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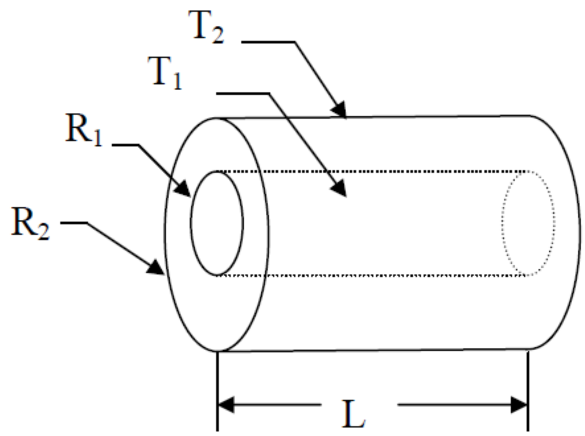
3rd Lucture

Radial Conduction in Bodies

1- One Dimensione Radial Conduction through a Cylinder.

Assume no heat sources within the wall of the tube. If $T_1 > T_2$, heat will flow outward, radially, from the inside radius, R_1, to the outside radius, R_2. The process will be described by the Fourier Law.



The differential equation governing heat diffusion is: $\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)=0$

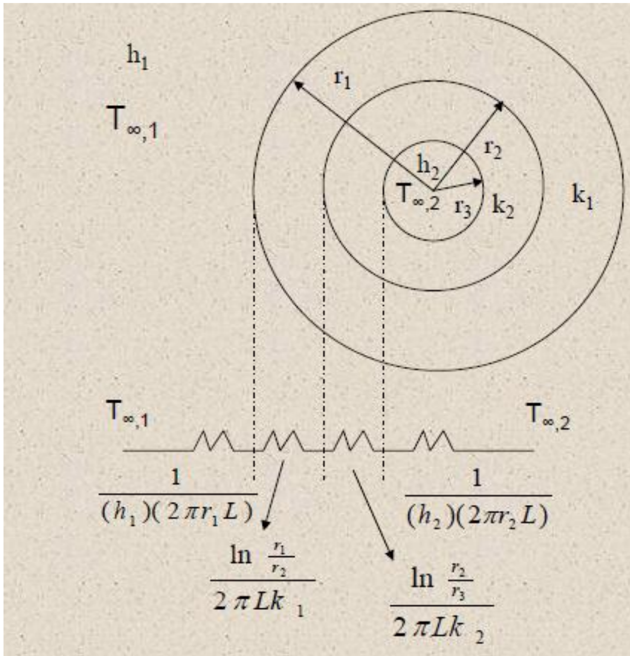
With constant k, the solution is

The heat flow rate across the wall is given by:

$$q_x = -kA\frac{dT}{dx} = \frac{kA}{L}\left(T_{s,1}-T_{s,2}\right) = \frac{T_{s,1}-T_{s,2}}{L/kA}$$

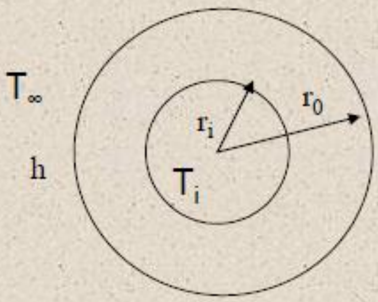
Hence, the thermal resistance in this case can be expressed as: $\frac{\ln\frac{r_1}{r_2}}{2\pi kL}$

2- One Dimensional Radial Conduction in a Composite Cylinder



$$q_r = \frac{T_{\infty,2} - T_{\infty,1}}{\sum R_t}$$

3- Critical Insulation Thickness.



Insulation Thickness : r_o-r_i

$$R_{tot} = \frac{\ln(\frac{r_0}{r_i})}{2\pi kL} + \frac{1}{(2\pi r_0 L)h}$$

Objective : decrease q , increases R_{tot}

Vary r_0 ; as r_0 increases ,first term increases, second term decreases.

Maximum – Minimum problem

Set $\frac{dR_{tot}}{dr_0} = 0$

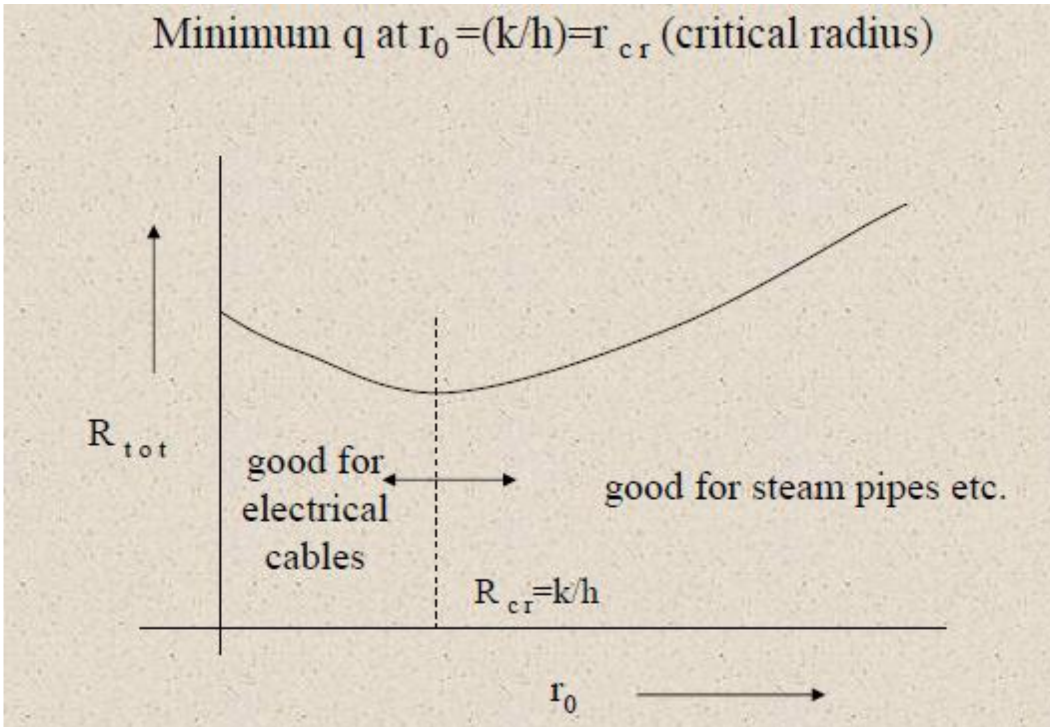
$$\frac{1}{2\pi k r_0 L} - \frac{1}{2\pi h L r_0^2} = 0$$

$$r_0 = \frac{k}{h}$$

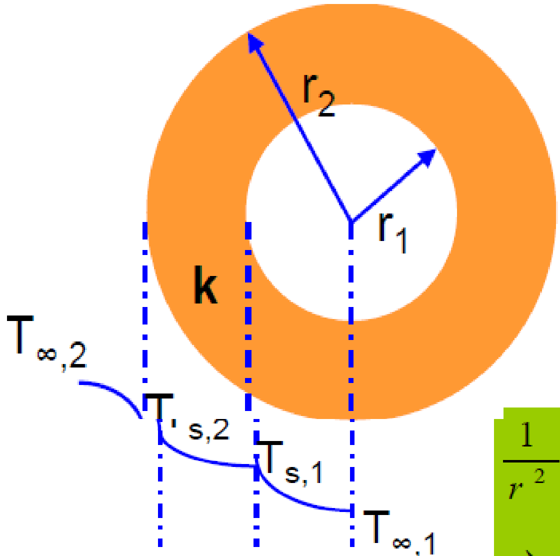
Max or Min. ? Take : $\frac{d^2 R_{tot}}{dr_0^2} = 0$ at $r_0 = \frac{k}{h}$

$$\frac{d^2 R_{tot}}{dr_0^2} = \frac{-1}{2\pi k r_0^2 L} + \left.\frac{1}{\pi r_0^2 hL}\right|_{r_0 = \frac{k}{h}}$$

$$= \frac{h^2}{2\pi L k^3} > 0$$



4- One-Dimension Conduction in Sphere.



Inside Solid:

$$\frac{1}{r^2}\frac{d}{dr}\left(kr^2\frac{dT}{dr}\right) = 0$$

$$\rightarrow T(r) = T_{s,1} - \{T_{s,1} - T_{s,2}\}\left[\frac{1-(r_1/r)}{1-(r_1/r_2)}\right]$$

$$\rightarrow q_r = -kA\frac{dT}{dr} = \frac{4\pi k\left(T_{s,1} - T_{s,2}\right)}{\left(1/r_1 - 1/r_2\right)}$$

$$\rightarrow R_{t,cond} = \frac{1/r_1 - 1/r_2}{4\pi k}$$

5-Conduction with Thermal Energy Generation.

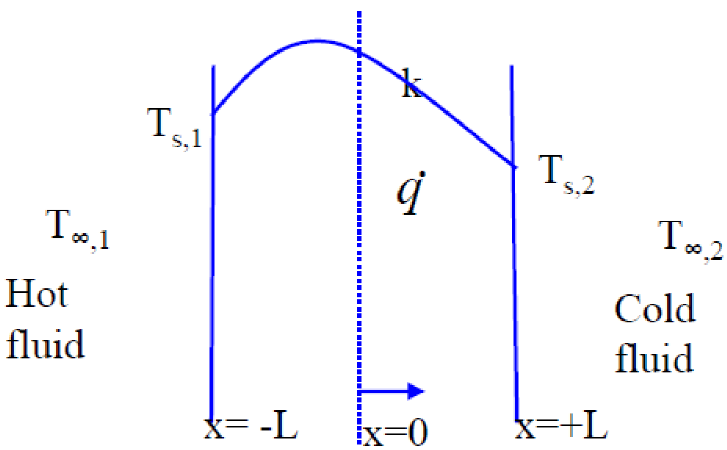
$$\dot{q} = \frac{\dot{E}}{V} \text{ = Energy generation per unit volume}$$

Applications: * current carrying conductors

* chemically reacting systems

* nuclear reactors

The Plane Wall :



Assumptions:

1D, steady state, constant k,

uniform $\dot{q}$

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0$$

$$Boundary\ cond.: \quad x = -L, \quad T = T_{s,1}$$

$$x = +L, \quad T = T_{s,2}$$

$$Solution: \quad T = -\frac{\dot{q}}{2k} x^2 + C_1 x + C_2$$

Use boundary conditions to find C_1 and C_2

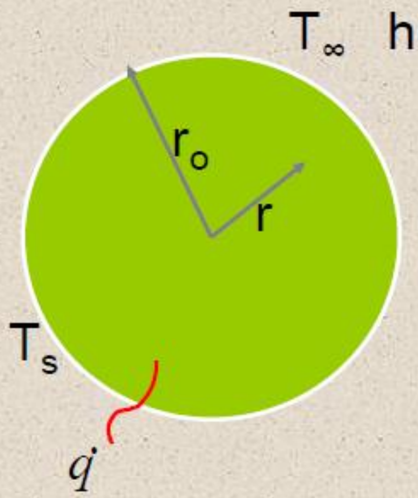
$$Final\ solution: T = \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + \frac{T_{s,2} - T_{s,1}}{2}\frac{x}{L} + \frac{T_{s,2} + T_{s,1}}{2}$$

No more linear

$$Heat\ flux: \quad q_x'' = -k\frac{dT}{dx}$$

Derive the expression and show that it is no more independent of x

6- Cylinder with Heat Source.



Assumptions:

1D, steady state, constant k, uniform $\dot{q}$

Start with 1D heat equation in cylindrical co-ordinates:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\dot{q}}{k}=0$$

$$Boundary\ cond.:\quad r=r_0,\quad T=T_s$$

$$r=0,\quad \frac{dT}{dr}=0$$

$$Solution:\quad T(r)=\frac{\dot{q}}{4k}r_0^2\left(1-\frac{r^2}{r_0^2}\right)+T_s$$

Example:

A current of 200A is passed through a stainless steel wire having a thermal conductivity K=19W/mK, diameter 3mm, and electrical resistivity R = 0.99 Ω. The length of the wire is 1m. The wire is submerged in a liquid at 110°C, and the heat transfer coefficient is 4W/m^2K. Calculate the centre temperature of the wire at steady state condition.