Mechanics

Static

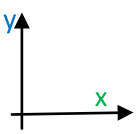
Dynamic

Kinematics

Kinetics

Rectilinear

Curvilinear



Rectangular Component

Position (x)

Velocity (v_x, v_y)

Acceleration $(a_x, a_{y,})$

$v_x = (v_o)_x$

$x = x_o + (v_o)_x t$

$v_y = (v_o)_y + at$

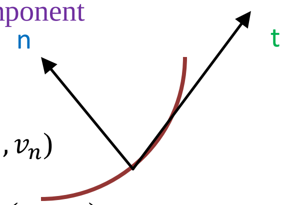
$y = y_o + (v_o)_y\, t + \frac{1}{2} a_c t^2$

$v^2{}_y = (v_o)_y{}^2 + 2a_c(\,y - y_o)$

$v = \sqrt{v_x{}^2 + v_y{}^2}$

$a = \sqrt{a_x{}^2 + a_y{}^2}$

Tangential and Normal Component



Position (r)

Velocity $(v_t\ , v_n)$

Acceleration (a_t, a_n)

$v_t = \frac{d_r}{d_t} = r^{\cdot}\ \ , v_n = 0$

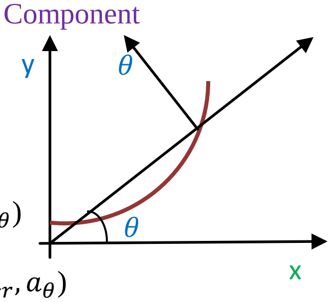
$a_t = \frac{d_v}{d_t} = v^{\cdot}, a_n = \frac{v^2}{\rho}$

$\rho = \left[\frac{1+\left({}^{dy}/_{dx}\right)^2}{(d_y^2/d_x^2)}\right]^{\frac{3}{2}}$

$v = \sqrt{v_n{}^2 + v_t{}^2}$

$a = \sqrt{a_t{}^2 + a_n{}^2}$

Radial or Polar Component



Position (r)

Velocity $(v_r\ , v_\theta)$

Acceleration (a_r, a_θ)

$v_r = r^{\cdot}\ \ , v_\theta = r\theta^{\cdot}$

$v = r^{\cdot} e_r + r\theta^{\cdot} e_\theta$

$v = \sqrt{v_r{}^2 + v_\theta{}^2}$

$a_r = r^{\cdot\cdot} - r\theta^{\cdot 2}$

$a_\theta = r\theta^{\cdot\cdot} + 2r^{\cdot}\theta^{\cdot}$

$a = \sqrt{a_r{}^2 + a_\theta{}^2}$

$\theta^o = \frac{\pi}{180}\, \theta_{rad}$

θ is always in radian

Units:

Position ⟶ (m, ft)

Velocity ⟶ (m/s, ft/s)

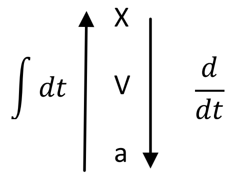
Acceleration ⟶ (m/s^2, ft/s^2)

Example (1)

The velocity of a particle is given by $v = \{16\, t^2 i + 4t^3 j + (5t + 2)k\}$ m/s, where t is in seconds. If the particle is at the origin when t = 0, determine the magnitude of the particle's acceleration when t = 2 s. Also, what is the x, y, z coordinate position of the particle at this instant?

solution:

$$a = \frac{dv}{dt} = \{16\, t^2 i + 4t^3 j + (5t + 2)k\}dt$$



$$a = 32\, ti + 12t^2 j + 5k$$

When t= 2s

$$a = 32x2i + 12x\, 4j + 5k \;\; m/s^2$$

$$a = 64i + 48j + 5k \;\; m/s^2$$

$$a = \sqrt{64^2 + 48^2 + 5^2} = 80.2 \;\; m/s^2$$

$a = 80.2 \;\; m/s^2$ ….Ans.

$$\int dx = \int vdt$$

$$\int_0^x dx = \int_0^t \{16\, t^2 i + 4t^3 j + (5t + 2)k\}dt$$

$$x = \frac{16}{3}\; t^3 i + t^4 j + \left(\frac{5}{2} t^2 + 2t\right) k$$

When t=2s

$$x = \frac{16}{3}\; 2^3 i + 2^4 j + \left(\frac{5}{2} 2^2 + 2x5\right) k$$

$$x = (42i + 16j + 14k)\; m$$

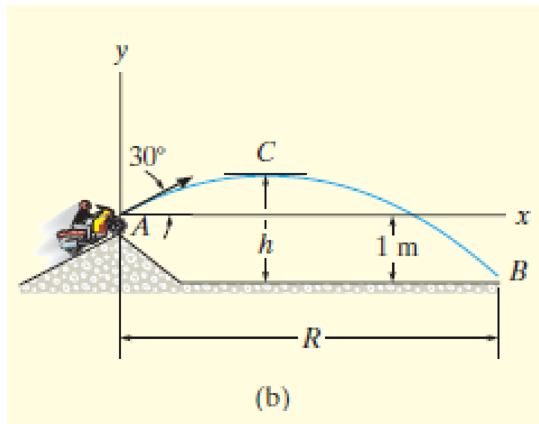
Thus, the coordinate of the particle is

(42,16,14) ….Ans.

Example (2)

The track for this racing event was designed so that riders jump off the slope at 30°, from a height of 1 m. During a race it was observed that the rider shown in Figure remained in mid air for 1.5 s. Determine the speed at which he was traveling off the ramp, the horizontal distance he travels before striking the ground, and the maximum height he attains. Neglect the size of the bike and rider.

solution:



$v_x = v_A \cos\ (30)$

$v_y = v_A \sin (30)$

$y = \ y_o + (v_o)_y\, t_{AB} + \frac{1}{2} a_c {t_{AB}}^2$

$-1 = \ 0 + v_A \sin (30)\ (1.5) + \frac{1}{2}(-9.81)(1.5)^2$

$v_A = 13.4\ m/s$ …Ans

$x_B = \ x_A + (v_o)_x t_{AB}$

$v_x = 13.4\ \cos\ (30) = 11.6\ m/s$

$R = \ 0 + 11.6\ x 1.5$

$R = \ 17.4\ m$ …Ans

$(v_c)^2{}_y = \ (v_A)_y{}^2 + 2a_c(\,y_c - y_A)$

$y_c = h - 1$

$v_y = 13.4 \sin (30) = 6.7\ m/s$

$0 = \ 6.7^2 + 2(-9.81)(\,(h-1) - 0)$

$h = 3.28\ m$ …Ans

Example (3)

The position of a crate sliding down a ramp is given by x = ($0.25t^3$) m, y = ($1.5t^2$) m, z = ($6 - 0.75t^{5/2}$) m, where t is in seconds. Determine the magnitude of the crate's velocity and acceleration when t = 2 s.

solution:

$v_x = \dot{x} = 0.75\, t^2$ m/s

$v_y = \dot{y} = 3t$ m/s

$v_z = \dot{z} = -1.875\, t^{3/2}$ m/s

When t = 2 s

$v_x = 3$ m/s , $v_y = 6$ m/s, $v_z = -5.303$ m/s

$$v = \sqrt{{v_x}^2 + {v_y}^2 + {v_z}^2}$$

$$v = \sqrt{(3)^2 + (6)^2 + (-5.303)^2}$$

$v = 8.55$ m/s … Ans

$a_x = \dot{v_x} = 1.5t\ m/s^2$, $a_y = \dot{v_y} = 3\ m/s^2$, $a_z = \dot{v_z} = -2.815\, t^{1/2} m/s^2$

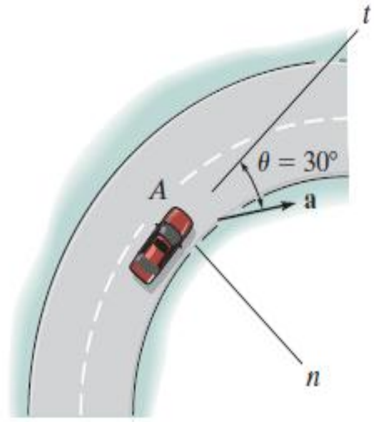
$a_x = 3\ m/s^2$, $a_y = 3\ m/s^2$, $a_z = -3.977\ m/s^2$

$a = \sqrt{{a_x}^2 + {a_y}^2 + {a_z}^2} = 5.82\ m/s^2$ … Ans

Example (4)

The automobile has a speed of 80 ft/s at point *A* and an acceleration having a magnitude of 10 ft/s^2, acting in the direction shown. Determine the radius of curvature of the path at point *A* and the tangential component of acceleration.

solution:



$a_t = a\cos(30)$

$a_t = 10\cos(30)$

$a_t = 8.66\ ft/s^2$

$a_n = a\sin(30)$

$a_n = 10\sin(30)$

$a_n = 5\ ft/s^2$

$a_n = \frac{v^2}{\rho}$

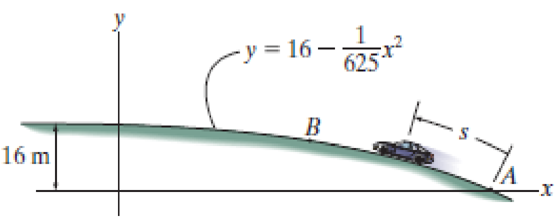
$\rho = \frac{v^2}{a_n} = \frac{80^2}{5}$

$\rho = 1280\ ft$ … Ans.

Example (5)

The car passes point *A* with a speed of 25 m/s after which its speed is defined by v = (25 - 0.15s) m/s. Determine the magnitude of the car's acceleration when it reaches point *B*, where *s* = 51.5 m and *x* = 50 m.

solution:



$$a_n = \frac{v^2}{\rho}$$

$$v_B = 25 - 0.15(51.5)$$

$$= 17.28 \frac{m}{s}$$

$$\rho = [\frac{1 + (^{dy}/_{dx})^2}{(d_y^2/d_x^2)}]^{\frac{3}{2}}$$

$$y = 16 - \frac{1}{625}x^2$$

$$y^{\cdot} = -3.2 * 10^{-3}x$$

$$y^{\cdot\cdot} = -3.2 * 10^{-3}$$

$$\rho = [\frac{1+(-3.2*10^{-3}x)^2}{(-3.2*10^{-3})}]^{\frac{3}{2}}$$ when x=50

$$\rho = 324.58\, m$$

$$a_n = \frac{v^2}{\rho}$$

$$a_n = \frac{17.28^2}{324.58} = 0.919\ m/s^2$$

$$a_t d_s = v\ d_v$$

$$a_t = v\frac{d_v}{d_s}$$ = (25-0.15*51.5) (-0.15)

$$a_t = -2.591\ m/s^2$$

$$a = \sqrt{{a_t}^2 + {a_n}^2}$$

$$a = \sqrt{0.919^2 + -2.591^2}$$

$a = 2.75\, m/s^2$ … Ans.