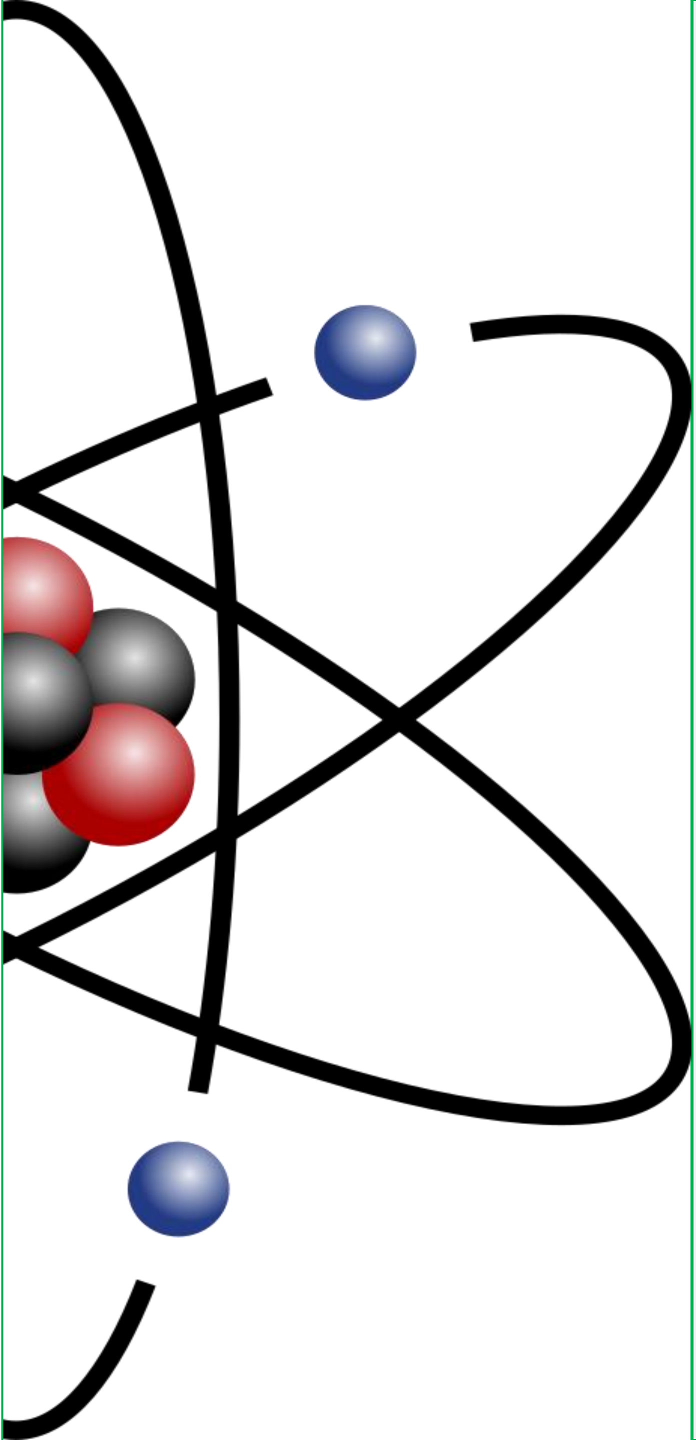


University of Al-Anbar
College of Applied Sciences-Heet
Biophysics Department

Modern Physics-Fourth Stage

Lecture One

Atomic Physics

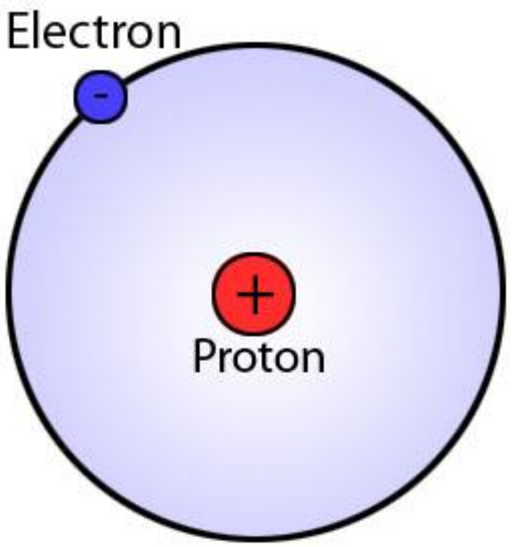
Dr. Nabeel F. Lattoofi

Introduction

the purpose of this chapter is: applying quantum mechanics to atomic systems (hydrogen atom).

Why we study H-atom?

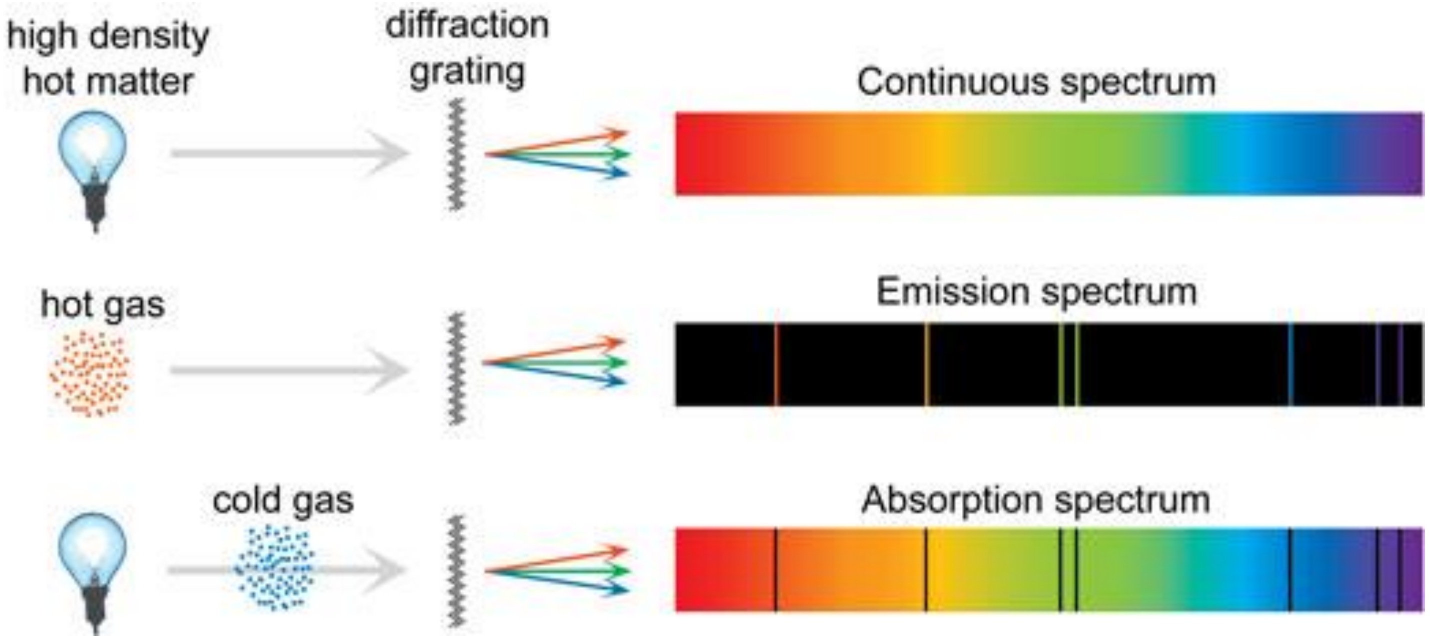
- It is the only atomic system that can be solved exactly.
- can be extended to such single-electron ions as He^+ and Li^+.
- The hydrogen atom is an ideal system for performing precise tests of theory against experiment
- The quantum numbers that are used to characterize the allowed states of hydrogen can also be used to investigate more complex atoms (periodic table).



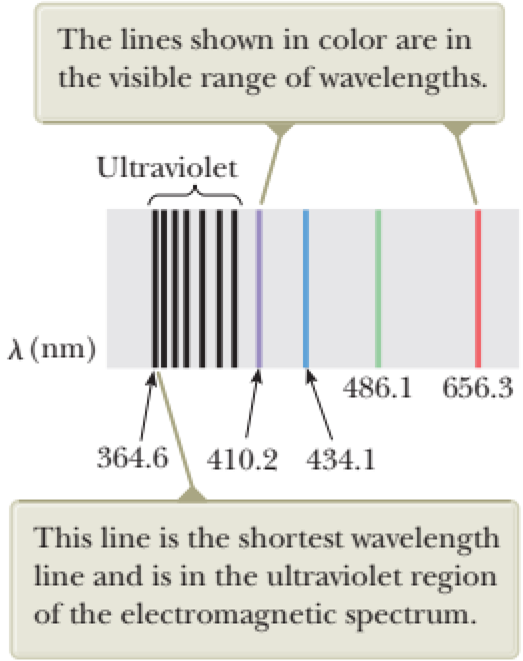
The full mathematical solution of the Schrödinger equation applied to the hydrogen atom gives a complete and beautiful description of the atom's properties.

Atomic Spectra of Gases

emission spectroscopy **Absorption spectroscopy**



- From 1860 to 1885, scientists accumulated a great deal of data on atomic emissions using spectroscopic measurements. In 1885, a Swiss schoolteacher, Johann Jacob Balmer (1825–1898), found an empirical equation that correctly predicted the wavelengths of four visible emission lines of hydrogen: Ha (red), Hb (bluegreen), Hg (blue-violet), and Hd (violet).



$$\frac{1}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \quad n = 3, 4, 5, \ldots$$

Other lines in the spectrum of hydrogen were found following Balmer's discovery. These spectra are called the Lyman, Paschen, and Brackett series after their discoverers.

$$\frac{1}{\lambda} = R_H\left(1 - \frac{1}{n^2}\right) \quad n = 2, 3, 4, \ldots \qquad (42.2)$$ Lyman series

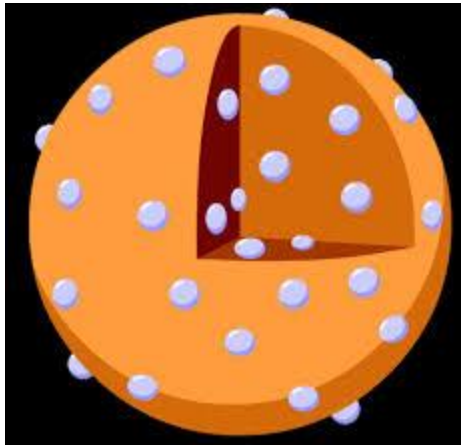
$$\frac{1}{\lambda} = R_H\left(\frac{1}{3^2} - \frac{1}{n^2}\right) \quad n = 4, 5, 6, \ldots \qquad (42.3)$$ Paschen series

$$\frac{1}{\lambda} = R_H\left(\frac{1}{4^2} - \frac{1}{n^2}\right) \quad n = 5, 6, 7, \ldots \qquad (42.4)$$ Brackett series

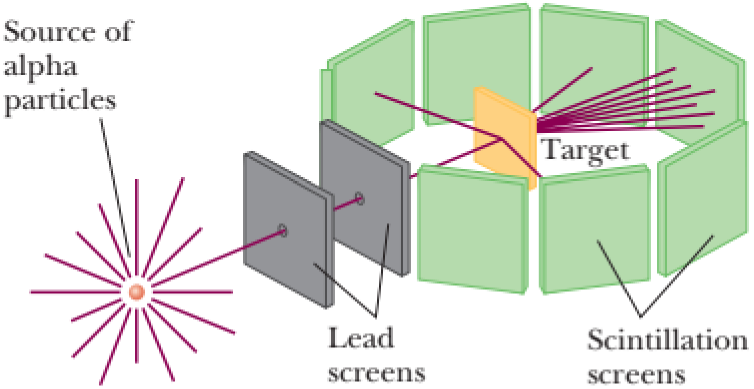
No theoretical basis existed for these equations;

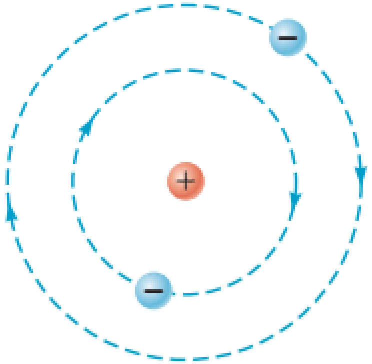
Early Models of the Atom

Thomson model



Rutherford model





Two basic difficulties exist with Rutherford's planetary model:

- An atom emits (and absorbs) certain characteristic frequencies of electromagnetic radiation and no others, but the Rutherford model cannot explain this phenomenon.
- A second difficulty is that Rutherford's electrons are described by the particle in uniform circular motion model; they have a centripetal acceleration. According to Maxwell's theory of electromagnetism, centripetally accelerated charges revolving with frequency f should radiate electromagnetic waves of frequency f.

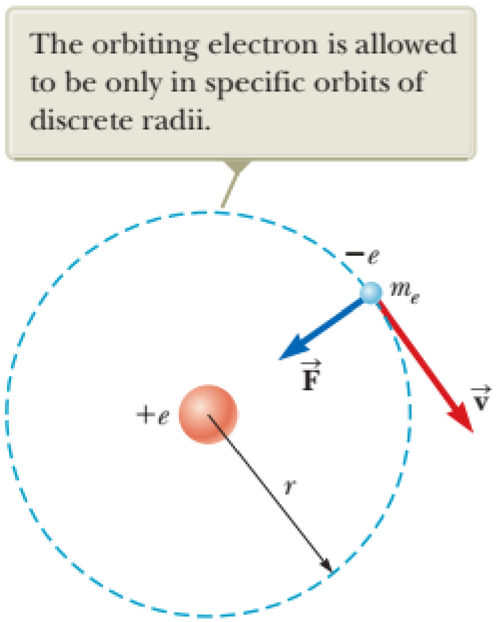
Bohr's Model of the Hydrogen Atom

Bohr applied Planck's ideas of quantized energy levels to Rutherford's orbiting atomic electrons.

Bohr combined ideas from Planck's original quantum theory, Einstein's concept of the photon, Rutherford's planetary model of the atom, and Newtonian mechanics to arrive at a semiclassical structural model based on some revolutionary ideas.

The structural model of the Bohr theory as it applies to the hydrogen atom has the following properties:

- The electron moves in circular orbits around the proton under the influence of the electric force of attraction
- the electron does not emit energy in the form of radiation, even though it is accelerating. Atom emits radiation when the electron makes a transition.



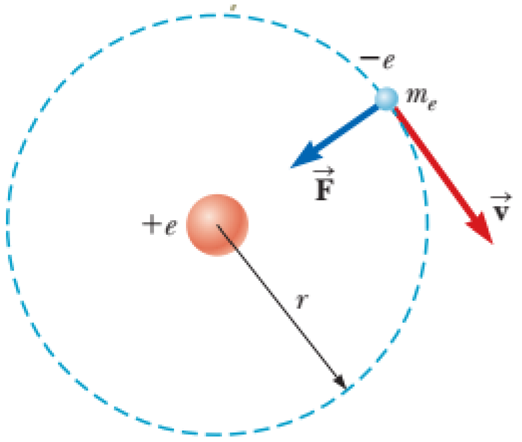
- the allowed orbits are those for which the electron's orbital angular momentum about the nucleus is quantized and equal to an integral multiple of " 5 h/2p

$$\boxed{m_e v r = n\hbar \qquad n = 1, 2, 3, \ldots}$$

let's calculate the allowed energy levels and find quantitative values of the emission wavelengths of the hydrogen atom.

The electric potential energy of the system

$$U = k_e q_1 q_2 / r = -k_e e^2 / r,$$



Therefore, the total energy of the atom, which consists of the electron's kinetic energy and the system's potential energy, is:

$$E = K + U = \tfrac{1}{2} m_e v^2 - k_e \frac{e^2}{r}$$

The electron is modeled as a particle in uniform circular motion

$$\frac{k_e e^2}{r^2} = \frac{m_e v^2}{r}$$

$$v^2 = \frac{k_e e^2}{m_e r}$$

$$\boxed{K = \tfrac{1}{2} m_e v^2 = \frac{k_e e^2}{2r}}$$

$$\boxed{E = -\frac{k_e e^2}{2r}}$$

Negative means bound (e, p) system

From $\boxed{m_e v r = n\hbar \quad n = 1, 2, 3, \ldots}$, one can find the radius *r* as by solve this eq. for *v* :

$$v^2 = \frac{n^2\hbar^2}{m_e^{\,2} r^2}$$

This equation is equal to the velocity found before

$$v^2 = \frac{n^2\hbar^2}{m_e^{\,2} r^2} = \frac{k_e e^2}{m_e r} \Rightarrow \boxed{r_n = \frac{n^2\hbar^2}{m_e k_e e^2} \quad n = 1, 2, 3, \ldots}$$

This equation shows that the radii of the allowed orbits have discrete values: they are quantized. The result is based on the *assumption* that the electron can exist only in certain allowed orbits determined by the integer *n* (Bohr's Property).

$$a_0 = \frac{\hbar^2}{m_e k_e e^2} = 0.052\,9 \text{ nm}$$

This constant (with n=1) is called Bohr radius (ao)

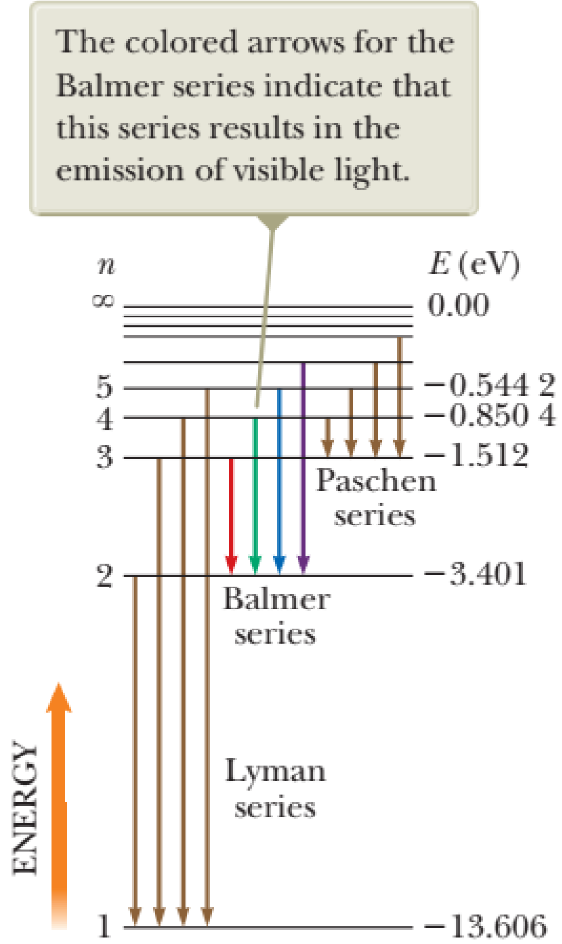
$$r_n = n^2 a_0 = n^2(0.052\,9 \text{ nm}) \quad n = 1, 2, 3, \ldots$$

$$E_n = -\frac{k_e e^2}{2a_0}\left(\frac{1}{n^2}\right) \quad n = 1, 2, 3, \ldots$$

OR

$$E_n = -\frac{13.606 \text{ eV}}{n^2} \quad n = 1, 2, 3, \ldots$$

The lowest allowed energy level, the ground state, has $n = 1$ and energy $E = -13.606$ eV. The next energy level, the first excited state, has $n = 2$ and energy $E = -3.401$ eV. The uppermost level corresponds to $n = \infty$ (or $r = \infty$) and $E = 0$



Notice how the allowed energies of the hydrogen atom differ from those of the particle in a box. The particle-in-a-box energies increase as n^2, so they become farther apart in energy as n increases. On the other hand, the energies of the hydrogen atom are inversely proportional to n^2, so their separation in energy becomes smaller as n increases. The separation between energy levels approaches zero as n approaches infinity and the energy approaches zero.

to calculate the frequency of the photon emitted when the electron makes a transition from an outer orbit to an inner orbit:

$$f = \frac{E_i - E_f}{h} = \frac{k_e e^2}{2a_0 h}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \rightarrow \frac{1}{\lambda} = \frac{f}{c} = \frac{k_e e^2}{2a_0 hc}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \rightarrow \boxed{\frac{1}{\lambda} = R_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)}$$

Remarkably, this expression, which is purely theoretical, is *identical* to the general form of the empirical relationships discovered by Balmer and Rydberg

Bohr extended his model for hydrogen to other elements in which all but one electron had been removed. These systems have the same structure as the hydrogen atom except that the nuclear charge is larger. Ionized elements such as He1, Li21, and Be31 were suspected to exist in hot stellar atmospheres, where atomic collisions frequently have enough energy to completely remove one or more atomic electrons. To describe a single electron orbiting a fixed nucleus of charge 1*Ze*, Bohr's theory gives

$$r_n = (n^2)\frac{a_0}{Z}$$

$$E_n = -\frac{k_e e^2}{2a_0}\left(\frac{Z^2}{n^2}\right) \quad n = 1, 2, 3, \ldots$$

Although the Bohr theory was triumphant in its agreement with some experimental results on the hydrogen atom, it suffered from some difficulties. One of the first indications that the Bohr theory needed to be modified arose when improved spectroscopic techniques were used to examine the spectral lines of hydrogen. It was found that many of the lines in the Balmer and other series were not single lines at all. Instead, each was a group of lines spaced very close together. An additional difficulty arose when it was observed that in some situations certain single spectral lines were split into three closely spaced lines when the atoms were placed in a strong magnetic field.

Bohr's Correspondence Principle

quantum physics agrees with classical physics when the difference between quantized levels becomes vanishingly small.

For example, consider an electron orbiting the hydrogen atom with $n > 10\ 000$. For such large values of n, the energy differences between adjacent levels approach zero; therefore, the levels are nearly continuous.

Example 42.1 Electronic Transitions in Hydrogen

(A) The electron in a hydrogen atom makes a transition from the $n = 2$ energy level to the ground level ($n = 1$). Find the wavelength and frequency of the emitted photon.

SOLUTION

Conceptualize Imagine the electron in a circular orbit about the nucleus as in the Bohr model in Figure 42.6. When the electron makes a transition to a lower stationary state, it emits a photon with a given frequency and drops to a circular orbit of smaller radius.

Categorize We evaluate the results using equations developed in this section, so we categorize this example as a substitution problem.

Use Equation 42.17 to obtain λ, with $n_i = 2$ and $n_f = 1$:

$$\frac{1}{\lambda} = R_H\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3R_H}{4}$$

$$\lambda = \frac{4}{3R_H} = \frac{4}{3(1.097 \times 10^7\ \text{m}^{-1})} = 1.22 \times 10^{-7}\ \text{m} = \boxed{122\ \text{nm}}$$

Use Equation 34.20 to find the frequency of the photon:

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^8\ \text{m/s}}{1.22 \times 10^{-7}\ \text{m}} = \boxed{2.47 \times 10^{15}\ \text{Hz}}$$

(B) In interstellar space, highly excited hydrogen atoms called Rydberg atoms have been observed. Find the wavelength to which radio astronomers must tune to detect signals from electrons dropping from the $n = 273$ level to the $n = 272$ level.

SOLUTION

Use Equation 42.17, this time with $n_i = 273$ and $n_f = 272$:

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = R_H\left(\frac{1}{(272)^2} - \frac{1}{(273)^2}\right) = 9.88 \times 10^{-8}\, R_H$$

Solve for λ:

$$\lambda = \frac{1}{9.88 \times 10^{-8} R_H} = \frac{1}{(9.88 \times 10^{-8})(1.097 \times 10^7 \text{ m}^{-1})} = \boxed{0.922 \text{ m}}$$

(C) What is the radius of the electron orbit for a Rydberg atom for which $n = 273$?

SOLUTION

Use Equation 42.12 to find the radius of the orbit:

$$r_{273} = (273)^2\,(0.052\,9 \text{ nm}) = \boxed{3.94\ \mu\text{m}}$$

This radius is large enough that the atom is on the verge of becoming macroscopic!

(D) How fast is the electron moving in a Rydberg atom for which $n = 273$?

SOLUTION

Solve Equation 42.8 for the electron's speed:

$$v = \sqrt{\frac{k_e e^2}{m_e r}} = \sqrt{\frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-19}\text{ C})^2}{(9.11 \times 10^{-31}\text{ kg})(3.94 \times 10^{-6}\text{ m})}}$$

$$= \boxed{8.01 \times 10^3 \text{ m/s}}$$

WHAT IF? What if radiation from the Rydberg atom in part (B) is treated classically? What is the wavelength of radiation emitted by the atom in the $n = 273$ level?

Answer Classically, the frequency of the emitted radiation is that of the rotation of the electron around the nucleus.

Calculate this frequency using the period defined in Equation 4.15:

$$f = \frac{1}{T} = \frac{v}{2\pi r}$$

Substitute the radius and speed from parts (C) and (D):

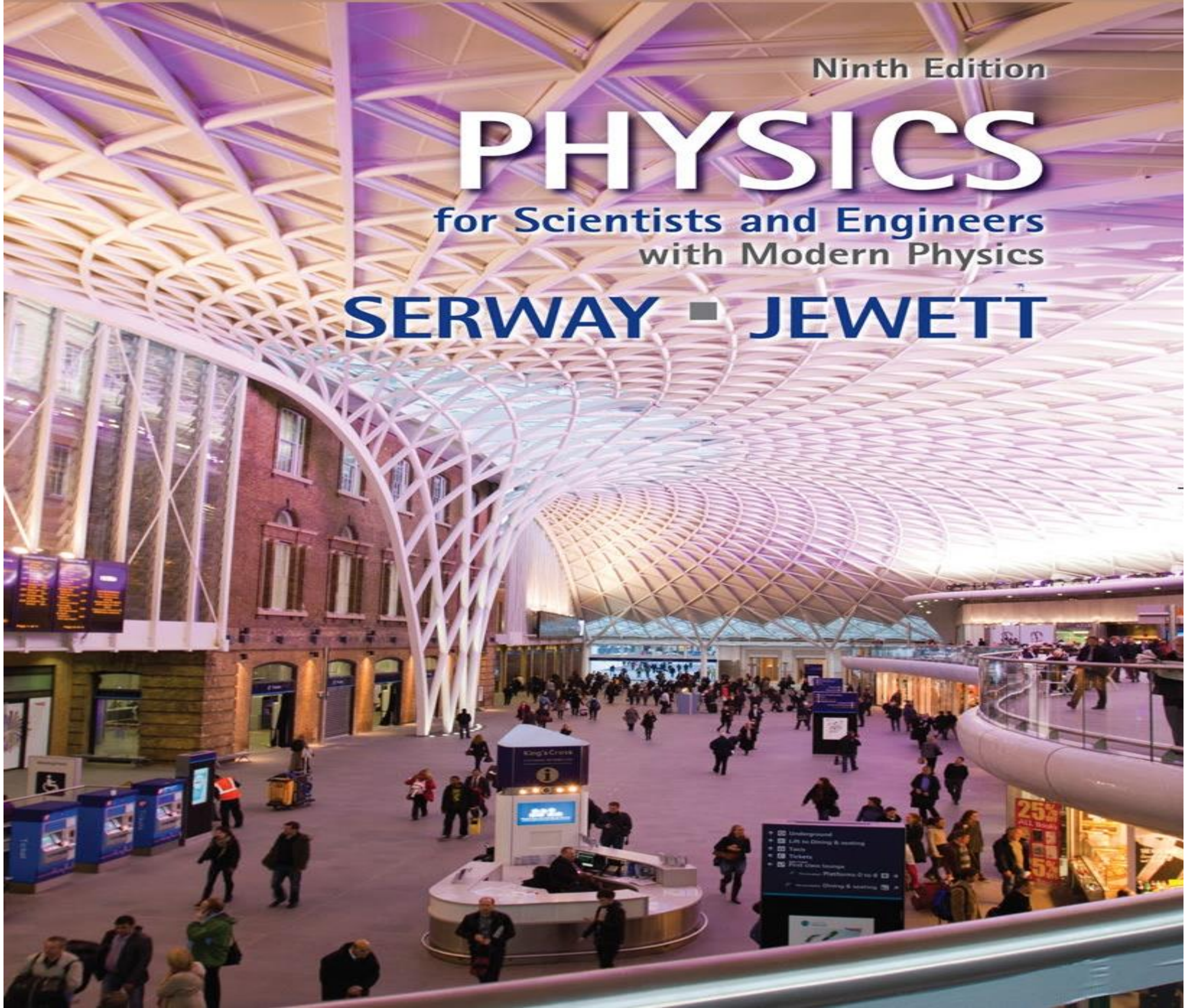
$$f = \frac{v}{2\pi r} = \frac{8.02 \times 10^3 \text{ m/s}}{2\pi(3.94 \times 10^{-6} \text{ m})} = 3.24 \times 10^8 \text{ Hz}$$

Find the wavelength of the radiation from Equation 34.20:

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{3.24 \times 10^8 \text{ Hz}} = 0.927 \text{ m}$$

This value is about 0.5% different from the wavelength calculated in part (B). As indicated in the discussion of Bohr's correspondence principle, this difference becomes even smaller for higher values of n.

Reference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