Engineering Mechanics

DYNAMICS

Fourteenth Edition

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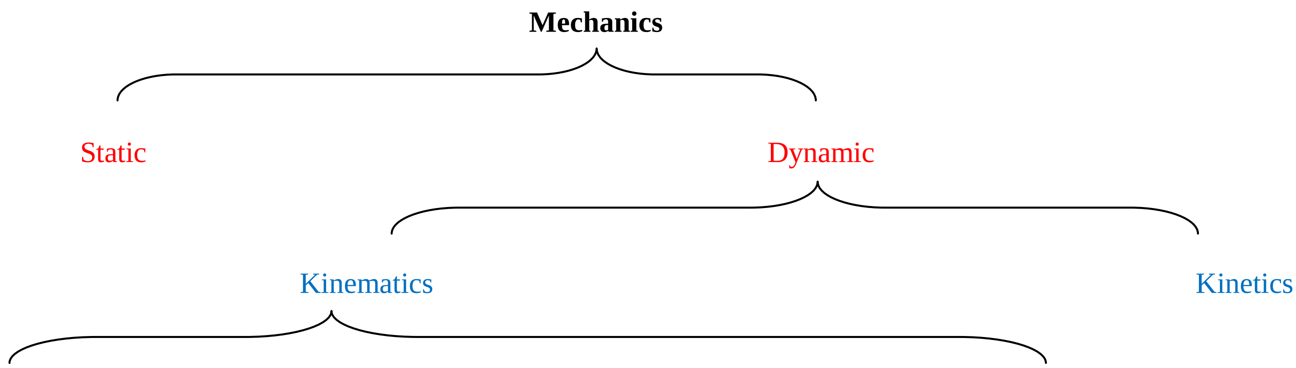


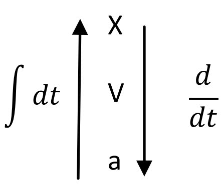
Rectilinear

$v = v_o + a_c t$

$x = x_o + v_o t + \frac{1}{2} a_c t^2$

$v^2 = {v_o}^2 + 2a_c(x - x_o)$

$a\, d_x = v\, d_v$



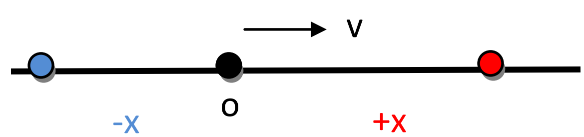
Units:

Position (x) ⟶ (m, ft)

Velocity (v) ⟶ (m/s, ft/s)

Acceleration (a) ⟶ (m/s^2, ft/s^2)

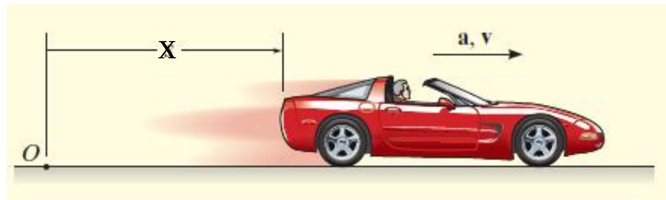
Deceleration (-a)



Example (1)

The car in Fig. shown moves in a straight line such that for a short time its velocity is defined by $v = (3t^2 + 2t)$ ft/s, where *t* is in seconds. Determine its position and acceleration when *t* = 3 s. When *t* = 0, *x* = 0.

solution:



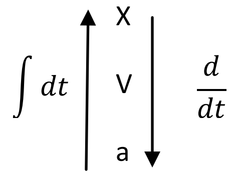
$v = (3t^2 + 2t)$

$\int_0^x dx = \int_0^t (3t^2 + 2t)dt$

$x|_0^x = t^3 + t^2|_0^3$

$x = (3^3 + 3^2)$

$x = 36\,ft$ ....Ans.



$a = \frac{dv}{dt} = \frac{d}{dt}(3t^2 + 2t)$

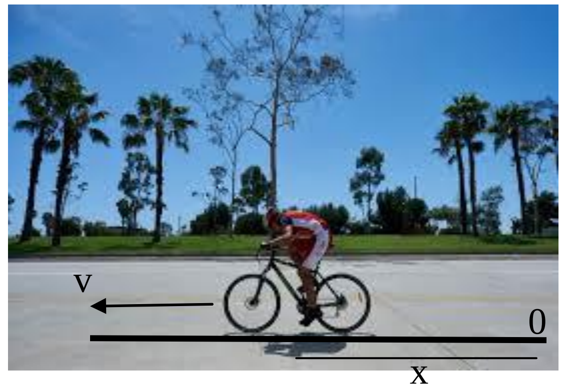
$a = (6t + 2)$ when: t=3s

$a = (6x3 + 2)$

$a = 20\,ft/s^2$ ....Ans.

Example (2)

The bicyclist in fig. has a constant acceleration of (2ft/s^2). If he starts from rest, determine his velocity and position when t=5s.



solution:

$$v = v_o + a_c t$$

$$v = 0 + 2x\,5$$

$$v = 10\ ft/s \leftarrow$$ ….Ans.

$$x = x_o + v_o t + \frac{1}{2} a_c t^2$$

$$x = 0 + 0x5 + \frac{1}{2} x2x5^2$$

$$x = 25\ ft$$ ….Ans.

or

$$v^2 = {v_o}^2 + 2a_c(\,x - x_o)$$

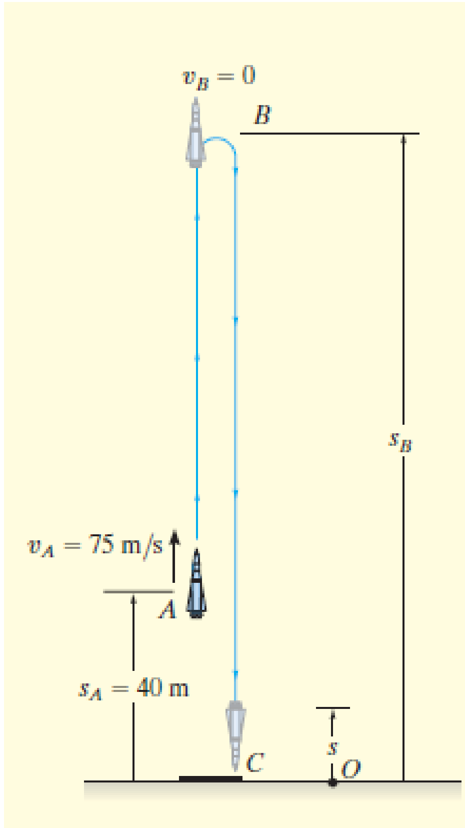
$$10^2 = 0 + 2x2\ (\,x - 0)$$

$$x = \frac{100}{4} = 25\ ft$$ ….Ans.

Example (3)

During a test a rocket travels upward at 75 m/s, and when it is 40 m from the ground its engine fails. Determine the maximum height X_B reached by the rocket and its speed just before it hits the ground. While in motion the rocket is subjected to a constant downward acceleration of 9.81 m/s^2 due to gravity. Neglect the effect of air resistance.

solution:



$$v^2 = {v_o}^2 + 2a_c(x - x_o)$$

Between A & B

$${v^2}_B = {v_a}^2 + 2a_c(x_B - x_a)$$

$$0 = 75^2 + 2\,(-9.81)(x_B - 40)$$

$$0 = 5625 - 19.62(x_B) + 784.8$$

$$x_B = 326.7\,m \qquad \ldots.\text{Ans.}$$

Between B& C

$${v^2}_c = {v_B}^2 + 2a_c(x_c - x_B)$$

$${v^2}_c = 0 + 2\,(-9.81)(0 - 326.7)$$

$$v_c = 253.2\frac{m}{s} \downarrow$$

Or Between A & C

$${v^2}_c = {v_a}^2 + 2a_c(x_c - x_a)$$

$${v^2}_c = 75^2 + 2\,(-9.81)(0 - 40)$$

$$v_c = 253.2\frac{m}{s} \downarrow$$

Absolute Dependent Motion Analysis of Two Particles:

Procedure for Analysis

The above method of relating the dependent motion of one particle to that of another can be performed using algebraic scalars or position coordinates provided each particle moves along a rectilinear path. When this is the case, only the magnitudes of the velocity and acceleration of the particles will change, not their line of direction.

Position-Coordinate Equation.

- Establish each position coordinate with an origin located at a *fixed* point or datum.
- It is *not necessary* that the *origin* be the *same* for each of the coordinates; however, it is *important* that each coordinate axis selected be directed along the *path of motion* of the particle.
- Using geometry or trigonometry, relate the position coordinates to the total length of the cord, l_T, or to that portion of cord, l, which *excludes* the segments that do not change length as the particles move—such as arc segments wrapped over pulleys.
- If a problem involves a *system* of two or more cords wrapped around pulleys, then the position of a point on one cord must be related to the position of a point on another cord using the above procedure. Separate equations are written for a fixed length of each cord of the system and the positions of the two particles are then related by these equations (see Examples 12.22 and 12.23).

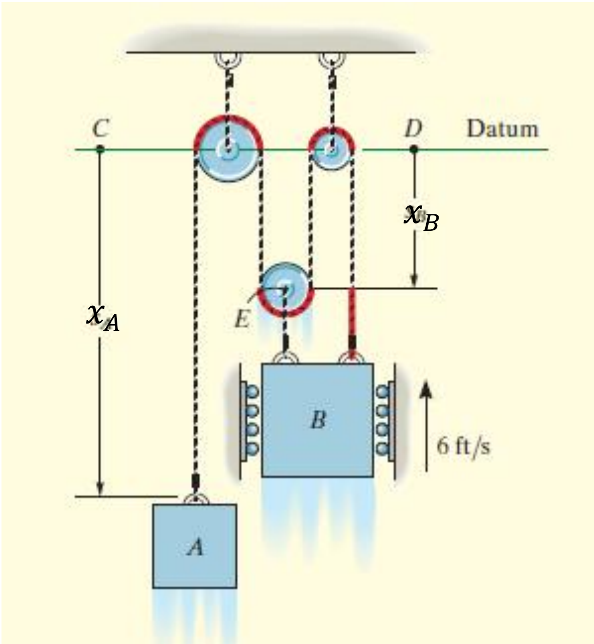
Time Derivatives.

- Two successive time derivatives of the position-coordinate equations yield the required velocity and acceleration equations which relate the motions of the particles.
- The signs of the terms in these equations will be consistent with those that specify the positive and negative sense of the position coordinates.

Example (4)

Determine the speed of block A in Fig. shown if block B has an upward speed of 6 ft/s.

solution:



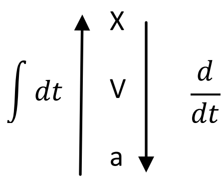
$x_A + 3x_B = l$

$v_A + 3v_B = 0$

$v_B = 6\frac{ft}{s}$

$\therefore v_A = -18\frac{ft}{s} \uparrow$

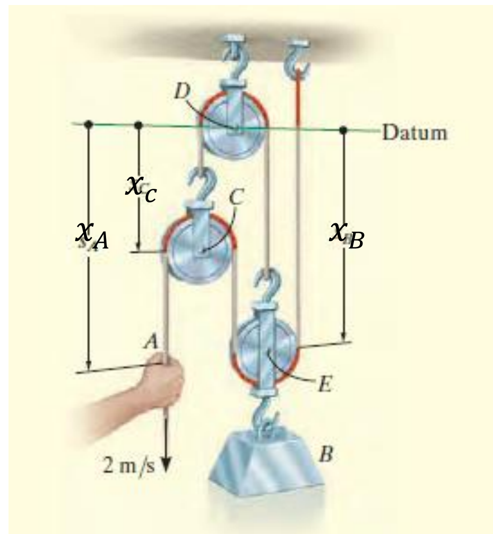
$\therefore v_A = 18\frac{ft}{s} \downarrow$ …Ans.



Example (5)

Determine the speed of block B in Fig. down if the end of the cord at A is pulled down with a speed of 2 m/s.

solution:



$x_c + x_B = l1$ …………..(1)

$(x_A - x_c) + (x_B - x_c) = l2$ …………..(2)

$v_c + v_B = 0$

$v_c = -v_B$ ……….(3)

$(v_A - v_c) + (v_B - v_c) + v_B = 0$

$v_A - 2v_c + 2v_B = 0$ …………..(4)

Sub. (3) into (4)

$v_A + 4v_B = 0$

$v_A = -2\frac{ft}{s}$ (downward)

$\therefore v_B = 0.5\ \frac{ft}{s} \uparrow$ …Ans.