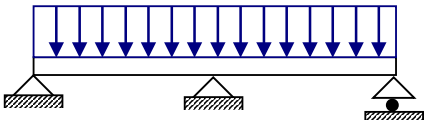
Structural Applications:



Y = 0
Y' ≠ 0
M ≠ 0
V ≠ 0

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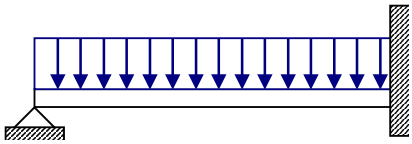

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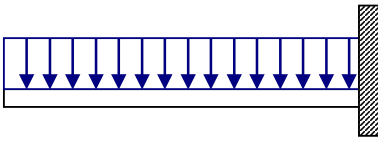

Y ≠ 0
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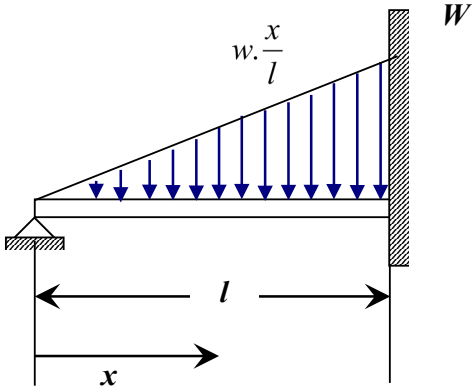
EI y = Deflection
EI y' = Rotation
EI y'' = Moment
EI y''' = Shear
EI y^{IV} = Load

Example: Find the equation of the deflection curve of the beam shown below?



Solution:

$$E\,I\,y^{IV} = w.\frac{x}{l} \quad Load\ equation$$

$$E\,I\,y''' = \frac{w.x^2}{2l} + c_1$$

$$E\,I\,y'' = \frac{w.x^3}{6l} + c_1 x + c_2$$

$$E\,I\,y' = \frac{w.x^4}{24l} + c_1\frac{x^2}{2} + c_2 x + c_3$$

$$E\,I\,y = \frac{w.x5}{120l} + c_1\frac{x3}{6} + c_2\frac{x^2}{2} + c_3 x + c_4$$

Initial Conditions:

at x = 0 y = 0

$$0 = \frac{w(0)}{120l} + \frac{c_1(0)}{6} + \frac{c_2(0)}{2} + c_3(0) + c_4 \quad \Rightarrow \quad c_4 = 0$$

at x = 0 y'' = 0

$$0 = \frac{w(0)}{6l} + c_1(0) + c_2 \quad \Rightarrow \quad c_2 = 0$$

at x = l y = 0

$$0 = \frac{w(l^5)}{120\,l} + \frac{c_1(l^3)}{6} + \frac{0(l^2)}{2} + c_3(l) + 0 \quad \Rightarrow \quad \frac{wl^4}{120} + c_1\frac{l^3}{6} + c_3 l = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

at x = l y' = 0

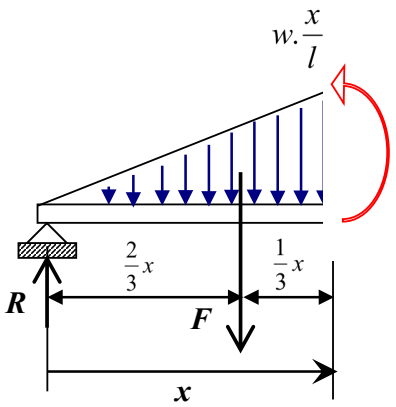
$$0 = \frac{w(l^4)}{24l} + \frac{c_1(l^2)}{2} + (0)l + c_3 \quad \Rightarrow \quad \frac{wl^3}{24} + c_1\frac{l^2}{2} + c_3 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Solve equation (1) and (2) to find c_1 and c_3

$$\frac{wl^3}{120}+c_1\frac{l^2}{6}+c_3=0 \quad \text{.....} \quad (1') \quad \textit{dividing eq (1) by } l$$

$$\frac{wl^3}{24}+c_1\frac{l2}{2}+c_3=0 \quad \text{.....} \quad (2)$$

Another way to solve the previous example is by starting with the moment equation:



$$E\,I\,y''=-M$$

$$E\,I\,y''=-\left[\left(R\,x-\frac{1}{2}w\frac{x^2}{l}(\frac{1}{3}x)\right)\right]$$

$$E\,I\,y''=-R\,x+\frac{wx^3}{6l}$$

$$E\,I\,y'=-R\frac{x^2}{2}+\frac{wx^4}{24l}+c_1$$

$$E\,I\,y=-R\frac{x^3}{6}+\frac{wx^5}{120l}+c_1\,x+c_2$$

at x = 0 y = 0

$$0=0+0+0+c_2 \quad \Rightarrow \quad c_2=0$$

at x = l y = 0

$$0=-\frac{Rl^3}{6}+\frac{wl^5}{120l}+c_1\,l+0 \quad \Rightarrow \quad -\frac{Rl^3}{6}+\frac{wl^4}{120}+c_1\,l=0 \quad \text{.....} \quad (1)$$

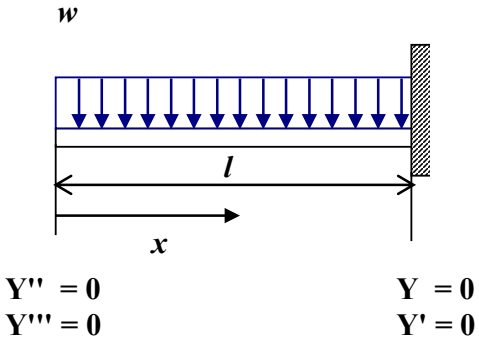
at x = l y' = o

$$0=-R\frac{l^2}{2}+\frac{wl^4}{24l}+c_1 \quad \Rightarrow \quad -R\frac{l^2}{2}+\frac{wl^3}{24}+c_1=0 \quad \text{.....} \quad (2)$$

Solve Equation (1) and (2) to find c_1 and substitute in the deflection equation.

Example: For the cantilever beam shown below find the deflection curve?

Solution:



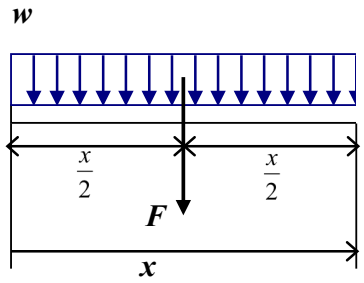
$$EI\,y'' = -wx\left(\frac{x}{2}\right)$$

$$EI\,y'' = -\frac{wx^2}{2}$$

$$y'' = -\frac{wx^2}{2EI}$$

$$y' = -\frac{wx^3}{6EI} + A$$

$$y = -\frac{wx^4}{24EI} + Ax + B$$



at x = l y = 0

$$0 = -\frac{wl^4}{24EI} + Al + B \quad \text{.....} \quad (1)$$

at x = l y' = 0

$$0 = -\frac{wl^3}{6EI} + A \quad \Rightarrow \quad A = \frac{wl^3}{6EI}$$

Substitute in eq. (1)

$$0 = -\frac{wl^4}{24EI} + \frac{wl^3}{6EI}(l) + B \quad \Rightarrow \quad B = \frac{wl^4}{24EI} - \frac{wl^4}{6EI} = \frac{3}{24}\cdot\frac{wl^4}{EI}$$

$$\therefore \quad y = -\frac{wx^4}{24EI} + \frac{wl^3}{6EI}(x) - \frac{3}{24}\cdot\frac{wl^4}{EI} \qquad \textit{complete solution}$$