Chapter 1

First Order Ordinary Differential Equations

Introduction:

A differential equation is an equation that involves one or more derivatives, or differentials. Differential equations are classified as:

a) **Type** (ordinary or partial),
b) **Order** (which is the highest order derivative that occurs in the equation),
c) **Degree** (the exponent of the highest power of the highest order derivative, after the equation has been cleared of fractions and radicals in the dependent variable and its derivatives).

For Example:

$$\left(\frac{d^3y}{dx^3}\right)^2+\left(\frac{d^2y}{dx^2}\right)^5+\frac{y}{x^2+1}=e^x$$

is an ordinary differential equation, of order three and degree two.

Only "ordinary" derivatives occur when the dependent variable y is a function of a single independent variable x. On the other hand, if the dependent variable y is a function of two or more independent variables, like

$$y=f(x,t),$$

where x and t are independent variables, then partial derivatives of y may occur. For example,

$$\frac{\partial^2 y}{\partial t^2}=a^2\frac{\partial^2 y}{\partial x^2}$$

is a partial differential equation, of order two and degree one, (It is one-dimensional "wave equation").

Many physical problems, when formulated in mathematical terms, lead to differential equations. For example,

$$m\frac{d^2x}{dt^2}=0, \qquad m\frac{d^2y}{dt^2}=-mg$$

describes the motion of a projectile (neglecting air resistance).

Indeed, one of the chief sources of differential equation is Newton's second law:

$$F=\frac{d}{dt}(mv),$$

where F is the resultant of the forces acting on a body of mass m and v is its velocity.

Solutions of Differential Equations (D.E):

A function

$$y = f(x)$$

is said to be a solution of a D.E if the latter is satisfied when *y* and its derivatives are replaced by *f(x)* and its corresponding derivatives. For example, if C_1 and C_2 are any constants, then

$$y = C_1 \cos x + C_2 \sin x \qquad (1a)$$

is a solution of the D.E

$$\frac{d^2 y}{dx^2} + y = o \qquad (1b)$$

A physical problem that translates into a D.E usually involves additional conditions not expressed by the D.E itself. In mechanics, for example, the initial position and velocity of the moving body are usually prescribed, as well as the forces. The D.E, or equations, of motion will usually have solutions in which certain arbitrary constants occur, as shown in (Eq. 1a) above. Specific values are then assigned to these arbitrary constants to meet the prescribed initial conditions.

A D.E of order *n* will generally have a solution involving *n* arbitrary constants. This solution is called the ***general*** solution. Once the general solution is known, it is only a matter of algebra to determine specific values of the constants if initial conditions are also prescribed.

The following topics will be considered for ordinary differential equations solution.

1. **First order.**
   a) Variable separable.
   b) Exact differentials.
   c) Homogeneous.
   d) Linear.
2. **Special types of second order.**
3. **Linear equations with constant coefficients.**
   a) Homogeneous.
   b) Inhomogeneous.