Note that the acceleration component a_x is independent of the mass of the car! It depends only on the angle of inclination and on g.

(B) Suppose the car is released from rest at the top of the incline and the distance from the car's front bumper to the bottom of the incline is d. How long does it take the front bumper to reach the bottom of the hill, and what is the car's speed as it arrives there?

Solution:

Apply equation: $x_f = x_i + v_{xi}t + \frac{1}{2} a_x t^2$

$x_f = d$, $x_i = 0$ and $v_{xi} = 0$ then $d = \frac{1}{2} a_x t^2$

Solve for t : $$t = \sqrt{\frac{2d}{a_x}} = \sqrt{\frac{2d}{g\sin\theta}}$$

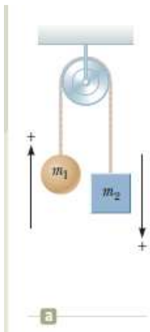
Use equation: $v_{xf}{}^2 = v_{xi}{}^2 + 2a_x(x_f - x_i)$

with $v_{xi} = 0$, to find the final velocity of the car:

$$v_{xf}{}^2 = 2\, a_x\, d$$

$$v_{xf} = \sqrt{2a_x d} = \sqrt{2gd\sin\theta}$$

Example (5.3):



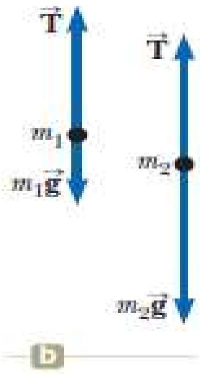
When two objects of unequal mass are hung vertically over a frictionless pulley of negligible mass as in the figure a, the arrangement is called an *Atwood machine.*

The device is sometimes used in the lab. to determine the value of g. Determine the magnitude of the acceleration of the two objects and the tension in the lightweight cord.

Solution:

The objects in the Atwood machine are subject to the gravitational force as well as to the forces exerted by the strings connected to them.

Two forces act on each object: the upward force $\vec{T}$ (tension) exerted by the string and the downward gravitational force.



Apply Newton's second law to object 1:

$$\sum F_y = T - m_1 g = m_1 a_y$$

Apply Newton's second law to object 2:

$$\sum F_y = m_2 g - T = m_2 a_y$$

$$-m_1 g + m_2 g = m_1 a_y + m_2 a_y$$

Solve for the acceleration: $a_y = \left(\dfrac{m_2 - m_1}{m_1 + m_2}\right)g$

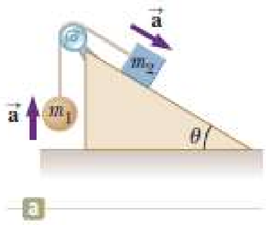
To find the tension T of the string: $T = m_1(g + a_y) = \left(\dfrac{2m_1 m_2}{m_1 + m_2}\right)g$

Example (5.4):

A ball of mass m_1 and a block of mass m_2 are attached by a lightweight cord that passes over a frictionless pulley of negligible mass as in the figure a. The block lies on a frictionless incline of angle θ.

Find the magnitude of the acceleration of the two objects and the tension in the cord.

Solution:



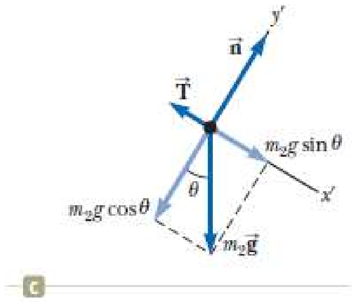
If m_2 moves down the incline, then m_1 moves upward. Because the objects are connected by a cord (which we assume does not stretch), their accelerations have the same magnitude.

Apply Newton's second law in component form to the ball, choosing the upward direction as positive:

$$(1)\ \sum F_x = 0$$
$$(2)\ \sum F_y = T - m_1 g = m_1 a_y = m_1 a$$

For the ball to accelerate upward, it is necessary that $T > m_1 g$.

Apply Newton's second law in component form to the block:



$$(3)\ \sum F_{x'} = m_2 g \sin\theta - T = m_2 a_{x'} = m_2 a$$
$$(4)\ \sum F_{y'} = n - m_2 g \cos\theta = 0$$

We replaced ($a_{x'}$) with (a) because the two objects have accelerations of equal magnitude (a).

Solve equation (2) for T : $$(5)\ T = m_1(g + a)$$

Substitute this expression for T into equation (3):

$$m_2 g \sin\theta - m_1(g + a) = m_2 a$$

Solve for (a):

$$(6)\ a = \left(\frac{m_2 \sin\theta - m_1}{m_1 + m_2}\right) g$$

Substitute this expression for (a) into equation (5) to find T :

$$(7)\ T = \left(\frac{m_1 m_2(\sin\theta + 1)}{m_1 + m_2}\right) g$$

5.5 Forces of Friction

When an object is in motion either on a surface or in a viscous medium such as air or water, there is resistance to the motion because the object interacts with its surroundings. We call such resistance a **force of friction.**

- If we apply an external horizontal force $\vec{F}$ to a block for example, acting to the right, the block can remains stationary when $\vec{F}$ is small. The force on the block that counteracts $\vec{F}$ and keeps it from moving acts toward the left and is called the **force of static friction** $\vec{f}_s$. As long as the block is not moving, $f_s = F$. Therefore, if $\vec{F}$ is increased, $\vec{f}_s$ also increases. Likewise, if $\vec{F}$ decreases, $\vec{f}_s$ also decreases.
- We call the friction force for an object in motion the **force of kinetic friction** $\vec{f}_k$.
- The magnitude of the force of static friction between any two surfaces in contact can have the values:

$$f_s \le \mu_s n \qquad (5.5)$$

Where the dimensionless constant (μ_s) is called the **coefficient of static friction** and (n) is the magnitude of the normal force exerted by one surface on the other.

- The equality in equation (5.5) holds when the surfaces are on the verge of slipping, that is, when $f_s = f_{s,\max} = \mu_s n$. This situation is called *impending motion.*
- The inequality holds when the surfaces are not on the verge of slipping.
- The magnitude of the force of kinetic friction acting between two surfaces is:

$$f_k = \mu_k n \qquad (5.6)$$

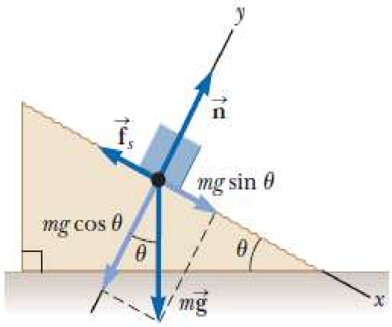
Where (μ_k) is the **coefficient of kinetic friction.**

- The values of μ_k and μ_s depend on the nature of the surfaces.
- **μ_k is generally less than μ_s.**

  Typical values range from around (0.03 to 1).
- The direction of the friction force on an object is parallel to the surface with which the object is in contact and opposite to the actual motion (kinetic friction) or the impending motion (static friction) of the object relative to the surface.

Example (5.5):

A block is placed on a rough surface inclined relative to the horizontal as shown in the figure. The incline angle is increased until the block starts to move. Show that you can obtain μ_s by measuring the critical angle θ at which this slipping just occurs?



Solution:

$$(1)\quad \sum F_x = mg\sin\theta - f_s = 0$$
$$(2)\quad \sum F_y = n - mg\cos\theta = 0$$

Substitute ($mg = n/\cos\theta$) from equation (2) into equation (1):

$$(3)\quad f_s = mg\sin\theta = \left(\frac{n}{\cos\theta}\right)\sin\theta = n\tan\theta$$

When the incline angle is increased until the block is on the verge of slipping, the force of static friction has reached its maximum value $\mu_s\, n$.

$$\mu_s\, n = n \tan\theta$$

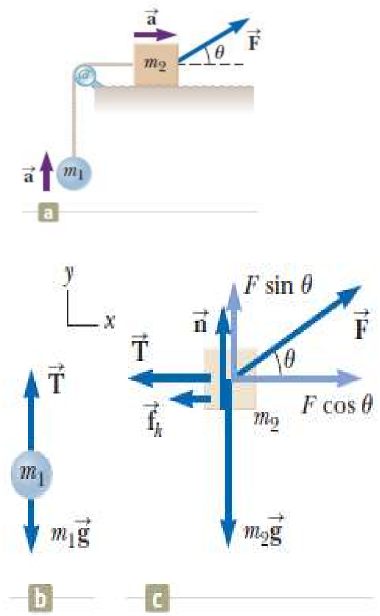
$$\mu_s = \tan\theta$$

Example (5.6):

A block of mass m_2 on a rough, horizontal surface is connected to a ball of mass m_1 by a lightweight cord over a lightweight, frictionless pulley as shown in the figure a. A force of magnitude F at an angle θ with the horizontal is applied to the block as shown, and the block slides to the right. The coefficient of kinetic friction between the block and surface is μ_k.

Determine the magnitude of the acceleration of the two objects.

Solution:



$$(1)\ \sum F_x = F\cos\theta - f_k - T = m_2 a_x = m_2 a$$

$$(2)\ \sum F_y = n + F\sin\theta - m_2 g = 0$$

$$(3)\ \sum F_y = T - m_1 g = m_1 a_y = m_1 a$$

Solve equation (2) for n:

$$n = m_2 g - F\sin\theta$$

Substitute (n) into $f_k = \mu_k n$:

$$(4)\ f_k = \mu_k(m_2 g - F\sin\theta)$$

Substitute equation (4) and the value of (T) from equation (3) into equation (1):

$$F\cos\theta - \mu_k(m_2 g - F\sin\theta) - m_1(a + g) = m_2 a$$

Solve for a:

$$(5)\ a = \frac{F(\cos\theta + \mu_k \sin\theta) - (m_1 + \mu_k m_2)g}{m_1 + m_2}$$

Chapter 6

(Uniform Circular Motion)

6.1 Particle in Uniform Circular Motion

If a car is moving on a circular path with *constant speed v,* we call it **uniform circular motion.** Even though an object moves at a constant speed in a circular path, *it still has acceleration*. To see why, consider the defining equation for acceleration,

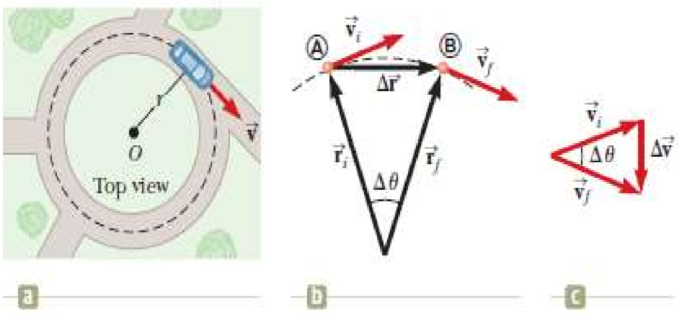
$$\vec{a} = \frac{d\vec{v}}{dt}$$

Notice that the acceleration depends on the change in the *velocity.* Because velocity is a vector quantity, acceleration can occur in two ways:

1- By a change in the *magnitude* of the velocity.

2- By a change in the *direction* of the velocity.

The constant-magnitude velocity vector is always tangent to the path of the object and perpendicular to the radius of the circular path. For *uniform* circular motion, the acceleration vector can only have a component perpendicular to the path, which is toward the center of the circle.



- Let us now find the magnitude of the acceleration of the particle.

The angle $\Delta\theta$ between the two position vectors in the figure (b) is the same as the angle between the velocities vectors in figure (c) because the velocity vector $\vec{v}$ is always perpendicular to the position vector $\vec{r}$. Therefore, the two triangles are *similar*; (Two triangles are similar if the angle between any two sides is the same for both triangles and if the ratio of the lengths of these sides is the same). We can now write a relationship between the lengths of the sides for the two triangles.

$$\frac{|\Delta\vec{v}|}{v} = \frac{|\Delta\vec{r}|}{r}$$

Where $v_i = v_f = v$ and $r = r_i = r_f$

The magnitude of the average acceleration over the time interval for the particle to move from A to B:

$$|\vec{a}_{avg}| = \frac{|\Delta\vec{v}|}{|\Delta t|} = \frac{v|\Delta\vec{r}|}{r\ \Delta t}$$

As A and B approach each other, Δt approaches zero, $|\Delta\vec{r}|$ approaches the distance traveled by the particle along the circular path, and the ratio $\frac{|\Delta\vec{r}|}{\Delta t}$ approaches the speed v. In addition, the average acceleration becomes the instantaneous acceleration at point A. Hence, in the limit $\Delta t \rightarrow 0$, the magnitude of the acceleration is:

$$a_c = \frac{v^2}{r}$$ **Centripetal acceleration** (6.1)

This acceleration is called a **centripetal acceleration** (*centripetal* means *center-seeking*) because $\vec{a}_c$ is directed toward the center of the circle. Furthermore, $\vec{a}_c$ is *always* perpendicular to $\vec{v}$.

In many situations, it is convenient to describe the motion of a particle moving with constant speed in a circle of radius r in terms of the **period** T, which is defined as (the time interval required for one complete revolution of the particle). In the time interval T, the particle moves a distance of $2\pi r$, which is equal to the circumference of the particle's circular path. Therefore, because its speed is equal to the circumference of the circular path divided by the period or $v = 2\pi r/T$, it follows that:

$$T = \frac{2\pi r}{v}$$ **Period of circular motion** (6.2)

Example (6.1):

What is the centripetal acceleration of the Earth as it moves in its orbit around the Sun?

Solution:

Combine equations (6.1) and (6.2):

$$a_c = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2}$$

The period of the Earth's orbit, which we know is one year, and the radius of the Earth's orbit around the Sun, which is (1.496×10^{11} m).

$$a_c = \frac{4\pi^2(1.496 \times 10^{11}\text{ m})}{(1\text{ yr})^2}\left(\frac{1\text{ yr}}{3.156 \times 10^7\text{ s}}\right)^2 = \boxed{5.93 \times 10^{-3}\text{ m/s}^2}$$

- Let us now consider a ball of mass m that is tied to a string of length r and moves at constant speed in a horizontal circular path:

According to Newton's first law, the ball would move in a straight line if there were no force on it; the string, however, prevents motion along a straight line by exerting on the ball a radial force $\vec{F}_r$ that makes it follow the circular path. This force is directed along the string toward the center of the circle. If Newton's second law is applied along the radial direction, the net force causing the centripetal acceleration can be related to the acceleration as follows:

$$\sum F = ma_c = m\frac{v^2}{r}$$ Centripetal force (6.3)

This force causing a centripetal acceleration acts toward the center of the circular path and causes a change in the direction of the velocity vector. If