University of Al-Anbar
College of Applied Sciences-Heet
Biophysics Department

Modern Physics-Fourth Stage

Nuclear Physics

Lecture Five

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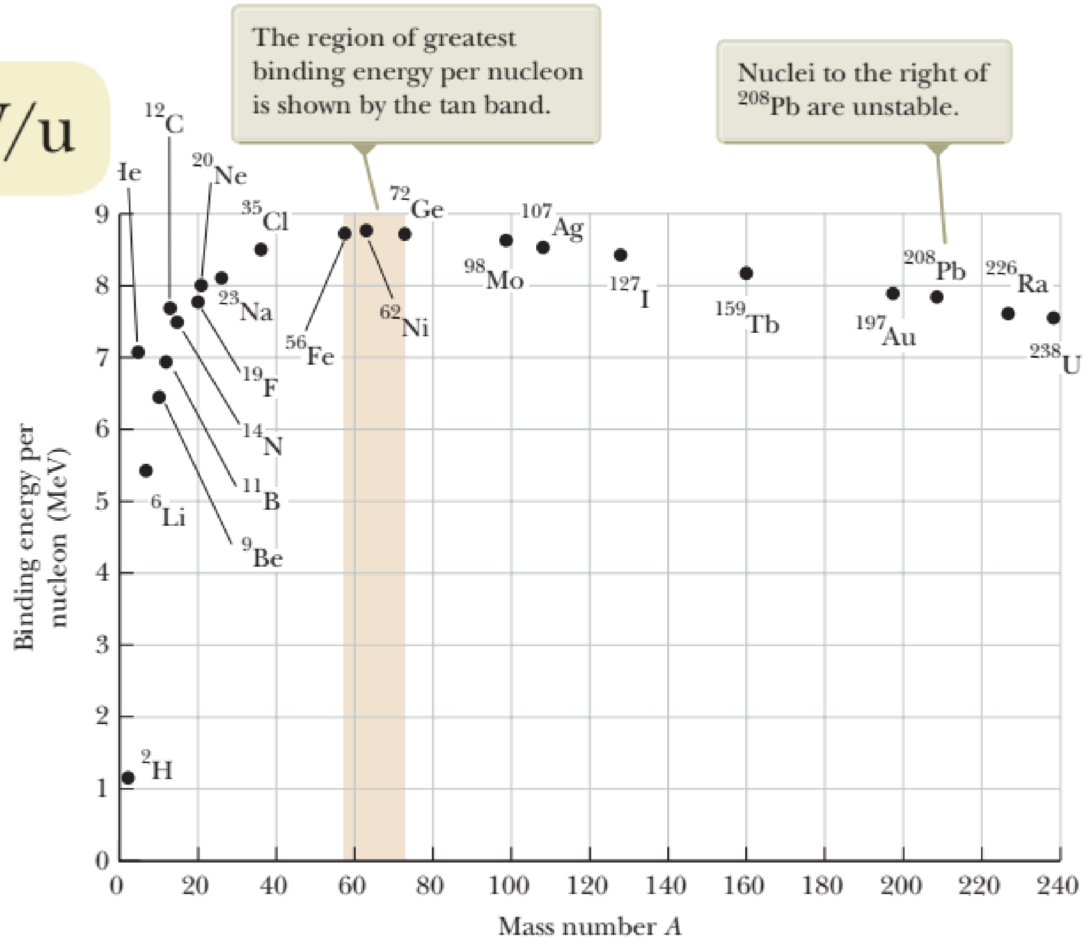
Nuclear Binding Energy

➢ The *binding energy* of the nucleus and can be interpreted as the energy that must be added to a nucleus to break it apart into its components. Its given by Einstein mass–energy equivalence relation as (mass of the Z electrons and its BE where canceled):

$$E_b = [ZM(\mathrm{H}) + Nm_n - M(^{A}_{Z}\mathrm{X})] \times 931.494\ \mathrm{MeV/u}$$



➢ the binding energy per nucleon is approximately constant at around 8 MeV per nucleon for all nuclei with A ≤ 50. For these nuclei, the nuclear forces are said to be *saturated*

Nuclear Models

All nuclei are composed of two types of particles: protons and neutrons

1- The Liquid-Drop Model

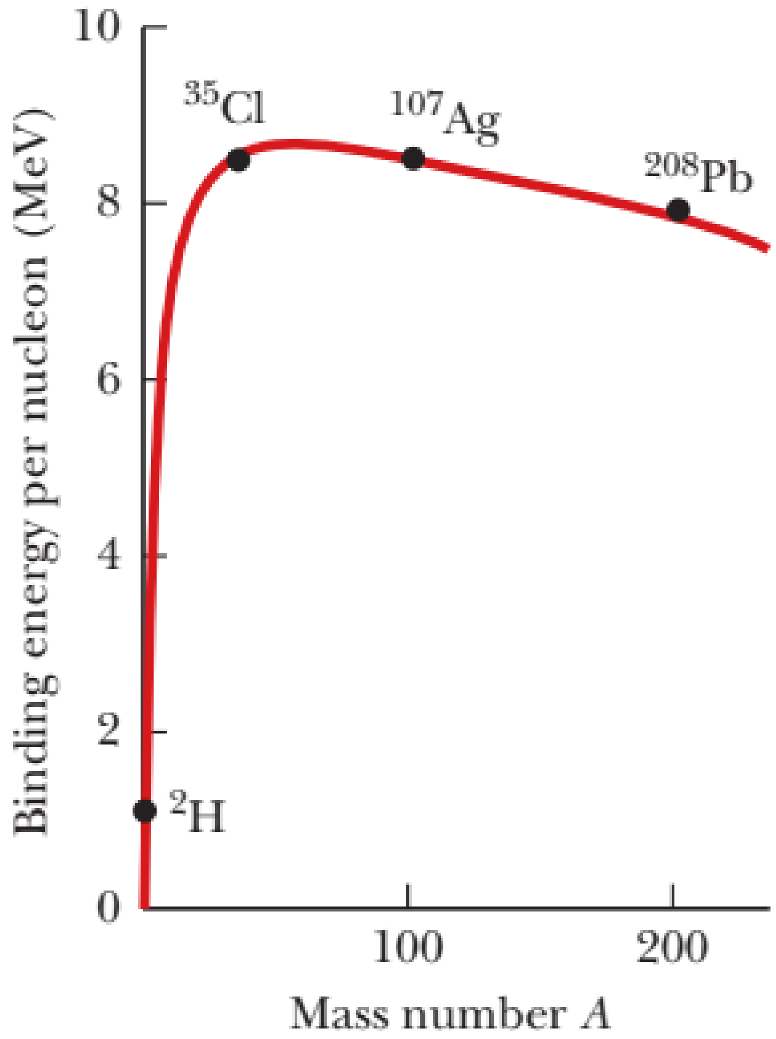
In 1936, Bohr proposed treating nucleons like molecules in a drop of liquid. Four major effects influence the binding energy of the nucleus in the liquid drop model:

The volume effect.

The surface effect.

The Coulomb repulsion effect.

The symmetry effect.



$$E_b = C_1 A - C_2 A^{2/3} - C_3 \frac{Z(Z-1)}{A^{1/3}} - C_4 \frac{(N-Z)^2}{A}$$

$C_1 = 15.7\ \text{MeV}$ $\quad C_2 = 17.8\ \text{MeV}$

$C_3 = 0.71\ \text{MeV}$ $\quad C_4 = 23.6\ \text{MeV}$

Example 44.3 Applying the Semiempirical Binding-Energy Formula

The nucleus ^{64}Zn has a tabulated binding energy of 559.09 MeV. Use the semiempirical binding-energy formula to generate a theoretical estimate of the binding energy for this nucleus.

SOLUTION

Conceptualize Imagine bringing the separate protons and neutrons together to form a ^{64}Zn nucleus. The rest energy of the nucleus is smaller than the rest energy of the individual particles. The difference in rest energy is the binding energy.

Categorize From the text of the problem, we know to apply the liquid-drop model. This example is a substitution problem.

For the ^{64}Zn nucleus, $Z = 30$, $N = 34$, and $A = 64$. Evaluate the four terms of the semiempirical binding-energy formula:

$$C_1 A = (15.7\text{ MeV})(64) = 1\,005\text{ MeV}$$

$$C_2 A^{2/3} = (17.8\text{ MeV})(64)^{2/3} = 285\text{ MeV}$$

$$C_3 \frac{Z(Z-1)}{A^{1/3}} = (0.71\text{ MeV})\frac{(30)(29)}{(64)^{1/3}} = 154\text{ MeV}$$

$$C_4 \frac{(N-Z)^2}{A} = (23.6\text{ MeV})\frac{(34-30)^2}{64} = 5.90\text{ MeV}$$

44.3 continued

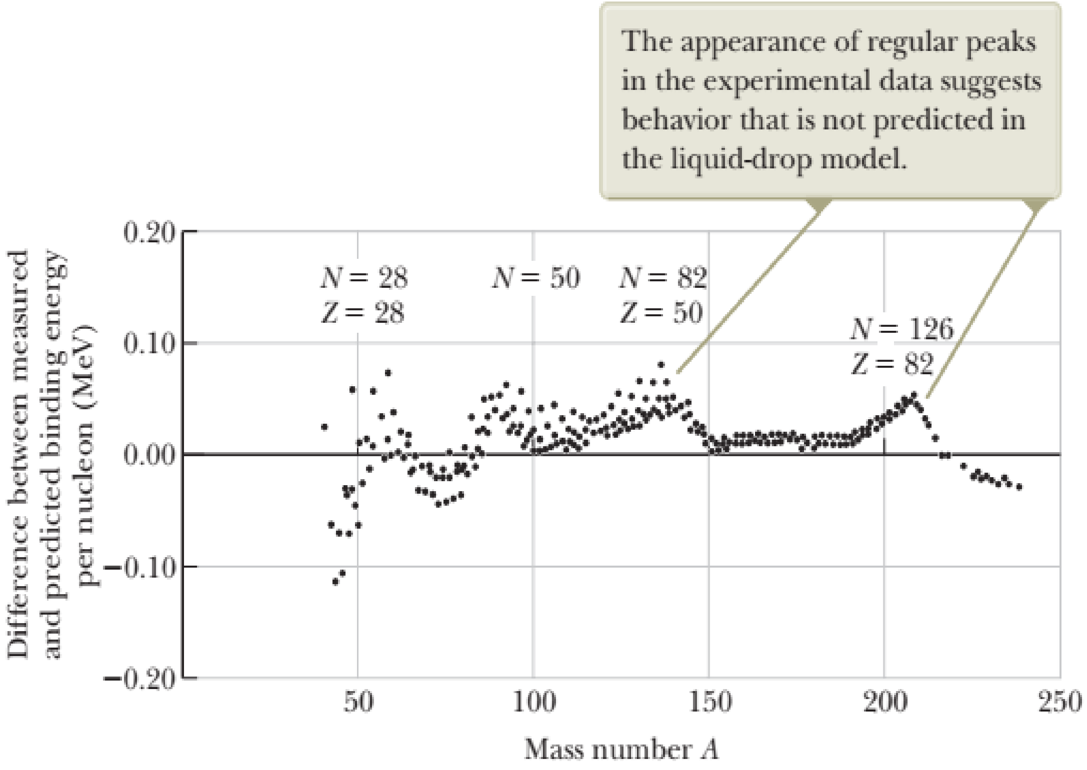
Substitute these values into Equation 44.3:

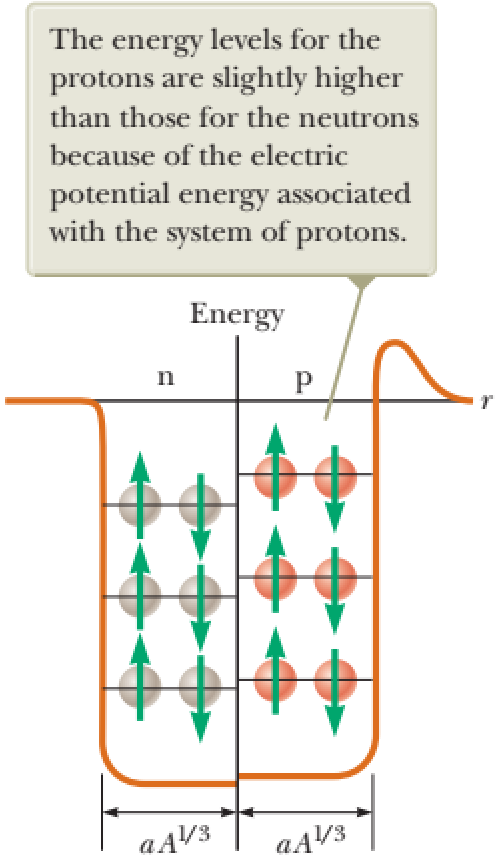
$$E_b = 1\,005\text{ MeV} - 285\text{ MeV} - 154\text{ MeV} - 5.90\text{ MeV} = \boxed{560\text{ MeV}}$$

This value differs from the tabulated value by less than 0.2%. Notice how the sizes of the terms decrease from the first to the fourth term. The fourth term is particularly small for this nucleus, which does not have an excessive number of neutrons.

2- The shell Model

When the binding energies are studied more closely we find the following features:
Most stable nuclei have an even value of A. Furthermore, only eight stable nuclei have odd values for both Z and N.





There is evidence for regularly spaced peaks in the data that are not described by the semiempirical binding-energy formula. The peaks occur at values of N or Z that have become known as magic numbers:

$$Z \text{ or } N = 2, 8, 20, 28, 50, 82$$

- In this model, each nucleon is assumed to exist in a shell, similar to an atomic shell for an electron. The nucleons exist in quantized energy states.

Radioactivity

Three types of radioactive decay occur in radioactive substances:

- Alpha decay (4He nuclei);
- Beta decay, (electrons or positrons);
- and gamma decay, (high-energy photons).

The rate at which a particular decay process occurs in a sample is proportional to the number of radioactive nuclei present (that is, the number of nuclei that have not yet decayed).

- If *N* is the number of undecayed radioactive nuclei present at some instant, the rate of change of *N* with time is
- The decay rate *R*, which is the number of decays per second
- The half-life of a radioactive substance is the time interval during which half of a given number of radioactive nuclei decay.

$$\frac{dN}{N} = -\lambda\, dt$$

$$N = N_0 e^{-\lambda t}$$

$$R = \left|\frac{dN}{dt}\right| = \lambda N = \lambda N_0 e^{-\lambda t} = R_0 e^{-\lambda t}$$

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

$$\boxed{N = N_0 \left(\tfrac{1}{2}\right)^n}$$

Example 44.4 How Many Nuclei Are Left?

The isotope carbon-14, ${}^{14}_{6}C$, is radioactive and has a half-life of 5 730 years. If you start with a sample of 1 000 carbon-14 nuclei, how many nuclei will still be undecayed in 25 000 years?

SOLUTION

Conceptualize The time interval of 25 000 years is much longer than the half-life, so only a small fraction of the originally undecayed nuclei will remain.

Categorize The text of the problem allows us to categorize this example as a substitution problem involving radioactive decay.

Analyze Divide the time interval by the half-life to determine the number of half-lives:

$$n = \frac{25\,000 \text{ yr}}{5\,730 \text{ yr}} = 4.363$$

Determine how many undecayed nuclei are left after this many half-lives using Equation 44.9:

$$N = N_0\left(\tfrac{1}{2}\right)^n = 1\,000\left(\tfrac{1}{2}\right)^{4.363} = \boxed{49}$$

Finalize As we have mentioned, radioactive decay is a probabilistic process and accurate statistical predictions are possible only with a very large number of atoms. The original sample in this example contains only 1 000 nuclei, which is certainly not a very large number. Therefore, if you counted the number of undecayed nuclei remaining after 25 000 years, it might not be exactly 49.

Example 44.5 The Activity of Carbon

At time $t = 0$, a radioactive sample contains 3.50 μg of pure $^{11}_{6}C$, which has a half-life of 20.4 min.

(A) Determine the number N_0 of nuclei in the sample at $t = 0$.

SOLUTION

Conceptualize The half-life is relatively short, so the number of undecayed nuclei drops rapidly. The molar mass of $^{11}_{6}C$ is approximately 11.0 g/mol.

Categorize We evaluate results using equations developed in this section, so we categorize this example as a substitution problem.

Find the number of moles in 3.50 μg of pure $^{11}_{6}C$:

$$n = \frac{3.50 \times 10^{-6}\ \text{g}}{11.0\ \text{g/mol}} = 3.18 \times 10^{-7}\ \text{mol}$$

Find the number of undecayed nuclei in this amount of pure $^{11}_{6}C$:

$$N_0 = (3.18 \times 10^{-7}\ \text{mol})(6.02 \times 10^{23}\ \text{nuclei/mol}) = \boxed{1.92 \times 10^{17}\ \text{nuclei}}$$

(B) What is the activity of the sample initially and after 8.00 h?

SOLUTION

Find the initial activity of the sample using Equations 44.7 and 44.8:

$$R_0 = \lambda N_0 = \frac{0.693}{T_{1/2}} N_0 = \frac{0.693}{20.4\ \text{min}} \left(\frac{1\ \text{min}}{60\ \text{s}}\right)(1.92 \times 10^{17})$$

$$= (5.66 \times 10^{-4}\ \text{s}^{-1})(1.92 \times 10^{17}) = \boxed{1.09 \times 10^{14}\ \text{Bq}}$$

44.5 continued

Use Equation 44.7 to find the activity at $t = 8.00\ \text{h} = 2.88 \times 10^4$ s:

$$R = R_0 e^{-\lambda t} = (1.09 \times 10^{14}\ \text{Bq})e^{-(5.66 \times 10^{-4}\ \text{s}^{-1})(2.88 \times 10^4\ \text{s})} = \boxed{8.96 \times 10^6\ \text{Bq}}$$

Example 44.6 A Radioactive Isotope of Iodine

A sample of the isotope ^{131}I, which has a half-life of 8.04 days, has an activity of 5.0 mCi at the time of shipment. Upon receipt of the sample at a medical laboratory, the activity is 2.1 mCi. How much time has elapsed between the two measurements?

SOLUTION

Conceptualize The sample is continuously decaying as it is in transit. The decrease in the activity is 58% during the time interval between shipment and receipt, so we expect the elapsed time to be greater than the half-life of 8.04 d.

Categorize The stated activity corresponds to many decays per second, so N is large and we can categorize this problem as one in which we can use our statistical analysis of radioactivity.

Analyze Solve Equation 44.7 for the ratio of the final activity to the initial activity:

$$\frac{R}{R_0} = e^{-\lambda t}$$

Take the natural logarithm of both sides:

$$\ln\left(\frac{R}{R_0}\right) = -\lambda t$$

Solve for the time t:

$$(1) \quad t = -\frac{1}{\lambda}\ln\left(\frac{R}{R_0}\right)$$

Use Equation 44.8 to substitute for λ:

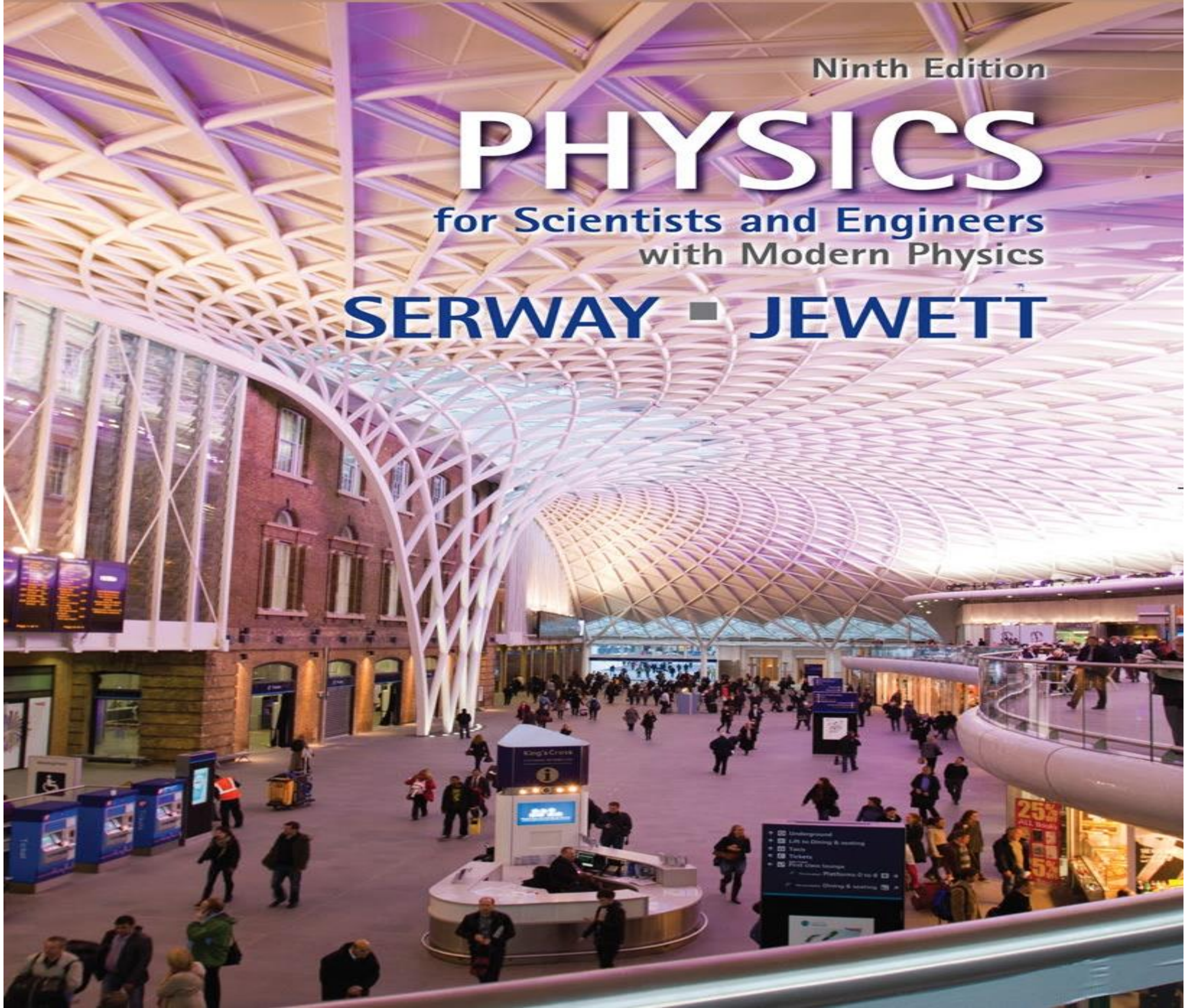
$$t = -\frac{T_{1/2}}{\ln 2}\ln\left(\frac{R}{R_0}\right)$$

Substitute numerical values:

$$t = -\frac{8.04\text{ d}}{0.693}\ln\left(\frac{2.1\text{ mCi}}{5.0\text{ mCi}}\right) = \boxed{10\text{ d}}$$

Thank you
For your Attention

Reference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