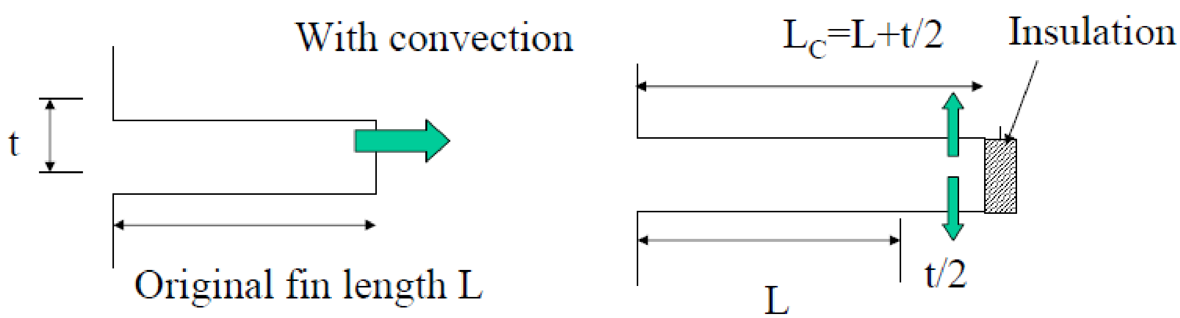
Lecture Five

Fin Specifications and Design

1- Correction Length.

- In some situations, it might be necessary to include the convective heat transfer at the tip. However, one would like to avoid using the long equation as described in case A, fins table. The alternative is to use case B instead and accounts for the convective heat transfer at the tip by extending the fin length L to $L_C = L + (t/2)$.



Then apply the adiabatic condition at the tip of the extended fin as shown above.

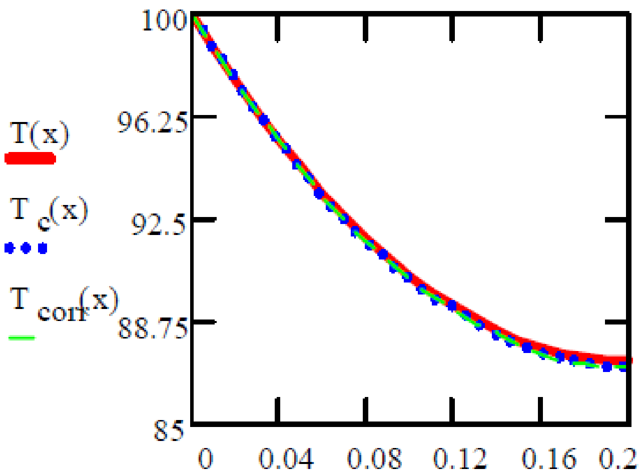
Use the same example: aluminum pot handle, m=3.138, the length will need to be corrected to

$L_C = l + (t/2) = 0.2 + 0.0025 = 0.2025 (m)$

$$\frac{T_{corr}(x) - T_\infty}{T_b - T_\infty} = \frac{\theta}{\theta_b} = \frac{\cosh m(L_c - x)}{\cosh mL_c}$$

$$\frac{T_{corr} - 25}{100 - 25} = \frac{\cosh[3.138(0.2025 - x)]}{\cosh(3.138 * 0.2025)},$$

$$T_{corr}(x) = 25 + 62.05 * \cosh[3.138(0.2025 - x)]$$



$T(0.2)=87.32\ °C$
$T_c(0.2)=87.09\ °C$
$T_{corr}(0.2025)=87.05\ °C$

slight improvement over the uncorrected solution

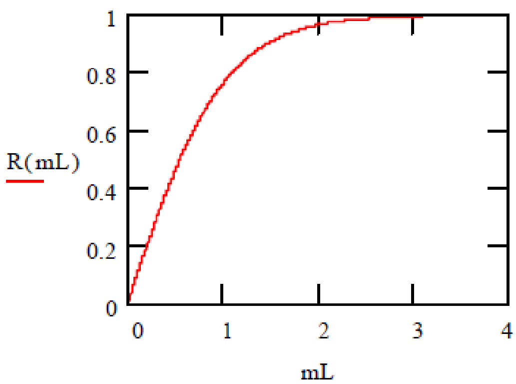
- The correction length can be determined by using the formula: $L_c=L+(A_c/P)$, where A_c is the cross-sectional area and P is the perimeter of the fin at the tip.

- Thin rectangular fin: $A_c=Wt$, $P=2(W+t)\approx 2W$, since $t \ll W$
$L_c=L+(A_c/P)=L+(Wt/2W)=L+(t/2)$

- Cylindrical fin: $A_c=(\pi/4)D^2$, $P=\pi D$, $L_c=L+(A_c/P)=L+(D/4)$

- Square fin: $A_c=W^2$, $P=4W$,
$L_c=L+(A_c/P)=L+(W^2/4W)=L+(W/4)$

❑ In general, the longer the fin, the higher the heat transfer. However, a long fin means more material and increased size and cost. Question: how do we determine the optimal fin length?
Use the rectangular fin as an example:



$q_f = M \tanh mL$, for an adiabatic tip fin

$(q_f)_\infty = M$, for an infinitely long fin

Their ratio: $R(mL) = \dfrac{q_f}{(q_f)_\infty} = \tanh mL$

Note: heat transfer increases with mL as expected. Initially the rate of change is large and slows down drastically when mL> 2.

R(1)=0.762, means any increase beyond mL=1 will increase no more than 23.8% of the fin heat transfer.

2- <u>Temperature Distribution.</u>

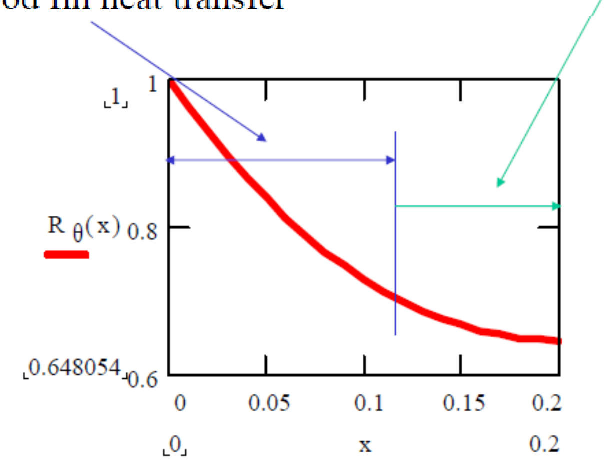
For an adiabatic tip fin case:

$$R_\theta = \frac{T - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L-x)}{\cosh mL}$$

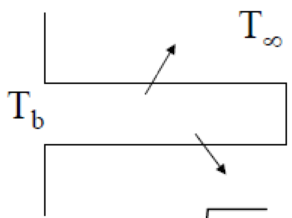
➢ Use m=5, and L=0.2 as an example:

High ΔT, good fin heat transfer

Low ΔT, poor fin heat transfer



3- Fin Design.



Total heat loss: q_f=Mtanh(mL) for an adiabatic fin, or q_f=Mtanh(mL_C) if there is convective heat transfer at the tip

where $m=\sqrt{\frac{hP}{kA_c}}$, and $M=\sqrt{hPkA_C}\,\theta_b=\sqrt{hPkA_C}\,(T_b-T_\infty)$

Use the thermal resistance concept:

$$q_f=\sqrt{hPkA_C}\tanh(mL)(T_b-T_\infty)=\frac{(T_b-T_\infty)}{R_{t,f}}$$

where $R_{t,f}$ is the thermal resistance of the fin.

For a fin with an adiabatic tip, the fin resistance can be expressed as

$$R_{t,f}=\frac{(T_b-T_\infty)}{q_f}=\frac{1}{\sqrt{hPkA_C}[\tanh(mL)]}$$

4- Fin Effectiveness.

fin effectiveness ε_f: Ratio of fin heat transfer and the heat transfer without the fin. For an adiabatic fin:

$$\varepsilon_f=\frac{q_f}{q}=\frac{q_f}{hA_C(T_b-T_\infty)}=\frac{\sqrt{hPkA_C}\tanh(mL)}{hA_C}=\sqrt{\frac{kP}{hA_C}}\tanh(mL)$$

If the fin is long enough, mL>2, tanh(mL) $\rightarrow$ 1,

it can be considered an infinite fin (case D of table3.4)

$$\varepsilon_f\rightarrow\sqrt{\frac{kP}{hA_C}}=\sqrt{\frac{k}{h}\left(\frac{P}{A_C}\right)}$$

In order to enhance heat transfer, $\varepsilon_f>1$.

However, $\varepsilon_f\geq 2$ will be considered justifiable

If $\varepsilon_f<1$ then we have an insulator instead of a heat fin

$$\varepsilon_f \rightarrow \sqrt{\frac{kP}{hA_C}} = \sqrt{\frac{k}{h}\left(\frac{P}{A_C}\right)}$$

- To increase ε_f, the fin's material should have higher thermal conductivity, k.
- It seems to be counterintuitive that the lower convection coefficient, h, the higher ε_f. But it is not because if h is very high, it is not necessary to enhance heat transfer by adding heat fins. Therefore, heat fins are more effective if h is low. Observation: If fins are to be used on surfaces separating gas and liquid. Fins are usually placed on the gas side. (Why?)
- P/AC should be as high as possible. Use a square fin with a dimension of W by W as an example: P=4W, AC=W2, P/AC=(4/W). The smaller W, the higher the P/AC, and the higher εf.
- Conclusion: It is preferred to use thin and closely spaced (to increase the total number) fins.

The effectiveness of a fin can also be characterized as

$$\varepsilon_f = \frac{q_f}{q} = \frac{q_f}{hA_C(T_b - T_\infty)} = \frac{(T_b - T_\infty)/R_{t,f}}{(T_b - T_\infty)/R_{t,h}} = \frac{R_{t,h}}{R_{t,f}}$$

It is a ratio of the thermal resistance due to convection to the thermal resistance of a fin. In order to enhance heat transfer, the fin's resistance should be lower than that of the resistance due only to convection.

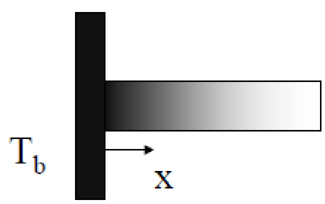
5- Fin Efficiency.

Define Fin efficiency: $\eta_f = \dfrac{q_f}{q_{max}}$

where q_{max} represents an idealized situation such that the fin is made up of material with infinite thermal conductivity. Therefore, the fin should be at the same temperature as the temperature of the base.

$$q_{max} = hA_f(T_b - T_\infty)$$

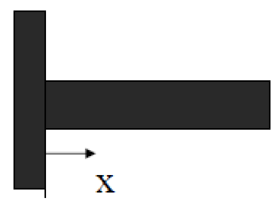
$T(x) < T_b$ for heat transfer to take place



Total fin heat transfer q_f

Real situation

For infinite k
$T(x) = T_b$, the heat transfer is maximum



Ideal heat transfer q_{max}

Ideal situation

Use an adiabatic rectangular fin as an example:

$$\eta_f = \frac{q_f}{q_{max}} = \frac{M \tanh mL}{hA_f(T_b - T_\infty)} = \frac{\sqrt{hPkA_c}(T_b - T_\infty)\tanh mL}{hPL(T_b - T_\infty)}$$

$$= \frac{\tanh mL}{\sqrt{\frac{hP}{kA_c}}L} = \frac{\tanh mL}{mL} \quad \text{(see Table 3.5 for } \eta_f \text{ of common fins)}$$

The fin heat transfer: $q_f = \eta_f q_{max} = \eta_f hA_f(T_b - T_\infty)$

$$q_f = \frac{T_b - T_\infty}{1/(\eta_f hA_f)} = \frac{T_b - T_\infty}{R_{t,f}}, \text{ where } R_{t,f} = \frac{1}{\eta_f hA_f}$$

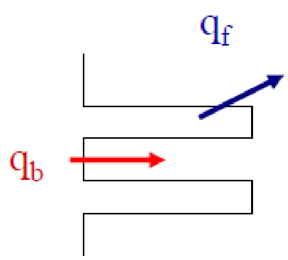
Thermal resistance for a single fin.

As compared to convective heat transfer: $R_{t,b} = \dfrac{1}{hA_b}$

In order to have a lower resistance as that is required to enhance heat transfer: $R_{t,b} > R_{t,f}$ or $A_b < \eta_f A_f$

6- Overall Fin Efficiency.

Overall fin efficiency for an array of fins:



Define terms: A_b: base area exposed to coolant

A_f: surface area of a single fin

A_t: total area including base area and total finned surface, $A_t = A_b + NA_f$

N: total number of fins

$$q_t = q_b + Nq_f = hA_b(T_b - T_\infty) + N\eta_f hA_f(T_b - T_\infty)$$

$$= h[(A_t - NA_f) + N\eta_f A_f](T_b - T_\infty) = h[A_t - NA_f(1-\eta_f)](T_b - T_\infty)$$

$$= hA_t[1 - \frac{NA_f}{A_t}(1-\eta_f)](T_b - T_\infty) = \eta_O hA_t(T_b - T_\infty)$$

Define overall fin efficiency: $\eta_O = 1 - \frac{NA_f}{A_t}(1-\eta_f)$

7- Heat Transfer from a Fin Array.

$$q_t = hA_t\eta_O(T_b - T_\infty) = \frac{T_b - T_\infty}{R_{t,O}} \quad \text{where } R_{t,O} = \frac{1}{hA_t\eta_O}$$

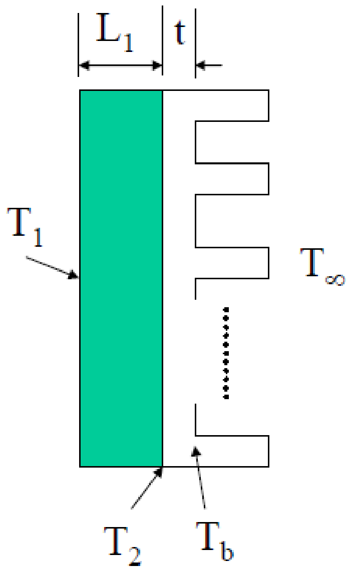
Compare to heat transfer without fins

$$q = hA(T_b - T_\infty) = h(A_b + NA_{b,f})(T_b - T_\infty) = \frac{1}{hA}$$

where $A_{b,f}$ is the base area (unexposed) for the fin

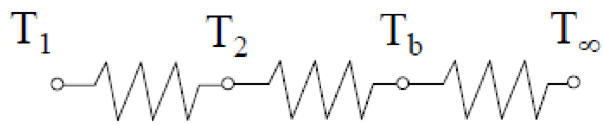
To enhance heat transfer $A_t\eta_O >> A$

That is, to increase the effective area $\eta_O A_t$.



$$A=A_b+NA_{b,f}$$

$$R_b=t/(k_b A)$$



$$R_1=L_1/(k_1 A) \qquad R_{t,O}=1/(hA_t\eta_O)$$

$$q=\frac{T_1-T_\infty}{\sum R}=\frac{T_1-T_\infty}{R_1+R_b+R_{t,O}}$$