If an applied force $\vec{F}$ is in the same direction as the displacement $\Delta\vec{r}$, then $\theta = 0$ and $\cos 0 = 1$. In this case, equation (7.1) gives:

$$W = F\,\Delta r$$

The units of work are those of force multiplied by those of length. Therefore, the SI unit of work is the **newton . meter** (N . m = kg . m^2/s^2). This combination of units is given a name, the **joule** (J).

- Work is an energy transfer. If W is the work done on a system and *W* is positive, energy is transferred *to* the system; if *W* is negative, energy is transferred *from* the system.

7.2 Work Done by a Varying Force

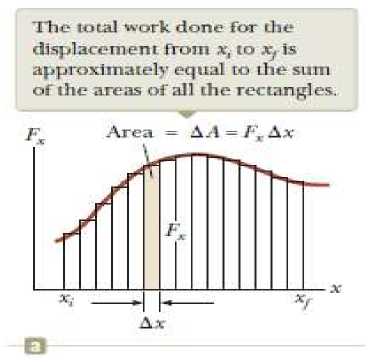
Consider a particle being displaced along the x - axis under the action of a force that varies with position. The particle is displaced in the direction of increasing x from $x = x_i$ to $x = x_f$. In such a situation, we cannot use ($W = F\,\Delta r\cos\theta$) to calculate the work done by the force because this relationship applies only when $\vec{F}$ is constant in magnitude and direction. If, however, we imagine that the particle undergoes a very small displacement Δx, the x component F_x of the force is approximately constant over this small interval; for this small displacement, we can approximate the work done on the particle by the force as:

$$W \approx F_x\,\Delta x$$

Which is the area of the shaded rectangle in the figure a. If we imagine the F_x versus x curve divided into a large number of such intervals, the total work done for the displacement from x_i to x_f is approximately equal to the sum of a large number of such terms:

$$W \approx \sum_{x_i}^{x_f} F_x\,\Delta x$$

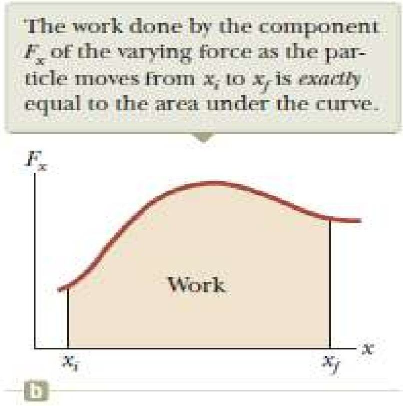
$$\lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x\,\Delta x = \int_{x_i}^{x_f} F_x\,dx$$



Therefore, we can express the work done by F_x on the particle as it moves from x_i to x_f as:

$$W = \int_{x_i}^{x_f} F_x \, dx \qquad (7.2)$$

This equation reduces to equation (7.1) when the component $F_x = F \cos \theta$ remains constant.



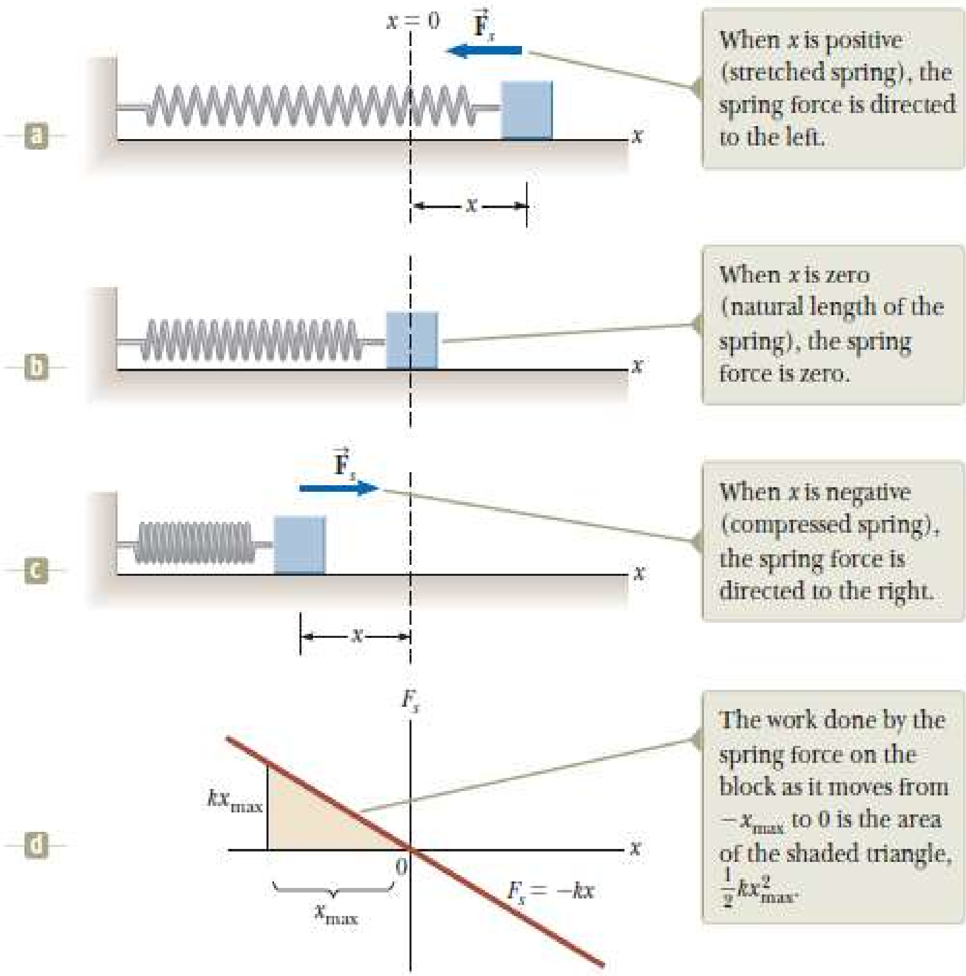
Work done by a spring

A model of a common physical system on which the force varies with position is shown in the figure.

The system is a block on a frictionless, horizontal surface and connected to a spring. For many springs, if the spring is either stretched or compressed a small distance from its unstretched (equilibrium) configuration, it exerts on the block a force that can be mathematically modeled as:

$$F_s = -kx \qquad \text{Spring force (\textbf{Hooke's law})} \qquad (7.3)$$

Where x is the position of the block relative to its equilibrium ($x = 0$) position and k is a positive constant called the **force constant** or the **spring constant** of the spring.



- The force required to stretch or compress a spring is proportional to the amount of stretch or compression x. This force law for springs is known as **Hooke's law.**

The value of k is a measure of the *stiffness* of the spring. Stiff springs have large k values, and soft springs have small k values. The units of k are N/m.

- The negative sign in equations (7.3) signifies that the force exerted by the spring is always directed ***opposite*** the displacement from equilibrium.

- When $x > 0$ as in the (figure a) so that the block is to the right of the equilibrium position, the spring force is directed to the left, in the negative x direction. When $x < 0$ as in the (figure c), the block is to the left of equilibrium and the spring force is directed to the right, in the positive x direction. When $x = 0$ as in the (figure b), the spring is unstretched and $F_s = 0$. Because the spring force always acts toward the equilibrium position ($x = 0$), it is sometimes called a ***restoring force***.
- The work W_s done by the spring force on the block as the block moves from $x_i = -x_{max}$ to $x_f = 0$:

$$W_s = \int_{-x_{\max}}^{0} (-kx)\,dx = \tfrac{1}{2}kx_{\max}^2 \qquad (7.4)$$

The work done by the spring force is positive because the force is in the same direction as its displacement (both are to the right).

- If the block undergoes a displacement from $x = x_i$ to $x = x_f$, the work done by the spring force on the block is:

$$W_s = \int_{x_i}^{x_f} (-kx)\,dx = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2$$ **Work done by a spring** (7.5)

We see that the work done by the spring force is zero for any motion that ends where it began ($x_i = x_f$).

Example (7.1):

A spring is hung vertically, and an object of mass m is attached to its lower end. The spring stretches a distance d from its equilibrium position.

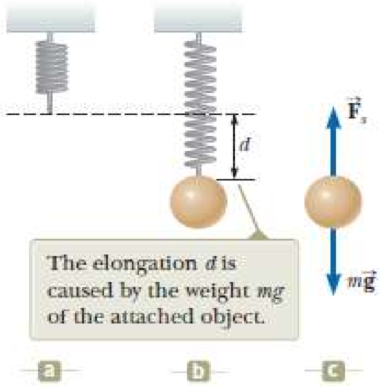
(A) If a spring is stretched 2 cm by a suspended object having a mass of 0.55 kg, what is the force constant of the spring?

(B) How much work is done by the spring on the object as it stretches through this distance?

Solution:

(A) Because the object is in equilibrium, the net force on it is zero and the upward spring force balances the downward gravitational force.

$$\vec{F}_s + m\vec{g} = 0 \rightarrow F_s - mg = 0 \rightarrow F_s = mg$$



Apply Hooke's law to give $F_s = kd$ and solve for k:

$$k = \frac{mg}{d} = \frac{(0.55\ \text{kg})(9.80\ \text{m/s}^2)}{2.0 \times 10^{-2}\ \text{m}} = \boxed{2.7 \times 10^2\ \text{N/m}}$$

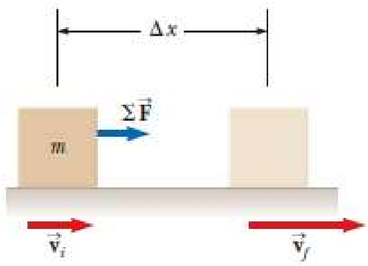
(B) To find the work done by the spring on the object:

$$W_s = 0 - \tfrac{1}{2}kd^2 = -\tfrac{1}{2}(2.7 \times 10^2\ \text{N/m})(2.0 \times 10^{-2}\ \text{m})^2$$
$$= \boxed{-5.4 \times 10^{-2}\ \text{J}}$$

7.3 Kinetic Energy and the Work–Kinetic Energy Theorem

Consider a system consisting of a single object. The figure shows a block of mass m moving through a displacement directed to the right under the action of a net force $\vec{F}$, also directed to the right. We know from Newton's second law that the block moves with an acceleration $\vec{a}$. If the block and (therefore the force) moves through a displacement $\Delta\vec{r} = \Delta x\hat{i} = (x_f - x_i)\hat{i}$, the net work done on the block by the external net force $\Sigma\vec{F}$ is:

$$W_{ext} = \int_{x_i}^{x_f} \sum F\, dx \qquad (7.6)$$



Using Newton's second law, we substitute for the magnitude of the net force $\Sigma\vec{F} = ma$.

$$W_{ext} = \int_{x_i}^{x_f} ma\, dx = \int_{x_i}^{x_f} m\frac{dv}{dt}\, dx = \int_{x_i}^{x_f} m\frac{dv}{dx}\frac{dx}{dt}\, dx = \int_{v_i}^{v_f} mv\, dv$$

$$W_{ext} = \tfrac{1}{2}mv_f^{\,2} - \tfrac{1}{2}mv_i^{\,2} \qquad (7.7)$$

Where v_i is the speed of the block when it is at $x = x_i$ and v_f is its speed at x_f.

- The quantity ($\frac{1}{2}mv^2$) represents the energy associated with the motion of the particle and it is called (Kinetic energy).

$$K = \frac{1}{2}mv^2 \qquad \text{Kinetic energy} \qquad (7.8)$$

- Kinetic energy is a scalar quantity and has the same units as work.
- Equation 7.7 states that the work done on a particle by a net force $\vec{F}$ acting on it equals the change in kinetic energy of the particle. It is often convenient to write equation 7.7 in the form:

$$W_{ext} = K_f - K_i = \Delta K \qquad (7.9)$$

Another way to write it is $K_f = K_i + W_{ext}$ which tells us that the final kinetic energy of an object is equal to its initial kinetic energy plus the change in energy due to the net work done on it.

Equation 7.9 is an important result known as the

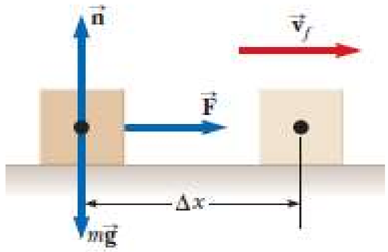
Work–kinetic energy theorem: (When work is done on a system and the only change in the system is in its speed, the net work done on the system equals the change in kinetic energy of the system).

- The work–kinetic energy theorem indicates that the speed of a system *increases* if the net work done on it is *positive* because the final kinetic energy is greater than the initial kinetic energy. The speed *decreases* if the net work is *negative* because the final kinetic energy is less than the initial kinetic energy.

Example (7.2):

A 6.0-kg block initially at rest is pulled to the right along a frictionless, horizontal surface by a constant horizontal force of 12 N. Find the block's speed after it has moved 3.0 m.

Solution:



The normal force balances the gravitational force on the block, and neither of these vertically acting forces does work on the block because their points of application are horizontally displaced.

The net external force acting on the block is the horizontal 12-N force.

Use the work–kinetic energy theorem for the block, noting that its initial kinetic energy is zero:

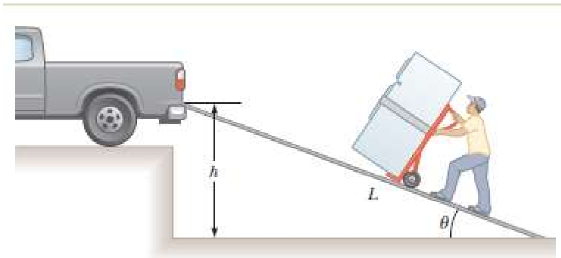
$$W_{\text{ext}} = K_f - K_i = \tfrac{1}{2}mv_f^{\,2} - 0 = \tfrac{1}{2}mv_f^{\,2}$$

$$v_f = \sqrt{\frac{2W_{\text{ext}}}{m}} = \sqrt{\frac{2F\Delta x}{m}}$$

$$v_f = \sqrt{\frac{2(12\text{ N})(3.0\text{ m})}{6.0\text{ kg}}} = 3.5\text{ m/s}$$

Example (7.3):

A man wishes to load a refrigerator onto a truck using a ramp at angle θ as shown in the figure. He claims that less work would be required to load the truck if the length L of the ramp were increased. Is his claim valid?



Solution:

The normal force exerted by the ramp on the system is directed at 90° to the displacement of its point of application and so does no work on the system. Because $\Delta K = 0$, the work–kinetic energy theorem gives:

$$W_{\text{ext}} = W_{\text{by man}} + W_{\text{by gravity}} = 0$$

The work done by the gravitational force equals the product of [the weight (mg) of the system, the distance (L) through which the refrigerator is displaced, and cos (θ + 90°)]. Therefore,

$$W_{\text{by man}} = -W_{\text{by gravity}} = -(mg)(L)[\cos(\theta + 90°)]$$
$$= mgL \sin\theta = mgh$$

Where ($h = L \sin\theta$) is the height of the ramp. Therefore, the man must do the same amount of work (mgh) on the system *regardless* of the length of the ramp. The work depends only on the height of the ramp.

7.4 Power

The time rate at which work is done by a force is said to be the **power** due to the force. If a force does an amount of work W in an amount of time Δt, the **average power** due to the force during that time interval is:

$$P_{\text{avg}} = \frac{W}{\Delta t} \quad \text{(average power).}$$

The SI unit of power is joules per second (J/s), also called the **watt** (W)

$$1\ \text{W} = 1\ \text{J/s} = 1\ \text{kg} \cdot \text{m}^2/\text{s}^3$$

Another unit of power is **horsepower** (hp): 1 hp = 746 W

Chapter 8

(Conservation of Energy)

8.1 Potential Energy of a System

We call the energy storage mechanism before the object is released **potential energy.** The amount of potential energy in the system is determined by the *configuration* of the system. The work represents a transfer of energy into the system and the system energy appears in a different form, which we have called potential energy. Therefore, we can identify the quantity (mgy) as the **gravitational potential energy** U_g:

$$U_g = mgy \quad \textbf{Gravitational potential energy} \qquad (8.1)$$

Where (y) is the height above the ground.

The units of gravitational potential energy are joules, the same as the units of work and kinetic energy. Potential energy, like work and kinetic energy, is a scalar quantity.

8.2 Conservative and Non-conservative Forces

Conservative Forces

Conservative forces have these two equivalent properties:

1. The work done by a conservative force on a particle moving between any two points is independent of the path taken by the particle.

2. The work done by a conservative force on a particle moving through any closed path is zero. (A closed path is one for which the beginning point and the endpoint are identical.)

The gravitational force is one example of a conservative force; the force that an ideal spring exerts on any object attached to the spring is another.

Non conservative Forces