Subject: Heat Transfer-I

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Second Lecture

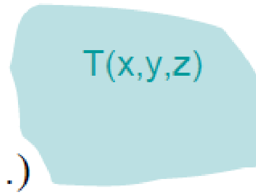
One Dimentional Steady State Heat Conduction

1- Objectives of Conduction Analysis.

To determine the temperature field, T(x,y,z,t), in a body
(i.e. how temperature varies with position within the body)

❑T(x,y,z,t) depends on:

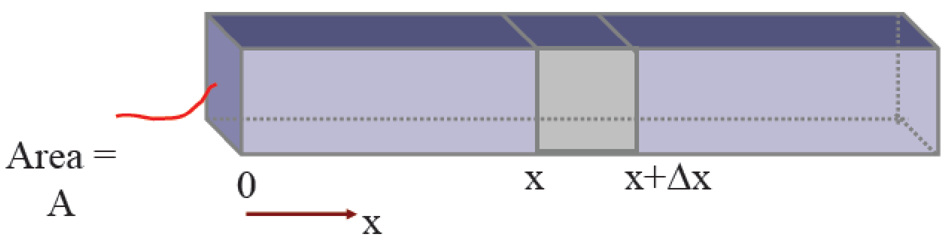
- boundary conditions
- initial condition
- material properties (k, c^p, ρ …)
- geometry of the body (shape, size)



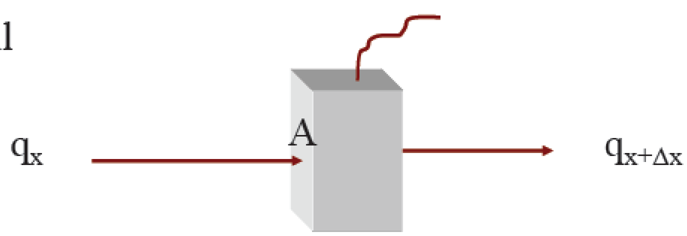
❑Why we need T(x,y,z,t) ?

- to compute heat flux at any location (using Fourier's eqn.)
- compute thermal stresses, expansion, deflection due to temp. etc.
- design insulation thickness
- chip temperature calculation
- heat treatment of metals

2- One Dimension Heat Conduction.



Solid bar, insulated on all long sides (1D heat conduction)



$\dot{q}$ = Internal heat generation per unit vol. (W/m³)

First Law (energy balance) $$(\dot{E}_{in} - \dot{E}_{out}) + \dot{E}_{gen} = \dot{E}_{st}$$

$$q_x - q_{x+\Delta x} + A(\Delta x)\dot{q} = \frac{\partial E}{\partial t}$$

$$E = (\rho A \Delta x) u$$

$$\frac{\partial E}{\partial t} = \rho A \Delta x \frac{\partial u}{\partial t} = \rho A \Delta x c \frac{\partial T}{\partial t}$$

$$q_x = -kA \frac{\partial T}{\partial x}$$

$$q_{x+\Delta x} = q_x + \frac{\partial q_x}{\partial x} \Delta x$$

$$-kA\frac{\partial T}{\partial x} + kA\frac{\partial T}{\partial x} + A\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)\Delta x + A\Delta x\dot{q} = \rho A c \Delta x \frac{\partial T}{\partial t}$$

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \dot{q} = \rho c \frac{\partial T}{\partial t}$$

Longitudinal conduction — Internal heat generation — Thermal inertia

If k is a constant $$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{k} = \frac{\rho c}{k}\frac{\partial T}{\partial t} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

- For T to rise, LHS must be positive (heat input is positive)
- For a fixed heat input, T rises faster for higher α
- In this special case, heat flow is 1D. If sides were not insulated, heat flow could be 2D, 3D.

3- Boundary and Initial Conditions.

- The objective of deriving the heat diffusion equation is to determine the temperature distribution within the conducting body.

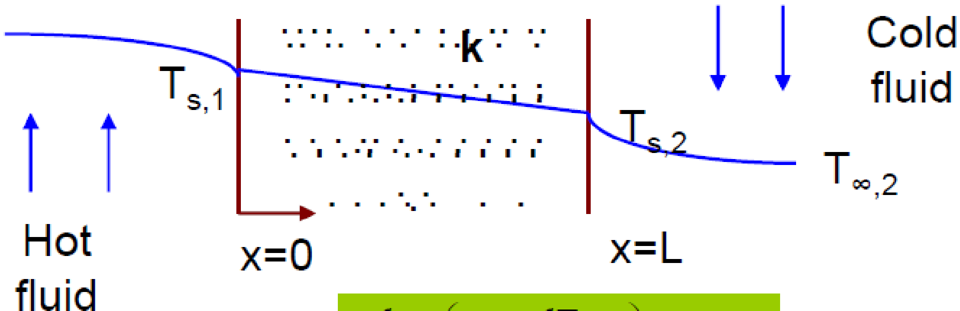
- We have set up a differential equation, with T as the dependent variable. The solution will give us T(x,y,z). Solution depends on boundary conditions (BC) and initial conditions (IC).

How many BC's and IC's ?

- Heat equation is second order in spatial coordinate. Hence, 2 BC's needed for each coordinate.
  - * 1D problem: 2 BC in x-direction
  - * 2D problem: 2 BC in x-direction, 2 in y-direction
  - * 3D problem: 2 in x-dir., 2 in y-dir., and 2 in z-dir.
- Heat equation is first order in time. Hence one IC needed

4- Plan Wall Heat Conduction.

The Plane Wall :



$$\frac{d}{dx}\left(k\frac{dT}{dx}\right) = 0$$

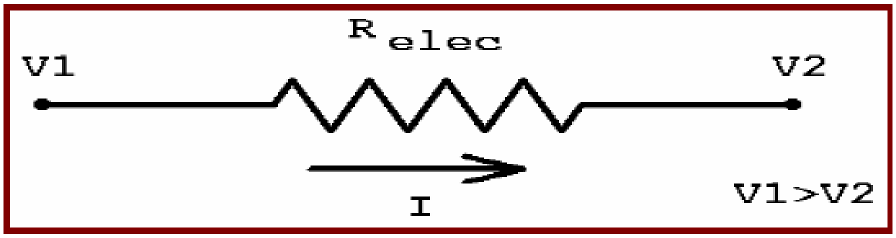
Const. K; solution is:

$$q_x = -kA\frac{dT}{dx} = \frac{kA}{L}\left(T_{s,1} - T_{s,2}\right) = \frac{T_{s,1} - T_{s,2}}{L/kA}$$

5- Thermal Resistance (Electrical Analogy).

OHM's LAW :Flow of Electricity

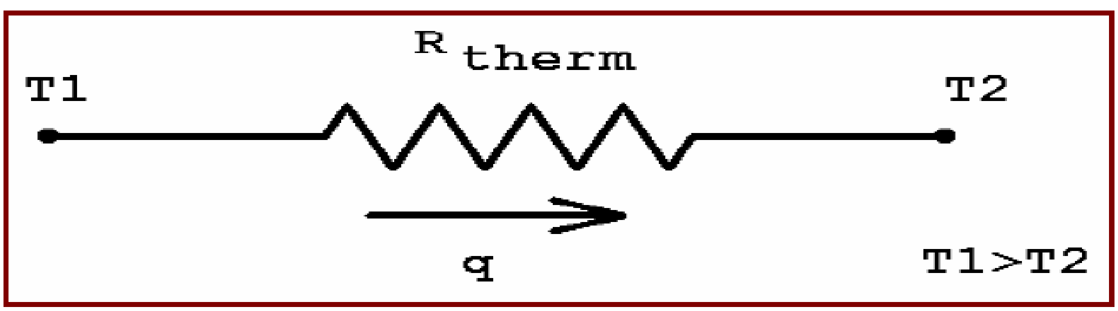
$$V = IR_{elect}$$

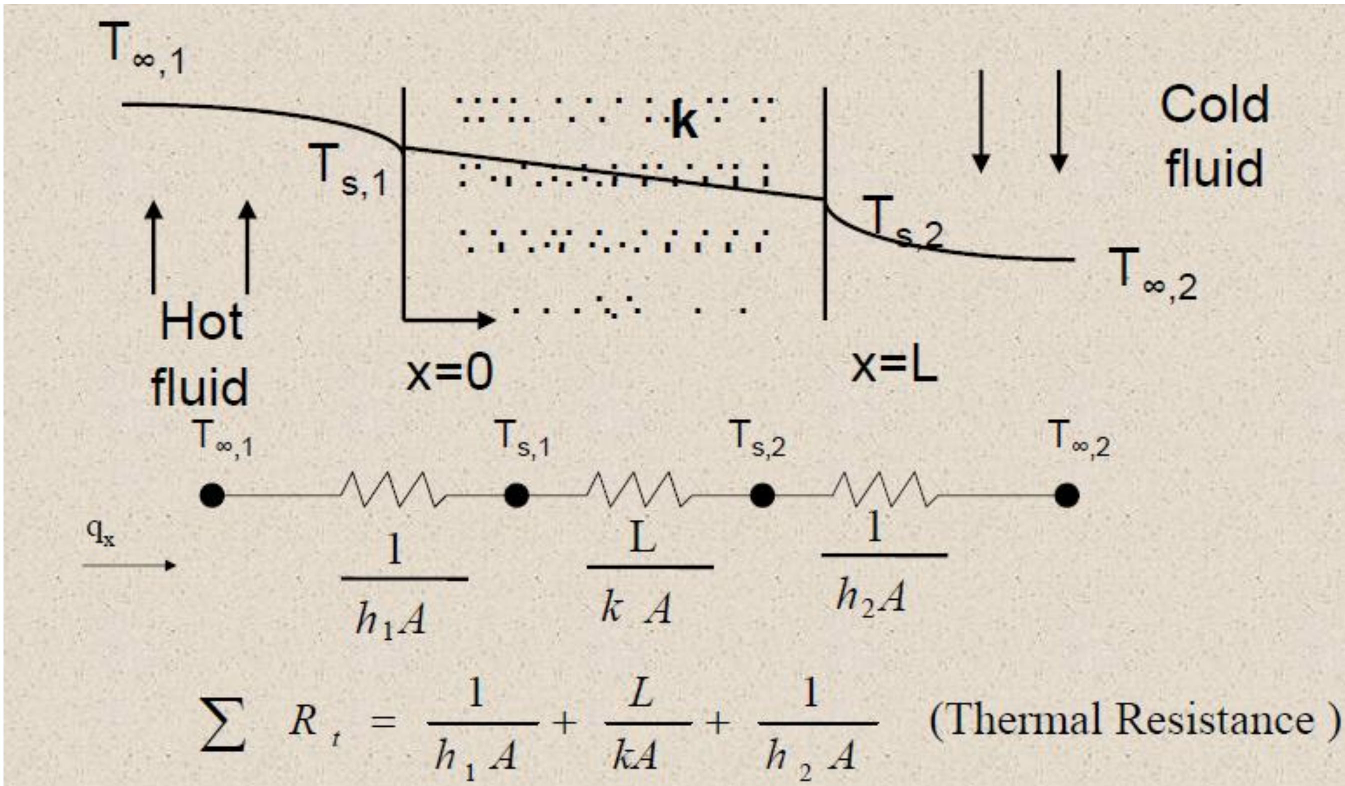


Voltage Drop = Current flow×Resistance

$$\Delta T = qR_{therm}$$

Temp Drop=Heat Flow×Resistance





THERMAL RESISTANCES

- Conduction

$$R_{cond} = \Delta x/kA$$

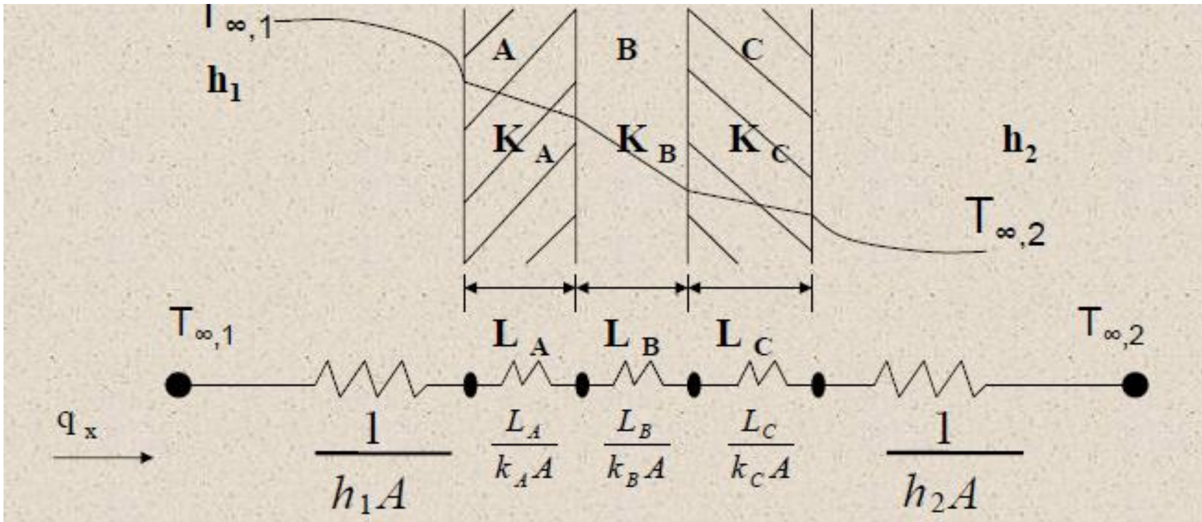
- Convection

$$R_{conv} = (hA)^{-1}$$

- Fins

$$R_{fin} = (h\eta A)^{-1}$$

6- Composit Wall.



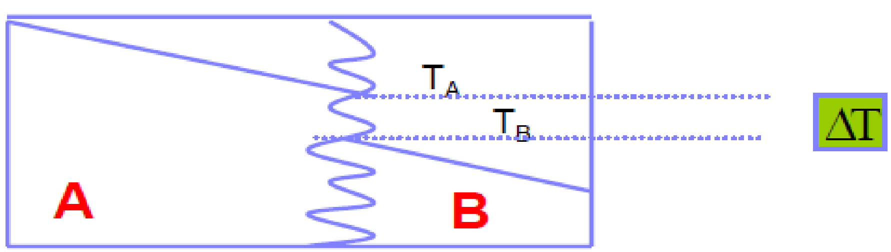
$$q_x = \frac{T_{\infty,1} - T_{\infty,2}}{\sum R_t} = \frac{T_{\infty,1} - T_{\infty,2}}{\frac{1}{h_1 A} + \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{1}{h_2 A}} = UA \;\Delta T$$

$$where, U = \frac{1}{R_{tot} A} = \text{Overall heat transfer coefficient}$$

7- Overall Heat Transfer Coefficient.

$$U = \frac{1}{R_{total} \; A} = \frac{1}{\frac{1}{h_1} + \Sigma \frac{L}{k} + \frac{1}{h_2}}$$

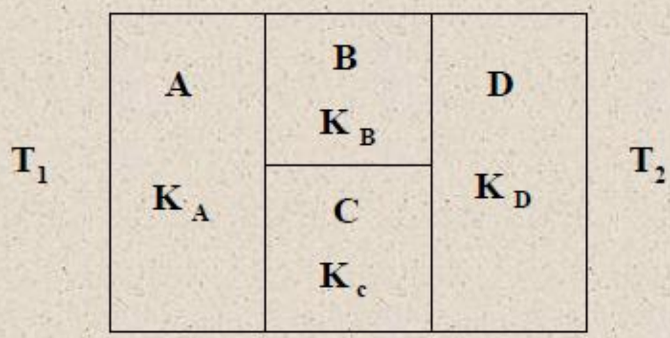
Contact Resistance :



$$R_{t,c} = \frac{\Delta T}{q_x}$$

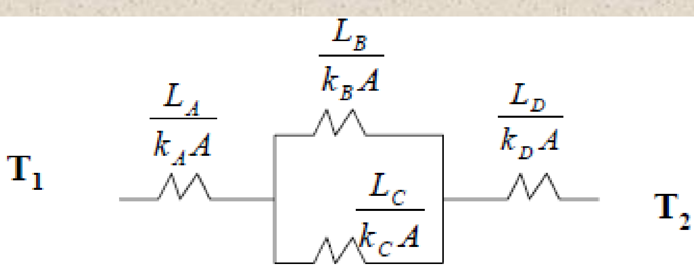
$$U = \frac{1}{\frac{1}{h_1} + \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{1}{h_2}}$$

Series-Parallel :



$A_B + A_C = A_A = A_D$

$L_B = L_C$



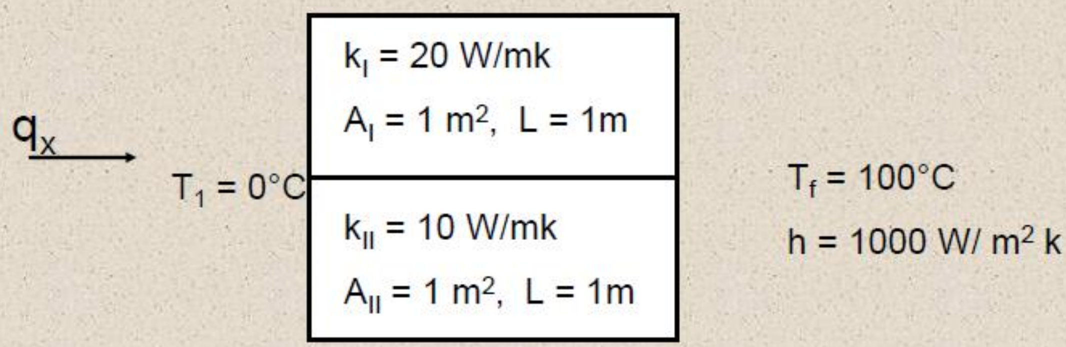
Assumptions :

(1) Face between B and C is insulated.

(2) Uniform temperature at any face normal to X.

Example:

Consider a composite plane wall as shown:



Develop an approximate solution for th rate of heat transfer through the wall.