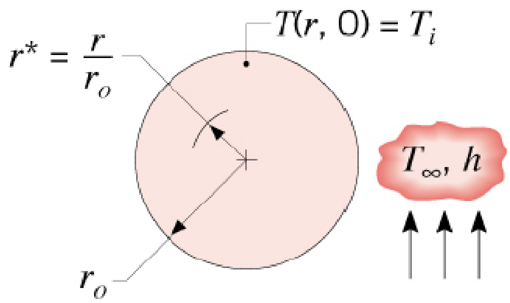
Lecture Nine

Unsteady Radial System

1- Radial System Coordinate.

Long Rods or Spheres Heated or Cooled by convection mechanism.

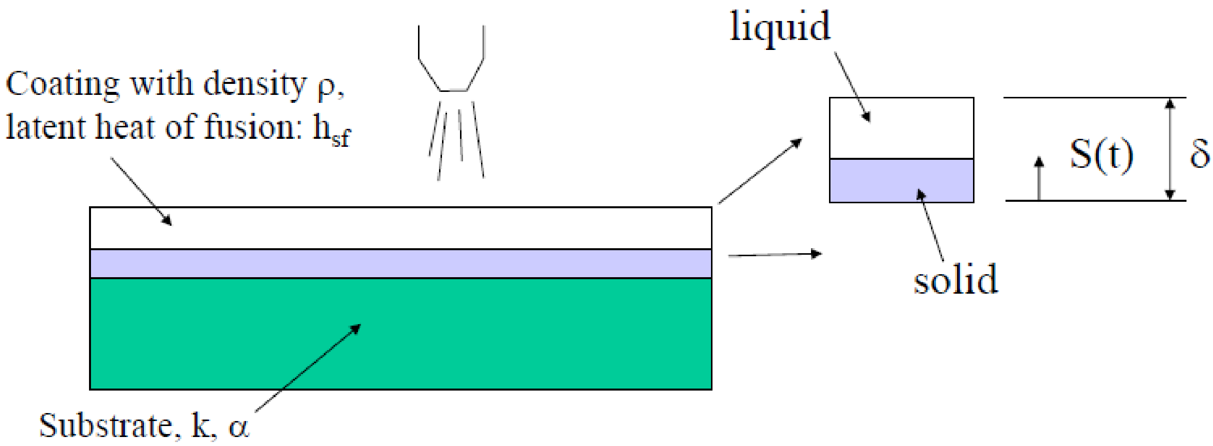


Similar Heisler charts are available for radial systems in standard text books.

Important tips: Pay attention to the length scale used in those charts, and calculate your Biot number accordingly.

2- Unsteady Heat Transfer in Semi-infinite Solids.

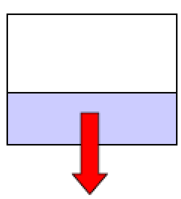
❑ Solidification process of the coating layer during a thermal spray operation is an unsteady heat transfer problem. As we discuss earlier, thermal spray process deposits thin layer of coating materials on surface for protection and thermal resistant purposes, as shown. The heated, molten materials will attach to the substrate and cool down rapidly. The cooling process is important to prevent the accumulation of residual thermal stresses in the coating layer.



Example

❑ As described in the previous slide, the cooling process can now be modeled as heat loss through a semi-infinite solid. (Since the substrate is significantly thicker than the coating layer) The molten material is at the fusion temperature T_f and the substrate is maintained at a constant temperature T_i. Derive an expression for the total time that is required to solidify the coating layer of thickness δ.

❑ Assume the molten layer stays at a constant temperature T_f throughout the process. The heat loss to the substrate is solely supplied by the release of the latent heat of fusion.



Heat transfer from the molten material to the substrate (q=q"A)

From energy balance:

$$h_{sf}\Delta m \text{(solidified mass during } \Delta t) = \Delta Q = q'' A \Delta t \text{ (energy input)}$$

$$h_{sf}\frac{dm}{dt} = q'' A, \text{ where } m = \rho V = \rho A S,$$

where S is solidified thickness

$$\rho\frac{dS}{dt} = q''$$

❑ Identify that the previous situation corresponds to the case of a semi-infinite transient heat transfer problem with a constant surface temperature boundary condition. This boundary condition can be modeled as a special case of convection boundary condition case by setting h=∞, therefore, $T_s=T_\infty$).

If the surface temperature is T_s and the initial temperature of the bolck is T_i, the analytical solution of the problem can be found: The temperature distribution and the heat transfer into the block are:

$\frac{T(x,t)-T_s}{T_i-T_s}=erf\left(\frac{x}{2\sqrt{\alpha t}}\right)$, where erf( ) is the Gaussian error function.

It is defined as $\text{erf}(w)=\frac{2}{\sqrt{\pi}}\int_0^w e^{-v^2}dv$

$$q_s''(t)=\frac{k(T_s-T_i)}{\sqrt{\pi\alpha t}}$$

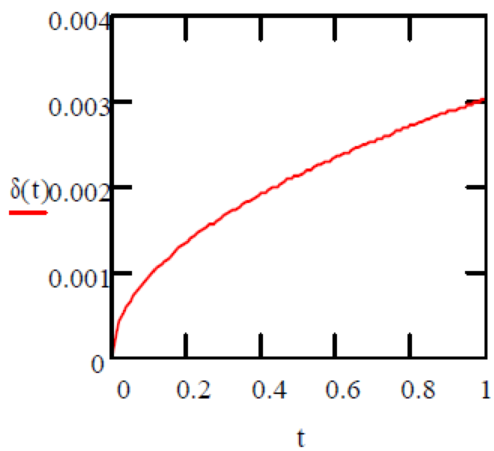
From the previous equation

$$\rho h_{sf}\frac{dS}{dt}=q''=\frac{k(T_f-T_i)}{\sqrt{\pi\alpha t}},\quad \text{and}\int_0^\delta dS=\frac{k(T_f-T_i)}{\rho h_{sf}\sqrt{\pi\alpha}}\int_0^t\frac{dt}{\sqrt{t}}$$

$\delta(t)=\frac{2k(T_f-T_i)}{\rho h_{sf}\sqrt{\pi\alpha}}\sqrt{t}$, therefore, $\delta\propto\sqrt{t}$. Cooling time $t=\frac{\pi\alpha}{4k^2}\left(\frac{\delta\rho h_{sf}}{T_f-T_i}\right)^2$

❑ Use the following values to calculate: k=120 W/m.K, $\alpha=4\times10^{-5}$ m^2/s, $\rho=3970$ kg/m^3, and $h_{sf}=3.577\times10^6$ J/kg, $T_f=2318$ K, $T_i=300$K, and $\delta=2$ mm

$$\delta(t)=\frac{2k(T_f-T_i)}{\rho h_{sf}\sqrt{\pi\alpha}}\sqrt{t}=0.00304\sqrt{t}$$



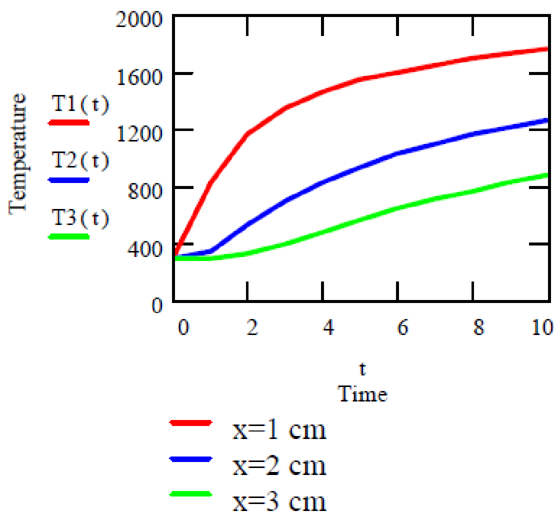
- $\delta(t) \propto t^{1/2}$
- Therefore, the layer solidifies very fast initially and then slows down as shown in the figure
- Note: we neglect contact resistance between the coating and the substrate and assume temperature of the coating material stays the same even after it solidifies.
- To solidify 2 mm thickness, it takes 0.43 seconds.
- What will be the substrate temperature as it varies in time? The temperature distribution is:

$$\frac{T(x,t)-T_S}{T_i-T_S} = erf\left(\frac{x}{2\sqrt{\alpha t}}\right),$$

$$T(x,t) = 2318 + (300-2318)erf\left(\frac{x}{2\sqrt{\alpha t}}\right) = 2318 - 2018 erf\left(79.06\frac{x}{\sqrt{t}}\right)$$

- For a fixed distance away from the surface, we can examine the variation of the temperature as a function of time. Example, 1 cm deep into the substrate the temperature should behave as:

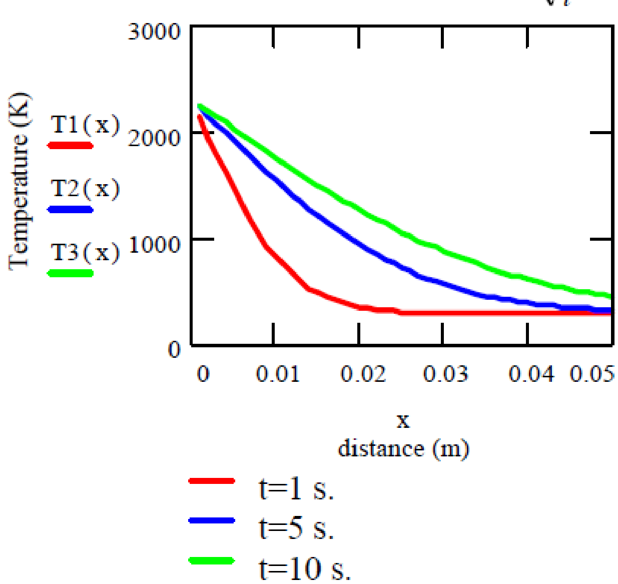
$$T(x=0.01,t) = 2318 - 2018 erf\ 79.06\frac{x}{\sqrt{t}} = 2318 - 2018 erf\ \frac{0.79}{\sqrt{t}}$$



- At x=1 cm, the temperature rises almost instantaneously at a very fast rate. A short time later, the rate of temp. increase slows down significantly since the energy has to distribute to a very large mass.
- At deeper depth (x=2 & 3 cm), the temperature will not respond to the surface condition until much later.

- We can also examine the spatial temperature distribution at any given time, say at t=1 second.

$$T(x,t=1) = 2318 - 2018\,erf\ 79.06\frac{x}{\sqrt{t}} = 2318 - 2018\,erf\ 79.06x$$



- Heat penetrates into the substrate as shown for different time instants.
- It takes more than 5 seconds for the energy to transfer to a depth of 5 cm into the substrate
- The slopes of the temperature profiles indicate the amount of conduction heat transfer at that instant.