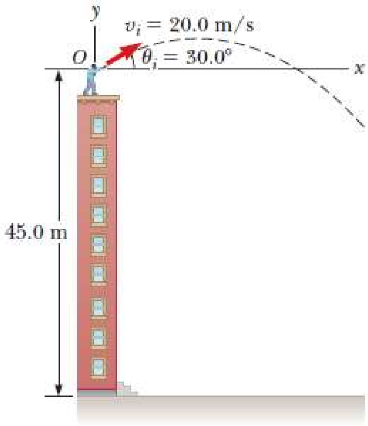


Solution: (A) We have the information

$x_i = y_i = 0$, $y_f = -45$ m, $a_y = -$ g , and $v_i = 20$ m/s

The initial x and y components of the stone's velocity:

$v_{xi} = v_i \cos \Theta i = 20 \cos 30° = 17.3$ m/s

$v_{yi} = v_i \sin \Theta i = 20 \sin 30° = 10$ m/s

The vertical position of the stone from the vertical component:

$y_f = y_i + v_{yi} t + \frac{1}{2} a_y t^2$

$-45 = 0 + 10\, t + \frac{1}{2}(-9.8)\, t^2$

$t = 4.22$ s

(B) What is the speed of the stone just before it strikes the ground?

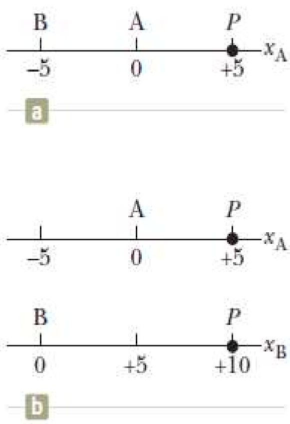
$v_{yf} = v_{yi} + a_y\, t$

$= 10 + (-9.8)(4.22) = -31.3$ m/s

4.4 Relative Velocity

We describe how observations made by different observers in different frames of reference are related to one another. A frame of reference can be described by a Cartesian coordinate system for which an observer is at rest with respect to the origin.

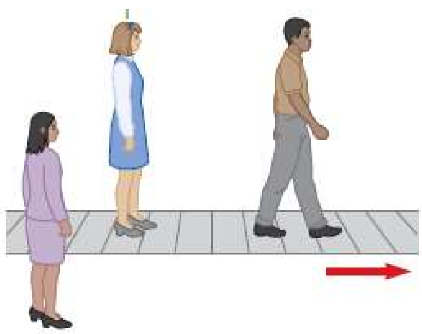
Consider the two observers A and B along the number line in figure a.



Observer A is located at the origin of a one-dimensional x_A axis, while observer B is at the position $x_A = -5$. We denote the position variable as x_A because observer A is at the origin of this axis. Both observers measure the position of point P, which is located at $x_A = +5$. Suppose observer B decides that he is located at the origin of an x_B axis as in Figure b. Notice that the two observers disagree on the value of the position of point *P*. Observer A claims point *P* is located at a position with a value of +5, whereas observer B claims it is located at a position with a value of +10. Both observers are correct, even though they make different measurements. Their measurements differ because they are making the measurement from different frames of reference.

Imagine now that observer B in (figure b) is moving to the right along the x_B axis. Now the two measurements are even more different. Observer A claims point *P* remains at rest at a position with a value of +5, whereas observer B claims the position of *P* continuously changes with time, even passing him and moving behind him! Again, both observers are correct, with the difference in their measurements arising from their different frames of reference.

We explore this phenomenon further by considering two observers watching a man walking on a moving beltway at an airport in figure below. The woman standing on the moving beltway sees the man moving at a normal walking speed. The woman observing from the stationary floor sees the man moving with a higher speed because the beltway speed combines with his walking speed. Both observers look at the same man and arrive at different values for his speed. Both are correct; the difference in their measurements results from the **relative velocity** of their frames of reference.



In a more general situation, consider a particle located at point *P* in the figure. Imagine that the motion of this particle is being described by two observers, observer A in a reference frame S_A fixed relative to the Earth and a second observer B in a reference frame S_B moving to the right relative to S_A (and therefore relative to the Earth) with a constant velocity $\vec{v}_{BA}$. In this discussion of relative velocity, we use a double-subscript notation; the first subscript represents what is being observed, and the second represents who is doing the observing. Therefore, the notation $\vec{v}_{BA}$ means the velocity of observer B (and the attached frame S_B) as measured by observer A. With this notation, observer B measures A to be moving to the left with velocity ($\vec{v}_{AB} = -\vec{v}_{BA}$). For purposes of this discussion, let us place each observer at his respective origin. We define the time $t = 0$ as the instant at which the origins of the two reference frames coincide in space. Therefore, at time t, the origins of the reference frames will be separated by a distance ($v_{BA}\ t$). We label the position P of the particle relative to observer A with the position vector $\vec{\mathbf{r}}_{PA}$ and that relative to observer B with the position vector $\vec{\mathbf{r}}_{PB}$, both at time t. We see that the vectors $\vec{\mathbf{r}}_{PA}$ and $\vec{\mathbf{r}}_{PB}$ are related to each other through the expression:

$$\vec{\mathbf{r}}_{PA} = \vec{\mathbf{r}}_{PB} + \vec{v}_{BA}\, t \qquad (4.13)$$

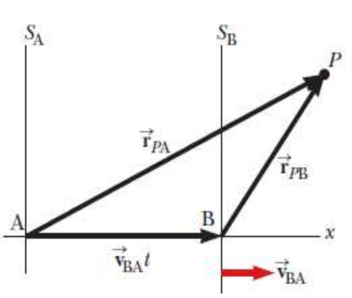
By differentiating (equation 4.13) with respect to time, noting that $\vec{v}_{BA}$ is constant, we obtain: $\frac{d\vec{\mathbf{r}}PA}{dt} = \frac{d\vec{\mathbf{r}}PB}{dt} + \vec{v}_{BA}$

$$\vec{\mathbf{u}}_{PA} = \vec{\mathbf{u}}_{PB} + \vec{v}_{BA} \qquad (4.14)$$

Where $\vec{\mathbf{u}}_{PA}$ is the velocity of the particle at P measured by observer A and $\vec{\mathbf{u}}_{PB}$ is its velocity measured by B. (We use the symbol $\vec{\mathbf{u}}$ for particle velocity rather than ($\vec{v}$), which we have already used for the relative velocity of two reference frames.) Equations 4.13 and 4.14 are known as **Galilean transformation equations.**

Although observers in two frames measure different velocities for the particle, they measure the *same acceleration* when $\vec{v}_{BA}$ is constant.

We can verify that by taking the time derivative of equation 4.14:



$$\frac{d\vec{\mathbf{u}}PA}{dt} = \frac{d\vec{\mathbf{u}}PB}{dt} + \frac{d\vec{\mathrm{v}}BA}{dt}$$

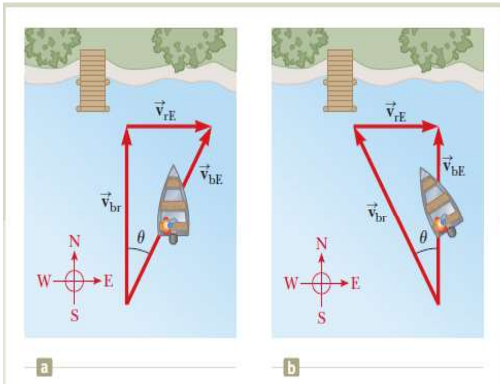
Because $\vec{\mathrm{v}}_{\mathrm{BA}}$ is constant, $\frac{d\vec{\mathrm{v}}\mathrm{BA}}{dt} = 0$.
Therefore, we conclude that ($\vec{\mathrm{a}}_{PA} = \vec{\mathrm{a}}_{PB}$). That is, the acceleration of the particle measured by an observer in one frame of reference is the same as that measured by any other observer moving with constant velocity relative to the first frame.

Example (4.4):

A boat crossing a wide river moves with a speed of 10 km/h relative to the water. The water in the river has a uniform speed of 5km/h due east relative to the Earth.

(A) If the boat heads due north, determine the velocity of the boat relative to an observer standing on either bank.

Solution:



We know $\vec{\mathrm{v}}_{\mathrm{br}}$, the velocity of the boat relative to the river, and $\vec{\mathrm{v}}_{\mathrm{rE}}$, the velocity of the river relative to the Earth.

What we must find is $\vec{\mathrm{v}}_{\mathrm{bE}}$, the velocity of the boat relative to the Earth. The relationship between these three quantities is $\vec{\mathrm{v}}_{\mathrm{bE}} = \vec{\mathrm{v}}_{\mathrm{br}} + \vec{\mathrm{v}}_{\mathrm{rE}}$.

The quantity $\vec{\mathrm{v}}_{\mathrm{br}}$ is due north; $\vec{\mathrm{v}}_{\mathrm{rE}}$ is due east; and the vector sum of the two, $\vec{\mathrm{v}}_{\mathrm{bE}}$, is at an angle θ as defined in the figure a.

$$v_{\mathrm{bE}} = \sqrt{v_{br}^2 + v_{rE}^2} = \sqrt{(10)^2 + (5)^2} = 11.2 \text{ km/h}$$

Find the direction of $\vec{\mathrm{v}}_{\mathrm{bE}}$: $\theta = \tan^{-1}\frac{v_{rE}}{v_{br}} = \tan^{-1}\frac{5}{10} = 26.6°$

The boat is moving at a speed of 11.2 km/h in the direction 26.6° east of north relative to the Earth.

Chapter 5

(Force and Motion)

The Laws of Motion

5.1 Newton's First Law of Motion:

Newton's First Law of Motion Sometimes called the (*law of inertia*). The term inertia is described as (the tendency of an object to resist changes in its motion). Another statement of Newton's first law is

(In the absence of external forces, an object at rest remains at rest and an object in motion continue in motion with a constant velocity in a straight line).

In other words, when no force acts on an object, the acceleration of the object is zero; the object is treated with the **particle in equilibrium** model. In this model, the net force on the object is zero:

$$\boldsymbol{\Sigma}\vec{\mathbf{F}} = \mathbf{0} \qquad (5.1)$$

- **Force**: From the first law, we can define **force** as that which causes a change in motion of an object.
- **Mass:** we can define mass is that property of an object that specifies how much resistance an object exhibits to changes in its velocity. Mass is a scalar quantity. The SI unit of mass is the kilogram. Mass should not be confused with weight. Mass and weight are two different quantities. The mass of an object is the same everywhere.
- **Weight:** The weight of an object is equal to the magnitude of the gravitational force exerted on the object and varies with location.

  For example, a person weighing (84 kg) on the Earth weighs only about (14 kg) on the Moon, that means (1/6) his weighs on the Earth.

5.2 Newton's Second Law

Newton's first law explains what happens to an object when no forces act on it: it either remains at rest or moves in a straight line with constant speed. Newton's second law answers the question of what happens to an object when one or more forces act on it.

- The acceleration of an object is directly proportional to the force acting on it: $\vec{\mathbf{F}} \propto \vec{\mathbf{a}}$
- The magnitude of the acceleration of an object is inversely proportional to its mass: $|\vec{\mathbf{a}}| \propto \mathbf{1}/\boldsymbol{m}$

Newton's second law: The acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass:

$$\vec{\mathbf{a}} \propto \frac{\Sigma \vec{\mathbf{F}}}{m}$$

If we choose a proportionality constant of 1, we can relate mass, acceleration, and force through the following mathematical statement of Newton's second law:

$$\boxed{\Sigma \vec{\mathbf{F}} = \boldsymbol{m}\, \vec{\mathbf{a}}} \qquad \text{Newton's second law} \qquad (5.2)$$

- The net force $(\Sigma \vec{\mathrm{F}})$ on an object is the vector sum of all forces acting on the object.
- The SI unit of force is the **newton** (N).
- The definition of the newton is: A force of 1 N is the force that, when acting on an object of mass 1 kg, produces an acceleration of 1 m/s^2.

$$1 \text{ N} = 1 \text{ kg . m/s}^2$$

5.3 The Gravitational Force and Weight

All objects are attracted to the Earth. The attractive force exerted by the Earth on an object is called the **gravitational force** $\vec{F}_g$. This force is directed toward the center of the Earth, and its magnitude is called the **weight** of the object.

A freely falling object experiences an acceleration ($\vec{g}$) acting toward the center of the Earth. Applying Newton's second law $\boldsymbol{\Sigma}\vec{F} = m\,\vec{a}$ to a freely falling object of mass *m*, with $\vec{a} = \vec{g}$ and $\boldsymbol{\Sigma}\vec{F} = \vec{F}_g$, gives

$$\vec{\mathbf{F}}_g = \boldsymbol{m}\,\vec{\mathbf{g}} \qquad (5.3)$$

- The weight of an object is equal to *mg*: $F = m\,g$
- Because it depends on *g*, weight varies with geographic location. Because *g* decreases with increasing distance from the center of the Earth, objects weigh less at higher altitudes than at sea level.

5.4 Newton's Third Law

When your finger pushes on the book, the book pushes back on your finger. This important principle is known as **Newton's third law:**

(If two objects interact, the force $\vec{F}_{12}$ exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force $\vec{F}_{21}$ exerted by object 2 on object 1):

$$\vec{\mathbf{F}}_{12} = -\,\vec{\mathbf{F}}_{21} \qquad (5.4)$$

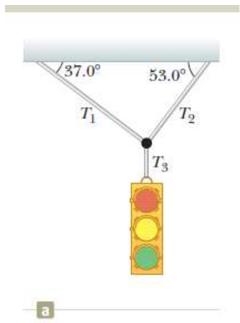
- The force that object 1 exerts on object 2 is popularly called the *action force,* and the force of object 2 on object 1 is called the *reaction force.*

- The action and reaction forces act on *different* objects and must be of the same type (gravitational, electrical, etc.).

Some Applications of Newton's laws:

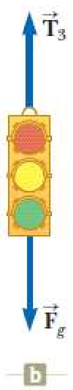
Example (5.1):

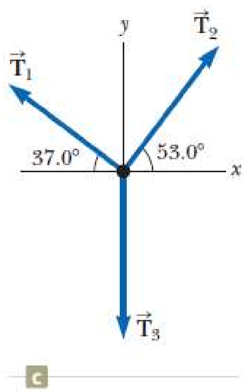
A traffic light weighing 122 N hangs from a cable tied to two other cables fastened to a support as in the figure a. The upper cables make angles of 37° and 53° with the horizontal. These upper cables are not as strong as the vertical cable and will break if the tension in them exceeds 100 N. Does the traffic light remain hanging in this situation, or will one of the cables break?



Solution:

We construct a diagram of the forces acting on the traffic light, shown in the figure b, and a free-body diagram for the knot that holds the three cables together, shown in the figure c. This knot is a convenient object to choose because all the forces of interest act along lines passing through the knot.





- Apply equation (5.1) for the traffic light in the y direction:

$$\sum F_y = 0 \rightarrow T_3 - F_g = 0$$

$$T_3 = F_g = 122 \text{ N}$$

Choose the coordinate axes as shown in the figure c and resolve the forces acting on the knot into their components:

Force	x Component	y Component
$\vec{\mathbf{T}}_1$	$-T_1 \cos 37.0°$	$T_1 \sin 37.0°$
$\vec{\mathbf{T}}_2$	$T_2 \cos 53.0°$	$T_2 \sin 53.0°$
$\vec{\mathbf{T}}_3$	0	-122 N

Apply the particle in equilibrium model to the knot:

$$(1)\ \sum F_x = -T_1 \cos 37.0° + T_2 \cos 53.0° = 0$$

$$(2)\ \sum F_y = T_1 \sin 37.0° + T_2 \sin 53.0° + (-122\text{ N}) = 0$$

$$(3)\ T_2 = T_1\left(\frac{\cos 37.0°}{\cos 53.0°}\right) = 1.33T_1$$

Substitute this value for T_2 into equation (2):

$$T_1 \sin 37.0° + (1.33T_1)(\sin 53.0°) - 122\text{ N} = 0$$

$$T_1 = 73.4\text{ N}$$

$$T_2 = 1.33T_1 = 97.4\text{ N}$$

Both values are less than 100 N, so the cables will not break.

Example (5.2):

A car of mass *m* is on an icy driveway inclined at an angle u as in the figure a.

(A) Find the acceleration of the car, assuming the driveway is frictionless.

Solution:

$$(1)\ \sum F_x = mg\sin\theta = ma_x$$

$$(2)\ \sum F_y = n - mg\cos\theta = 0$$

$$(3)\ a_x = g\sin\theta$$