, B_x + \hat{\jmath}\, B_y) \quad \text{or}$$

$$\vec{R} = \hat{\imath}\,(A_x + B_x) + \hat{\jmath}\,(A_y + B_y)$$

Because $\vec{R} = \hat{\imath}\, R_x + \hat{\jmath}\, R_y$, we see that the components of the resultant vector are: $R_x = (A_x + B_x)$ and $R_y = (A_y + B_y)$.

The magnitude of $\vec{R}$ and the angle it makes with the (x- axis) are obtained from its components using the relationships:

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2} \qquad \text{Magnitude of } \vec{R}$$

$$\tan\theta = \frac{R_y}{R_x} = \frac{A_y + B_y}{A_x + B_x} \qquad \text{Direction of } \vec{R}$$

- If $\vec{A}$ *and* $\vec{B}$ both have three components (x,y,z), they can be expressed in the form:

$$\vec{A} = \hat{\imath}\, A_x + \hat{\jmath}\, A_y + \hat{k}\, A_z$$

$$\vec{B} = \hat{\imath}\, B_x + \hat{\jmath}\, B_y + \hat{k}\, B_z$$

The sum of $\vec{A}$ *and* $\vec{B}$ is: $\vec{R} = \vec{A} + \vec{B}$ or

$$\vec{R} = \hat{\imath}\,(A_x + B_x) + \hat{\jmath}\,(A_y + B_y) + \hat{k}\,(A_z + B_z)$$

Example (3.1):

Find the sum of two displacement vectors $\vec{A}$ *and* $\vec{B}$ lying in the xy plane and given by: $\vec{A} = (\,2\hat{\imath} + 2\hat{\jmath})$ m and $\vec{B} = (\,2\hat{\imath} - 4\hat{\jmath})$ m.

Solution:

The resultant vector $\vec{R}$: $\vec{R} = \vec{A} + \vec{B} = \hat{\imath}\,(A_x + B_x) + \hat{\jmath}\,(A_y + B_y)$

$= \hat{\imath}\,(2+2) + \hat{\jmath}\,(2-4\,)$

The components of $\vec{R}$: $R_x = 4$ m and $R_y =$ -2 m

The magnitude of $\vec{R}$: $R = \sqrt{R_x^2 + R_y^2} = \sqrt{(4)^2 + (-2)^2} = \sqrt{20} = 4.5$ m

The direction of $\vec{R}$: $\tan\theta = \frac{R_y}{R_x} = \frac{-2}{4} =$ - 0.5

$$\theta = -27°$$

This answer is correct if we interpret it to mean 27° clockwise from the (x – axis).

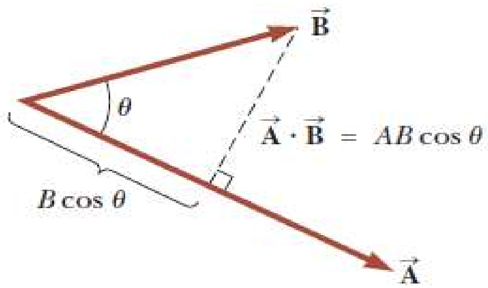
3.4 Scalar Product

The scalar product of any two vectors $\vec{A}$ *and* $\vec{B}$ is defined as (a scalar quantity equal to the product of the magnitudes of the two vectors and the cosine of the angle θ between them):

$$\vec{A} \bullet \vec{B} = AB\cos\theta$$

We write scalar product of vectors $\vec{A}$ *and* $\vec{B}$ as $\vec{A} \bullet \vec{B}$ (Because of the dot symbol, the scalar product is often called the **dot product**).

- The scalar product ($\vec{A} \bullet \vec{B}$) equals the magnitude of $\vec{A}$ multiplied by the projection of $\vec{B}$ onto $\vec{A}$:($B\cos\theta$) as shown in the figure below.



Properties of the scalar product:

1. Scalar product is **commutative**:
$$\vec{A} \bullet \vec{B} = \vec{B} \bullet \vec{A}$$
2. Scalar product obeys the **distributive law of multiplication**:
$$\vec{A} \bullet (\vec{B} + \vec{C}) = \vec{A} \bullet \vec{B} + \vec{A} \bullet \vec{C}$$
3. If $\vec{A}$ is perpendicular to $\vec{B}$ ($\theta = 90°$), then $\vec{A} \bullet \vec{B} = 0$.
4. If $\vec{A}$ is parallel to $\vec{B}$ ($\theta = 0°$), then $\vec{A} \bullet \vec{B} = AB$.
5. If $\theta = 180°$, then $\vec{A} \bullet \vec{B} = -AB$.
6. The scalar product is negative when ($90° < \theta \leq 180°$).

$\hat{\imath} \bullet \hat{\imath} = \hat{\jmath} \bullet \hat{\jmath} = \hat{k} \bullet \hat{k} = 1$

$\hat{\imath} \bullet \hat{\jmath} = \hat{\imath} \bullet \hat{k} = \hat{\jmath} \bullet \hat{k} = 0$

Two vectors $\vec{A}$ and $\vec{B}$ can be expressed in unit vector form as:

$$\vec{A} = \hat{\imath}A_x + \hat{\jmath}A_y + \hat{k}A_z$$

$$\vec{B} = \hat{\imath}B_x + \hat{\jmath}B_y + \hat{k}B_z$$

so $\vec{A} \bullet \vec{B} = A_xB_x + A_yB_y + A_zB_z$ *and* $\vec{A} \bullet \vec{A} = A^2$.

Example (3.2):

The vectors $\vec{A}$ and $\vec{B}$ are given by: $\vec{A} = 2\hat{\imath} + 3\hat{\jmath}$ and $\vec{B} = -\hat{\imath} + 2\hat{\jmath}$

(A) Determine the scalar product $\vec{A} \bullet \vec{B}$

(B) Find the angle (θ) between $\vec{A}$ and $\vec{B}$

Solution:

(A) $\vec{A} \bullet \vec{B} = (2\hat{\imath} + 3\hat{\jmath}) \bullet (-\hat{\imath} + 2\hat{\jmath})$

$= -2\hat{\imath} \bullet \hat{\imath} + 2\,\hat{\imath} \bullet 2\hat{\jmath} - 3\hat{\jmath} \bullet \hat{\imath} + 3\hat{\jmath} \bullet 2\hat{\jmath}$

$= -2 + 0 - 0 + 6 = 4$

(B) The magnitude of $\vec{A}$: $A = \sqrt{A_x^2 + A_y^2} = \sqrt{(2)^2 + (3)^2} = \sqrt{13}$

The magnitude of $\vec{B}$: $B = \sqrt{B_x^2 + B_y^2} = \sqrt{(-1)^2 + (2)^2} = \sqrt{5}$

$$\cos\theta = \frac{\vec{A} \bullet \vec{B}}{AB} = \frac{4}{\sqrt{13}\sqrt{5}} = \frac{4}{\sqrt{65}}$$

$$\theta = \cos^{-1}\frac{4}{\sqrt{65}} = 60.3°$$