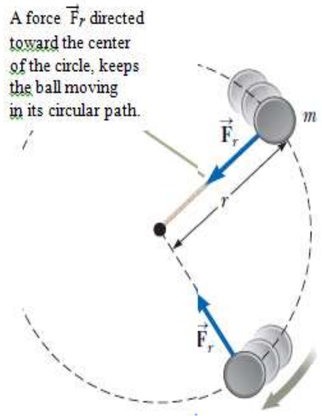
that force should vanish, the object would no longer move in its circular path; instead, it would move along a straight-line path tangent to the circle as shown in the figure.



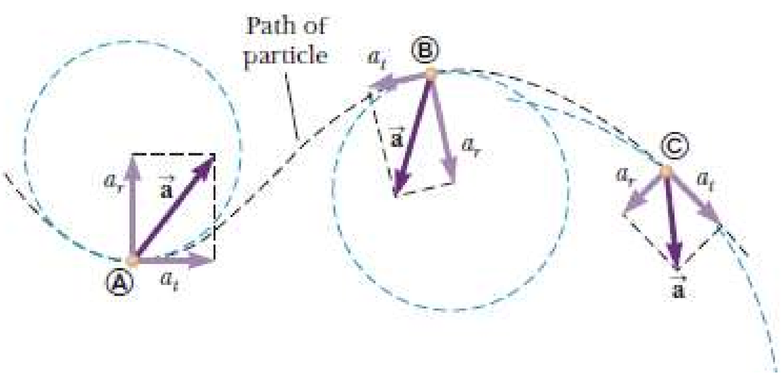
The magnitude of the centripetal force required to keep on the object in a circular path depends on the mass of the object and its acceleration.

6.2 Tangential and Radial Acceleration

Let us consider a more general motion:

A particle moves to the right along a curved path, and its velocity changes both in direction and in magnitude. In this situation, the velocity vector is always tangent to the path; the acceleration vector $\vec{a}$. The direction of the total acceleration vector $\vec{a}$ changes from point to point. At any instant, this vector can be resolved into two components as shown in the figure, based on an origin at the center of the dashed circle corresponding to that instant: a radial component a_r along the radius of the circle and a tangential component a_t perpendicular to this radius. The *total* acceleration vector $\vec{a}$ can be written as the vector sum of the component vectors:

$$\vec{a} = \vec{a}_r + \vec{a}_t \quad \text{Total acceleration} \quad (6.4)$$



Curved path

The tangential acceleration component causes a change in the speed v of the particle. This component is parallel to the instantaneous velocity, and its magnitude is given by:

$$\mathbf{a}_t = \left|\frac{dv}{dt}\right| \qquad \text{Tangential acceleration} \qquad (6.5)$$

The radial acceleration component arises from a change in direction of the velocity vector and is given by:

$$\mathbf{a}_r = -\,\mathbf{a}_c = -\,\frac{v^2}{r} \qquad \text{Radial acceleration} \qquad (6.6)$$

Where (r) is the radius of curvature of the path. The negative sign in equation (6.6) indicates that the direction of the centripetal acceleration is toward the center of the circle representing the radius of curvature. The direction is opposite that of the radial unit vector $\vec{r}$, which always points away from the origin at the center of the circle.

Because $\vec{a}_r$ and $\vec{a}_t$ are perpendicular component vectors of $\vec{a}$,

it follows that the magnitude of $\vec{a}$ is: $a=\sqrt{a_r^2 + a_t^2}$.

At a given speed, a_r is **large** when the radius of curvature **(r)** is **small** (as at points A and B) in the figure, and **small** when **(r)** is **large** (as at point C). The direction of $\vec{a}_t$ is either in the same direction as $\vec{v}$ (if v is increasing) or opposite $\vec{v}$ (if v is decreasing, as at point B).

- In uniform circular motion, where v is constant, $a_t = 0$ and the acceleration is always **completely radial.** In other words, uniform circular motion is a special case of motion along a general curved path.

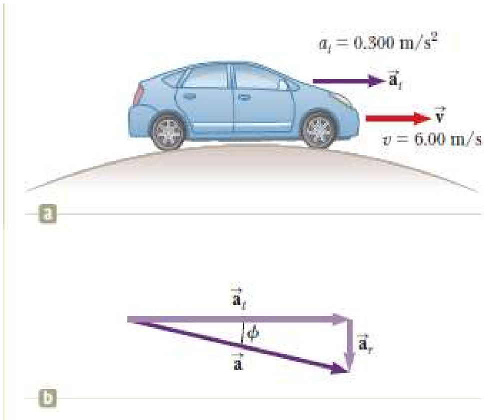
Example (6.2):

A car exhibits a constant acceleration of 0.3 m/s^2 parallel to the roadway. The car passes over a rise in the roadway such that the top of the rise is shaped like a circle of radius 500 m. At the moment the car is at the top of the rise, its velocity vector is horizontal and has a magnitude of

(6 m/s). What are the magnitude and direction of the total acceleration vector for the car at this instant?

Solution:

Because the accelerating car is moving along a curved path, the car has both tangential and radial acceleration.



The radial acceleration vector is directed straight downward, and the tangential acceleration vector has magnitude ($0.3 m/s^2$) and is horizontal.

The radial acceleration:

$$\mathbf{a}_r = -\frac{v^2}{r} = -\frac{(6)^2}{500} = -\mathbf{0.072\ m/s^2}$$

The total acceleration is:

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(-0.072)^2 + (0.3)^2}$$

$$= \mathbf{0.309\ m/s^2}$$

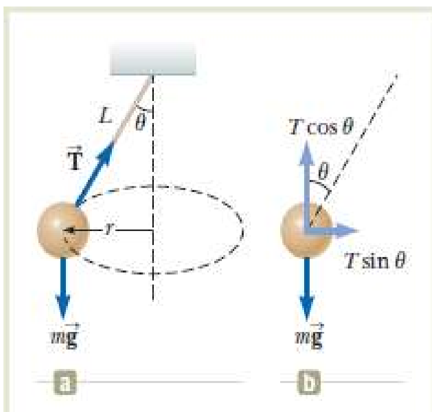
The direction of the total acceleration vector is:

$$\emptyset = \tan^{-1}\frac{a_r}{a_t} = \tan^{-1}\frac{(-0.072)}{(0.3)} = -\mathbf{13.5°}$$

Example (6.3):

A small ball of mass m is suspended from a string of length L. The ball revolves with constant speed v in a horizontal circle of radius r as shown in the figure. Find an expression for v.

Solution:



Let θ represents the angle between the string and the vertical.

The force $\vec{T}$ exerted by the string on the ball is resolved into a vertical component ($T\cos\theta$) and a horizontal component ($T\sin\theta$) acting toward the center of the circular path.

Apply the particle in equilibrium model in the vertical direction:

$$\sum F_y = T\cos\theta - mg = 0$$
$$(1) \quad T\cos\theta = mg$$

Use equation (6.3) in the horizontal direction:

$$(2) \quad \sum F_x = T\sin\theta = ma_c = \frac{mv^2}{r}$$

Divide equation (2) by equation (1) and use $\sin\theta / \cos\theta = \tan\theta$:

$$\tan\theta = \frac{v^2}{rg}$$

Solve for v:

$$v = \sqrt{rg\tan\theta}$$

Since ($r = L$ sin)

$$v = \sqrt{Lg\sin\theta\tan\theta}$$

Notice that the speed is independent of the mass of the ball.

Example (6.4):

A car moving on a flat, horizontal road negotiates a curve as shown in the figure a. If the radius of the curve is 35 m and the coefficient of static friction between the tires and dry pavement is 0.523, find the maximum speed the car can have and still make the turn successfully.

a

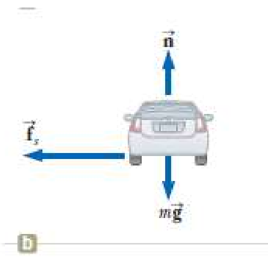
Solution:

Imagine that the curved roadway is part of a large circle so that the car is moving in a circular path.

The force that enables the car to remain in its circular path is the force of static friction. (It is *static* because no slipping occurs at the point of contact between road and tires). The maximum speed v_{max} the car can have around the curve is the speed at which it is on the verge of skidding outward. At this point, the friction force has its maximum value $f_{s,\max}=\mu_s n$.

$$(1)\quad f_{s,\max} = \mu_s n = m\frac{v_{\max}^2}{r}$$

$$\sum F_y = 0 \ \rightarrow\ n - mg = 0 \ \rightarrow\ n = mg$$



Solve equation (1) for the maximum speed and substitute for n:

$$(2)\quad v_{\max} = \sqrt{\frac{\mu_s n r}{m}} = \sqrt{\frac{\mu_s m g r}{m}} = \sqrt{\mu_s g r}$$

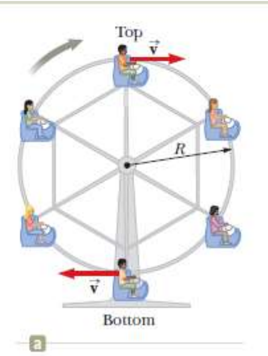
$$v_{\max} = \sqrt{(0.523)(9.80\ \mathrm{m/s^2})(35.0\ \mathrm{m})} = \boxed{13.4\ \mathrm{m/s}}$$

Notice that the maximum speed does not depend on the mass of the car.

Example (6.5):

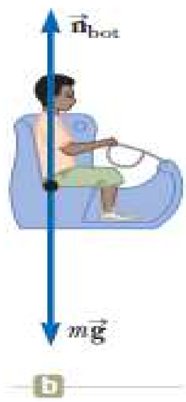
Child of mass m rides on a wheel as shown in the figure a. The child moves in a vertical circle of radius (10 m) at a constant speed of (3 m/s).

(A) Determine the force exerted by the seat on the child at the bottom of the ride.



Solution:

We draw a diagram of forces acting on the child at the bottom of the ride as shown in the figure b. The only forces acting on him are the downward gravitational force $\vec{F}_g = m\vec{g}$ and the upward force $\vec{n}_{bot}$ exerted by the seat. The net upward force on the child that provides his centripetal acceleration has a magnitude ($\vec{n}_{bot} - mg$). Apply Newton's second law to the child in the radial direction:



$$\sum F = n_{bot} - mg = m\frac{v^2}{r}$$

$$n_{bot} = mg + m\frac{v^2}{r} = mg\left(1 + \frac{v^2}{rg}\right)$$

$$n_{bot} = mg\left[1 + \frac{(3.00\ m/s)^2}{(10.0\ m)(9.80\ m/s^2)}\right]$$

$$= \boxed{1.09\ mg}$$

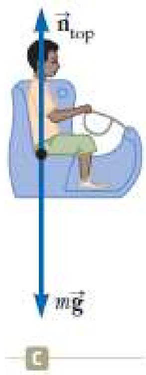
Hence, the magnitude of the force $\vec{n}_{bot}$ exerted by the seat on the child is *greater* than the weight of the child by a factor of (1.09).

(B) Determine the force exerted by the seat on the child at the top of the ride.

Solution:

The diagram of forces acting on the child at the top of the ride is shown in the figure c. The net downward force that provides the centripetal acceleration has a magnitude ($mg - \vec{n}_{top}$).

Apply Newton's second law to the child at this position:



$$\sum F = mg - n_{top} = m\frac{v^2}{r}$$

$$n_{top} = mg - m\frac{v^2}{r} = mg\left(1 - \frac{v^2}{rg}\right)$$

$$n_{top} = mg\left[1 - \frac{(3.00\ m/s)^2}{(10.0\ m)(9.80\ m/s^2)}\right]$$

$$= \boxed{0.908\ mg}$$

In this case, the magnitude of the force exerted by the seat on the child is *less* than his true weight by a factor of 0.908, and the child feels lighter.

6.3 Gravitation

Newton's Law of Universal Gravitation:

Newton's law of universal gravitation states that:

(Every particle in the Universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them).

$$F_g = G\frac{m_1 m_2}{r^2}$$ Gravitational force (6.7)

Where G is a constant, called the universal gravitational constant:

$$G = 6.67 \times 10^{-11}\ N.m^2/\ kg^2$$

Free-Fall Acceleration and the Gravitational Force

The magnitude of the gravitational force on an object near the Earth's surface is called the *weight* of the object:

$$mg = G\frac{M_E m}{R_E^{\ 2}}$$

$$g = G\frac{M_E}{R_E^{\ 2}} \qquad (6.8)$$

Where M_E is the Earth's mass and R_E is its radius.

According to equation (6.8), we see that the free - fall acceleration (g) near the Earth's surface is constant since the other quantities in this equation are also constants.

Example (6.6):

The surface of the Earth is approximately (6400 km) from its center and its mass is (6×10^{24} kg), what is the acceleration due to gravity (g) near the surface?

Solution:

Apply equation (6.8): $\boldsymbol{g} = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{(6.4 \times 10^{6})^{2}}$

$$\boldsymbol{g = 9.8\ m/s^2}$$

- Now consider an object of mass m located a distance h above the Earth's surface or a distance r from the Earth's center,
  Where ($\boldsymbol{r = R_E + h}$). The magnitude of the gravitational force acting on this object is:

$$F_g = G\frac{M_E m}{r^2} = G\frac{M_E m}{(R_E + h)^2}$$

The magnitude of the gravitational force acting on the object at this position is also $F_g = mg$, where g is the value of the free-fall acceleration at the altitude h. Substituting this expression for F_g into the last equation shows that g is given by:

$$g = \frac{GM_E}{r^2} = \frac{GM_E}{(R_E + h)^2} \qquad \textbf{Variation of } \boldsymbol{g} \textbf{ with altitude} \qquad (6.9)$$

Therefore, it follows that g *decreases* with *increasing altitude.*

Chapter 7

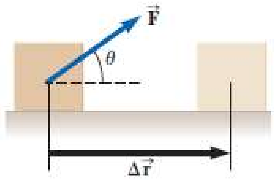
(Work and Energy)

7.1 Work Done by a Constant Force

The **work** W done on a system by exerting a constant force on the system is (the product of the magnitude F of the force, the magnitude Δr of the displacement of the point of application of the force, and $\cos\theta$, where θ is the angle between the force and displacement vectors):

$$W = F\Delta r\cos\theta \qquad \textbf{Work done by a constant force} \qquad (7.1)$$

Work is a scalar, even though it is defined in terms of two vectors; a force $\vec{F}$ and a displacement $\Delta\vec{r}$.



- A force does no work on an object if the force does not move through a displacement, that is if $\Delta r = 0$ then W=0.
- If the work done by a force on a moving object is zero when the force applied is perpendicular to the displacement of its point of application. That is, if θ= 90°, then W = 0 because cos 90°=0.
- The sign of the work depends on the direction of $\vec{F}$ relative to $\Delta\vec{r}$.

The work done by the applied force on a system is positive when the projection of $\vec{F}$ onto $\Delta\vec{r}$ is in the same direction as the displacement. For example, when an object is lifted, the work done by the applied force on the object is positive because the direction of that force is upward, in the same direction as the displacement of its point of application.

When the projection of $\vec{F}$ onto $\Delta\vec{r}$ is in the direction opposite the displacement, W is negative. For example, as an object is lifted, the work done by the gravitational force on the object is negative.