A force is **non-conservative** if it does not satisfy properties 1 and 2 for conservative forces. We define the sum of the kinetic and potential energies of a system as the **mechanical energy** of the system:

$$E_{mech} = K + U \qquad (8.2)$$

Where K is the kinetic energy of the system and U is the potential energy in the system.

- The force of kinetic friction is a non-conservative force.

8.3 Relationship between Conservative Forces and Potential Energy

A **potential energy function *U*** is defined as (the work done within the system by the conservative force equals the decrease in the potential energy of the system).

The work done by the force **F** as the particle moves along the *x* axis is:

$$W_{\text{int}} = \int_{x_i}^{x_f} F_x\, dx = -\Delta U \qquad (8.3)$$

Where F_x is the component of $\vec{F}$ in the direction of the displacement.

We can also express equation (8,3) as:

$$\Delta U = U_f - U_i = -\int_{x_i}^{x_f} F_x\, dx$$

We can then define the potential energy function as:

$$U_f(x) = -\int_{x_i}^{x_f} F_x\, dx + U_i$$

The value of U_i is often taken to be zero.

$$dU = -F_x\, dx$$

Therefore, the conservative force is related to the potential energy function through the relationship:

$$F_x = -\frac{dU}{dx}$$ **Relationship between Conservative Forces and Potential Energy** 8.4)

- The potential energy for a spring is:

$U_s \equiv \frac{1}{2}kx^2$ Elastic potential energy (8.5)

The elastic potential energy of the system can be thought of as the energy stored in the deformed spring (one that is either compressed or stretched from its equilibrium position). The elastic potential energy stored in a spring is zero whenever the spring is undeformed ($x = 0$). Energy is stored in the spring only when the spring is either stretched or compressed. Because the elastic potential energy is proportional to x^2, we see that U_s is always positive in a deformed spring.

- In the case of the deformed spring:

$$F_s = -\frac{dU_s}{dx} = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx$$

This is corresponding to the restoring force in the spring (Hooke's law).

$F_s = - k\,x$ Hooke's law

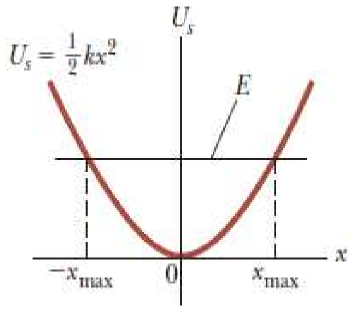
8.4 Potential Energy Diagram

Consider the potential energy function for a block–spring system, given by: $U_s \equiv \frac{1}{2}kx^2$

This function is plotted versus x in the figure. The force F_s exerted by the spring on the block is related to U_s through equation:

$$F_s = -\frac{dU_s}{dx} = -kx \qquad (8.6)$$

This means that the x component of the force is equal to the negative of the slope of the U_s versus x curve.



8.5 Conservation of Energy

The general statement of the principle of **conservation of energy** can be described mathematically with the **conservation of energy equation** as follows:

$$\Delta E_{system} = \sum T \qquad (8.7)$$

Where E_{system} is the total energy of the system, including all methods of energy storage (kinetic, potential, and internal), and T (for *transfer*) is the amount of energy transferred across the system boundary by some mechanism.

We can express the conservation of energy of the system as:

$$\Delta E_{system} = 0 \qquad (8.8)$$

Therefore, $\Delta K + \Delta U = 0$

Or $$(K_f - K_i) + (U_f - U_i) = 0$$

$$K_f + U_f = K_i + U_i \qquad (8.9)$$

Example (8.1):

A ball of mass m is dropped from a height h above the ground as shown in the figure.

(A) Neglecting air resistance, determine the speed of the ball when it is at a height y above the ground.

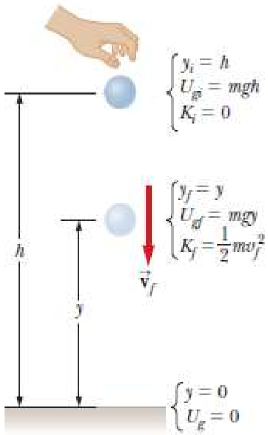
Solution: (A) At the instant the ball is released, its kinetic energy is

$K_i = 0$ and the gravitational potential energy of the system is $U_{gi} = mgh$. When the ball is at a position y above the ground, its kinetic energy is

$$K_f = 1/2\ mv_f^2$$

and the potential energy relative to the ground is

$$U_{gf} = mgy.$$



Apply equation (8.9):

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\tfrac{1}{2}mv_f^2 + mgy = 0 + mgh$$

$$v_f^2 = 2g(h - y) \rightarrow v_f = \boxed{\sqrt{2g(h - y)}}$$

(B): Determine the speed of the ball at y if at the instant of release it already has an initial upward speed v_i at the initial altitude h.

Solution:

In this case, the initial energy includes kinetic energy equal to $1/2\ mv_i^2$.

$$\tfrac{1}{2}mv_f^2 + mgy = \tfrac{1}{2}mv_i^2 + mgh$$

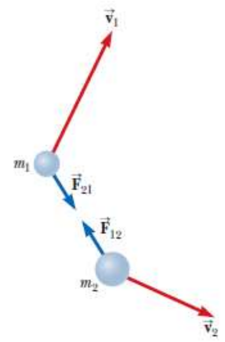
$$v_f^2 = v_i^2 + 2g(h - y) \rightarrow v_f = \boxed{\sqrt{v_i^2 + 2g(h - y)}}$$

Chapter 9

(Linear Momentum and Collisions)

9.1 Linear Momentum

Consider an isolated system of two particles as in the figure, with masses m_1 and m_2 moving with velocities $\vec{v}_1$ and $\vec{v}_2$ at an instant of time. Because the system is isolated, the only force on one particle is that from the other particle. If a force from particle 1 (for example, a gravitational force) acts on particle 2, there must be a second force—equal in magnitude but opposite in direction—that particle 2 exerts on particle 1. That is, the forces on the particles form



Newton's third law action–reaction pair, and $\vec{F}_{12} = -\vec{F}_{21}$.

We can express this condition as:

$$\vec{F}_{12} + \vec{F}_{21} = 0$$

The interacting particles in the system have accelerations corresponding to the forces on them.

Therefore, replacing the force on each particle with $m_1\vec{a}$ for the particle gives:

$$m_1\vec{a}_1 + m_2\vec{a}_2 = 0$$

Now we replace each acceleration with its definition:

$$m_1\frac{d\vec{v}_1}{dt} + m_2\frac{d\vec{v}_2}{dt} = 0$$

If the masses m_1 and m_2 are constant, we can bring them inside the derivative operation, which gives:

$$\frac{d(m_1\vec{v}_1)}{dt} + \frac{d(m_2\vec{v}_2)}{dt} = 0$$

$$\frac{d}{dt}(m_1\vec{v}_1 + m_2\vec{v}_2) = 0 \qquad (9.1)$$

Notice that the derivative of the sum $m_1\vec{v}_1 + m_2\vec{v}_2$ with respect to time is zero. Consequently, this sum must be constant.

We call the quantity $m\vec{v}$ of a particle as (*linear momentum*).

- The **linear momentum** of a particle or an object that can be modeled as a particle of mass m moving with a velocity $\vec{v}$ is defined to be the product of the mass and velocity of the particle:

$$\vec{p} = m\,\vec{v} \quad \textbf{linear momentum} \qquad (9.2)$$

- Linear momentum is a vector quantity because it equals the product of a scalar quantity m and a vector quantity $\vec{v}$. Its direction is along $\vec{v}$, and its SI unit is kg . m/s.
- Using Newton's second law of motion, we can relate the linear momentum of a particle to the resultant force acting on the particle. We start with Newton's second law and substitute the definition of acceleration:

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$$

In Newton's second law, the mass m is assumed to be constant. Therefore, we can bring m inside the derivative operation to give us:

$$\sum \vec{F} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{p}}{dt} \quad \text{Newton's second law for a particle} \quad (9.3)$$

This equation shows that **the time rate of change of the linear momentum of a particle is equal to the net force acting on the particle**.

- Using the definition of momentum, equation (9.1) can be written:

$$\frac{d}{dt}(\vec{p}_1 + \vec{p}_2) = 0$$

Because the time derivative of the total momentum $\vec{p}_{tot} = \vec{p}_1 + \vec{p}_2$ is *zero,* we conclude that the *total* momentum must remain constant:

$$\vec{p}_{tot} = \text{constant} \qquad (9.4)$$

or, equivalently, $\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$ (9.5)

9.2 Impulse

Let us assume a net force $\boldsymbol{\Sigma}\vec{F}$ acts on a particle and this force may vary with time. According to Newton's second law, $\boldsymbol{\Sigma}\vec{F} = d\vec{p}/dt$, or

$$d\vec{p} = \Sigma\vec{F}\,dt \qquad (9.6)$$

We can integrate this expression to find the change in the momentum of a particle when the force acts over some time interval.

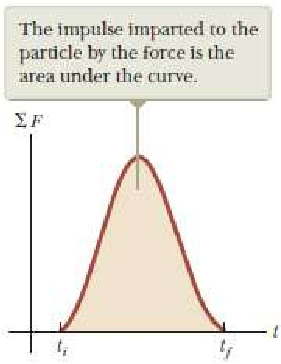
If the momentum of the particle changes from $\vec{p}_i$ at time t_i to $\vec{p}_f$ at time t_f, integrating equation (9.6) gives:

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = \int_{ti}^{tf} \Sigma\vec{F}\,dt \qquad (9.7)$$

- The quantity on the right side of this equation is a vector called the **impulse** of the net force $\Sigma\vec{F}$ acting on a particle over the time interval $\Delta t = t_f - t_i$:

$$\vec{I} = \int_{ti}^{tf} \Sigma\vec{F}\,dt \qquad \text{Impulse of a force} \qquad (9.8)$$

From its definition, we see that impulse $\vec{I}$ is a vector quantity having a magnitude equal to the area under the force–time curve as described in the figure.



- The direction of the impulse vector is the same as the direction of the change in momentum.
- Impulse has the dimensions of momentum.
- Impulse is *not* a property of a particle;

it is a measure of the degree to which an external force changes the particle's momentum.

- Combining equations (9.7) and (9.8) gives us an important statement known as the

Impulse–momentum theorem:

(The change in the momentum of a particle is equal to the impulse of the net force acting on the particle):

$\Delta\vec{p} = \vec{I}$ Impulse–momentum theorem for a particle (9.9)

This statement is equivalent to Newton's second law. When we say that an impulse is given to a particle, we mean that momentum is transferred from an external agent to that particle.

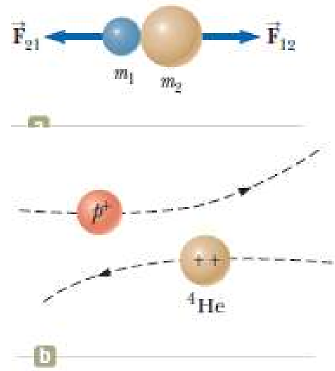
- Equation (9.9) is the most general statement of the principle of **conservation of momentum** and is called the **conservation of momentum equation**. The conservation of momentum equation is often identified as the special case of equation (9.5).

9.3 Collisions

Collisions in One Dimension

The term **collision** represents an event during which two particles come close to each other and interact by means of forces.

A collision may involve physical contact between two macroscopic objects as described in figure (a).



To understand this concept, consider a collision on an atomic scale as in the figure (b) such as the collision of a proton with an alpha particle (the nucleus of a helium atom). Because the particles are both positively charged, they repel each other due to the strong electrostatic force between them at close separations and never come into "physical contact."

- Collisions are categorized as being either *elastic* or *inelastic* depending on whether or not kinetic energy is conserved.
- An **elastic collision** between two objects is one in which the total kinetic energy (as well as total momentum) of the system is the

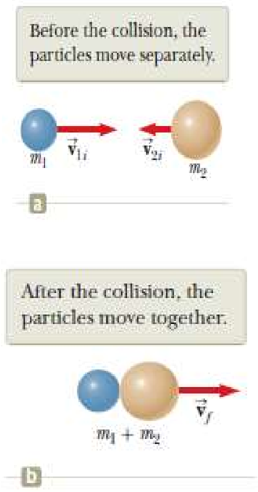
same before and after the collision. Elastic collisions occur between atomic and subatomic particles. There must be no transformation of kinetic energy into other types of energy within the system.

- An **inelastic collision** is one in which the total kinetic energy of the system is not the same before and after the collision (even though the momentum of the system is conserved).

Inelastic collisions are of two types. When the objects stick together after they collide, as happens when a meteorite collides with the Earth, the collision is called **perfectly inelastic.** When the colliding objects do not stick together but some kinetic energy is transformed or transferred away, as in the case of a rubber ball colliding with a hard surface, the collision is called **inelastic.**

Perfectly Inelastic Collisions

Consider two particles of masses m_1 and m_2 moving with initial velocities $\vec{v}_{1i}$ and $\vec{v}_{2i}$ along the same straight line as shown in the figure. The two particles collide head-on, stick together, and then move with some common velocity $\vec{v}_f$ after the collision.



Because the momentum of an isolated system is conserved in *any* collision, we can say that the total momentum before the collision equals the total momentum of the composite system after the collision:

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\vec{v}_f \qquad (9.10)$$

$$\vec{v}_f = \frac{m_1\vec{v}_{1i} + m_2\vec{v}_{2i}}{m_1 + m_2} \qquad (9.11)$$

Elastic Collisions

Consider two particles of masses m_1 and m_2 moving with initial velocities $\vec{v}_{1i}$ and $\vec{v}_{2i}$ along the same straight line as shown in the figure.