



Lecture Four

Extended Surface Heat Transfer

1- Extended Surface- Fins.

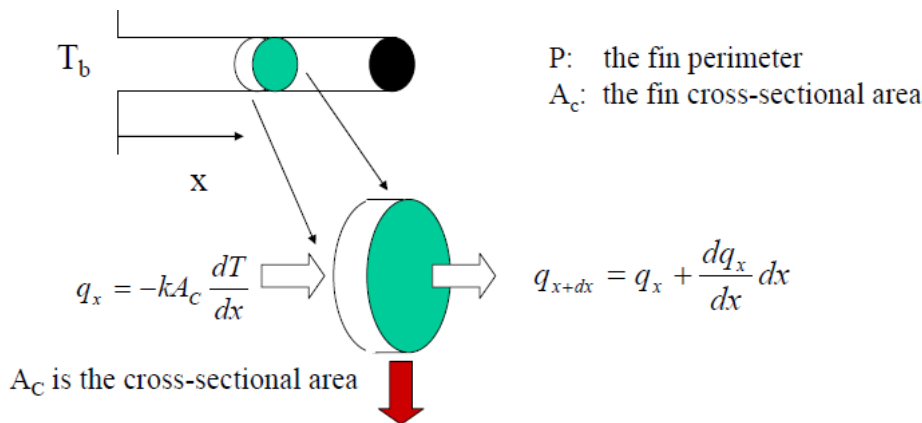
Convection: Heat transfer between a solid surface and a moving fluid is governed by the Newton's cooling law: $q = hA(T_s - T_\infty)$. Therefore, to increase the convective heat transfer, one can

❑ Increase the temperature difference $(T_s - T_\infty)$ between the surface and the fluid.

❑ Increase the convection coefficient h . This can be accomplished by increasing the fluid flow over the surface since h is a function of the flow velocity and the higher the velocity, the higher the h . Example: a cooling fan.

❑ Increase the contact surface area A . Example: a heat sink with fins.

2- Extended Surface Analysis.



$dq_{conv} = h(dA_s)(T - T_\infty)$, where dA_s is the surface area of the element

Energy Balance: $q_x = q_{x+dx} + dq_{conv} = q_x + \frac{dq_x}{dx} dx + h dA_s (T - T_\infty)$

$$-kA_c \frac{d^2 T}{dx^2} dx + hP(T - T_\infty) dx = 0, \text{ if } k, A_c \text{ are all constants.}$$



$$\frac{d^2 T}{dx^2} - \frac{hP}{kA_c} (T - T_\infty) = 0, \text{ A second - order, ordinary differential equation}$$

Define a new variable $\theta(x) = T(x) - T_\infty$, so that

$$\frac{d^2 \theta}{dx^2} - m^2 \theta = 0, \text{ where } m^2 = \frac{hP}{kA_c}, (D^2 - m^2)\theta = 0$$

Characteristics equation with two real roots: $+m$ & $-m$

The general solution is of the form

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

To evaluate the two constants C_1 and C_2 , we need to specify two boundary conditions:

The first one is obvious: the base temperature is known as $T(0) = T_b$

The second condition will depend on the end condition of the tip

For example: assume the tip is insulated and no heat transfer $d\theta/dx(x=L)=0$

The temperature distribution is given by

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\theta}{\theta_b} = \frac{\cosh m(L - x)}{\cosh mL}$$

The fin heat transfer rate is

$$q_f = -kA_c \frac{dT}{dx}(x = 0) = \sqrt{hPkA_c} \tanh mL = M \tanh mL$$

These results and other solutions using different end conditions are

Tabulated in HT Textbooks

3- Temperature Distribution for Fins.

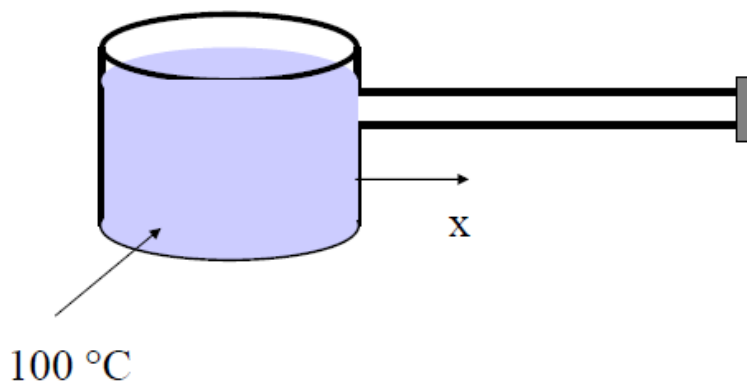
Case	Tip Condition	Temp. Distribution	Fin heat transfer
A	Convection heat transfer: $h\theta(L) = -k(d\theta/dx)_{x=L}$	$\frac{\cosh m(L-x) + (h/mk) \sinh m(L-x)}{\cosh mL + (h/mk) \sinh mL}$	$M\theta_0 \frac{\sinh mL + (h/mk) \cosh mL}{\cosh mL + (h/mk) \sinh mL}$
B	Adiabatic $(d\theta/dx)_{x=L} = 0$	$\frac{\cosh m(L-x)}{\cosh mL}$	$M\theta_0 \tanh mL$
C	Given temperature: $\theta(L) = \theta_L$	$\frac{(\theta_L/\theta_b) \sinh m(L-x) + \sinh m(L-x)}{\sinh mL}$	$M\theta_0 \frac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$
D	Infinitely long fin $\theta(L) = 0$	e^{-mx}	$M\theta_0$

$$\theta \equiv T - T_\infty, \quad m^2 \equiv \frac{hP}{kA_c}$$

$$\theta_b = \theta(0) = T_b - T_\infty, \quad M = \sqrt{hPkA_c} \theta_b$$

Example

□ An Aluminum pot is used to boil water as shown below. The handle of the pot is 20-cm long, 3-cm wide, and 0.5-cm thick. The pot is exposed to room air at 25°C, and the convection coefficient is 5 W/m² °C. Question: can you touch the handle when the water is boiling? (k for aluminum is 237 W/m °C)



$$T_\infty = 25 \text{ °C}$$

$$h = 5 \text{ W/m}^2 \text{ °C}$$



We can model the pot handle as an extended surface. Assume that there is no heat transfer at the free end of the handle. The condition matches that specified in the fins Table, case B.

$h=5 \text{ W/m}^2 \text{ }^\circ\text{C}$, $P=2W+2t=2(0.03+0.005)=0.07(\text{m})$, $k=237 \text{ W/m}^\circ\text{C}$, $A_C=Wt=0.00015(\text{m}^2)$, $L=0.2(\text{m})$

Therefore, $m=(hP/kA_C)^{1/2}=3.138$,

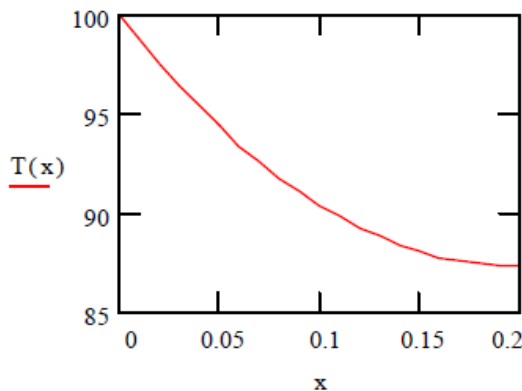
$M=\sqrt{(hPkA_C)}(T_b-T_\infty)=0.111\theta_b=0.111(100-25)=8.325(\text{W})$

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\theta}{\theta_b} = \frac{\cosh m(L - x)}{\cosh mL}$$

$$\frac{T - 25}{100 - 25} = \frac{\cosh[3.138(0.2 - x)]}{\cosh(3.138 * 0.2)}$$

$$T(x) = 25 + 62.32 * \cosh[3.138(0.2 - x)]$$

Plot the temperature distribution along the pot handle



As shown, temperature drops off very quickly. At the midpoint $T(0.1)=90.4^\circ\text{C}$. At the end $T(0.2)=87.3^\circ\text{C}$.

Therefore, it should not be safe to touch the end of the handle



The total heat transfer through the handle can be calculated also. $q_f = M \tanh(mL) = 8.325 * \tanh(3.138 * 0.2) = 4.632 \text{ (W)}$

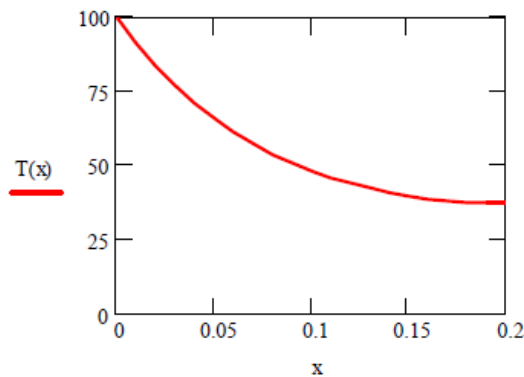
Very small amount: latent heat of evaporation for water: 2257 kJ/kg. Therefore, the amount of heat loss is just enough to vaporize 0.007 kg of water in one hour.

If a stainless steel handle is used instead, what will happen: For a stainless steel, the thermal conductivity $k = 15 \text{ W/m}^\circ\text{C}$. Use the same parameter as before:

$$m = \left(\frac{hP}{kA_C} \right)^{1/2} = 12.47, \quad M = \sqrt{hPkA_C} = 0.0281$$

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L - x)}{\cosh mL}$$

$$T(x) = 25 + 12.3 \cosh[12.47(L - x)]$$



Temperature at the handle ($x = 0.2 \text{ m}$) is only $37.3 \text{ }^\circ\text{C}$, not hot at all. This example illustrates the important role played by the thermal conductivity of the material in terms of conductive heat transfer.



□ If the pot from previous lecture is made of other materials other than the aluminum, what will be the temperature distribution? Try stainless steel ($k=15 \text{ W/m.K}$) and copper (385 W/m.K).

Recall: $h=5 \text{ W/m}^2\text{C}$, $P=2W+2t=2(0.03+0.005)=0.07(\text{m})$

$A_C=Wt=0.00015(\text{m}^2)$, $L=0.2(\text{m})$

Therefore, $m_{ss}=(hP/kA_C)^{1/2}=12.47$, $m_{cu}=2.46$

$M_{ss}=\sqrt{(hPk_{ss}A_C)}(T_b-T_\infty)=0.028(100-25)=2.1(\text{W})$

$M_{cu}=\sqrt{(hPk_{ss}A_C)}\theta_b=0.142(100-25)=10.66(\text{W})$

$$\text{For stainless steel, } \frac{T_{ss}(x) - T_\infty}{T_b - T_\infty} = \frac{\theta}{\theta_b} = \frac{\cosh m(L-x)}{\cosh mL}$$

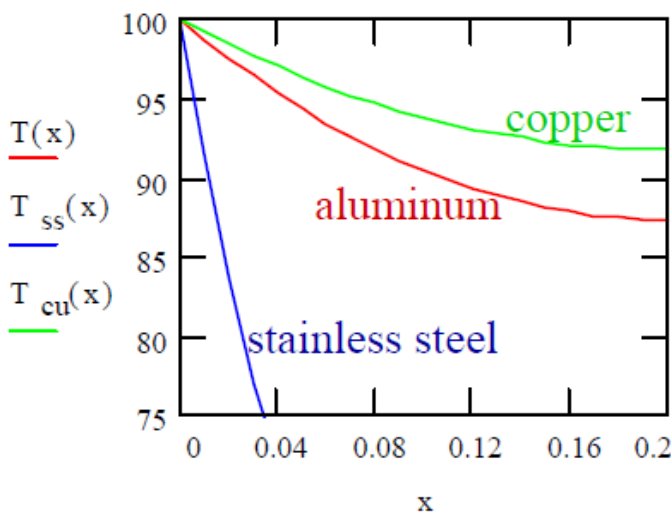
$$\frac{T_{ss} - 25}{100 - 25} = \frac{\cosh[12.47(0.2 - x)]}{\cosh(12.47 * 0.2)},$$

$$T_{ss}(x) = 25 + 12.3 * \cosh[12.47(0.2 - x)]$$

$$\text{For copper, } \frac{T_{cu}(x) - T_\infty}{T_b - T_\infty} = \frac{\theta}{\theta_b} = \frac{\cosh m(L-x)}{\cosh mL}$$

$$\frac{T_{cu} - 25}{100 - 25} = \frac{\cosh[2.46(0.2 - x)]}{\cosh(2.46 * 0.2)},$$

$$T_{cu}(x) = 25 + 66.76 * \cosh[2.46(0.2 - x)]$$



Subject: Heat Transfer-I

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- ❑ Inside the handle of the stainless steel pot, temperature drops quickly. Temperature at the end of the handle is 37.3°C . This is because the stainless steel has low thermal conductivity and heat can not penetrate easily into the handle.
- ❑ Copper has the highest k and, correspondingly, the temperature inside the copper handle distributes more uniformly. Heat easily transfers into the copper handle.
- ❑ Question? Which material is most suitable to be used in a heat sink?