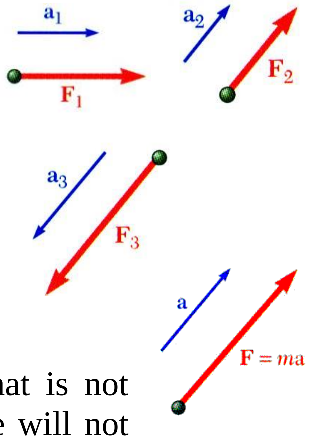
CHAPTER TWO
KINETICS OF PARTICLES: FORCE AND ACCELERATION

2-1 *Newton's Second Law of Motion*



If the resultant force acting on a particle is not zero, the particle will have an acceleration proportional to the magnitude of resultant and in the direction of the resultant.

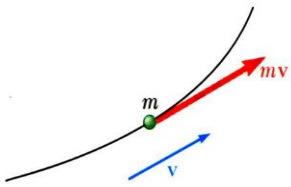
$$\vec{F} = m\vec{a}$$

If particle is subjected to several forces: $\sum \vec{F} = m\vec{a}$

We must use a Newtonian frame of reference, i.e., one that is not accelerating or rotating. If no force acts on particle, particle will not accelerate, i.e., it will remain stationary or continue on a straight line at constant velocity.

2-2 *Linear Momentum of a Particle*

The principle of conservation of linear momentum is:



$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$$

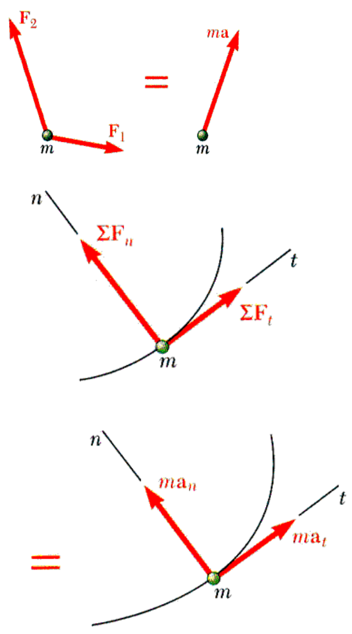
$$= \frac{d}{dt}\left(m\vec{v}\right) = \frac{d}{dt}\left(\vec{L}\right)$$

Where: $\vec{L} = m\vec{v}$ = Linear momentum

Sum of forces = rate of change of linear momentum $\sum \vec{F} = \dot{\vec{L}}$

If $\sum \vec{F} = 0$ then linear momentum is <u>constant</u>

2-3 Equations of Motion



- Newton's second law $\sum \vec{F} = m\vec{a}$
- Convenient to resolve into components:

$$\sum\left(F_x\vec{i} + F_y\vec{j} + F_z\vec{k}\right) = m\left(a_x\vec{i} + a_y\vec{j} + a_z\vec{k}\right)$$

$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum F_z = ma_z$$

$$\sum F_x = m\ddot{x} \qquad \sum F_y = m\ddot{y} \qquad \sum F_z = m\ddot{z}$$

- For tangential and normal components:

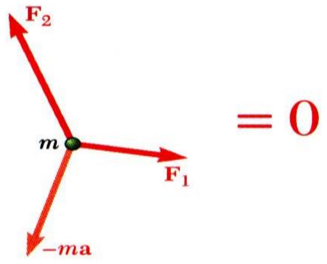
$$\sum F_t = ma_t \qquad \sum F_n = ma_n$$

$$\sum F_t = m\frac{dv}{dt} \qquad \sum F_n = m\frac{v^2}{\rho}$$

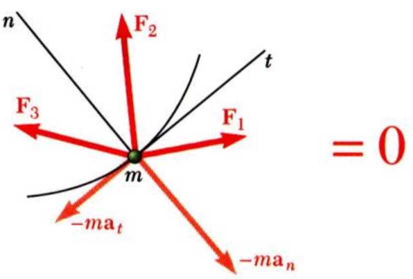
2-4 Dynamic Equilibrium

Alternate expression of Newton's law:: $\sum \vec{F} - m\vec{a} = 0$

Where: $-m\vec{a}$ = inertia vector



If we include inertia vector, the system of forces acting on particle is equivalent to zero. The particle is said to be in dynamic equilibrium.
Inertia vectors are often called *inertia forces* as they measure the resistance that particles offer to changes in motion.



2-5 Equation of Motion for a System of Particles

The summation of the internal forces, if carried out, will equal zero, since internal forces between any two particles occur in equal but opposite collinear pairs. Consequently, only the sum of the external forces will remain, and therefore the equation of motion, written for the system of particles, becomes

$$\sum F_i = \sum m_i a_i$$

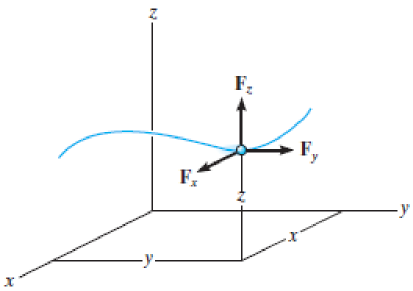
- The equation of motion is based on experimental evidence and is valid only when applied within an inertial frame of reference.
- The equation of motion states that the unbalanced force on a particle causes it to accelerate.
- An inertial frame of reference does not rotate, rather its axes either translate with constant velocity or are at rest.
- Mass is a property of matter that provides a quantitative measure of its resistance to a change in velocity. It is an absolute quantity and so it does not change from one location to another.
- Weight is a force that is caused by the earth's gravitation. It is not absolute; rather it depends on the altitude of the mass from the earth's surface.

2-6 Equations of Motion: Rectangular Coordinates

When a particle moves relative to an inertial *x, y, z* frame of reference, the forces acting on the particle, as well as its acceleration, can be expressed in terms of their **i**, **j**, **k** components. Applying the equation of motion, we have

$$\Sigma \mathbf{F} = m\mathbf{a}; \qquad \Sigma F_x\mathbf{i} + \Sigma F_y\mathbf{j} + \Sigma F_z\mathbf{k} = m(a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k})$$

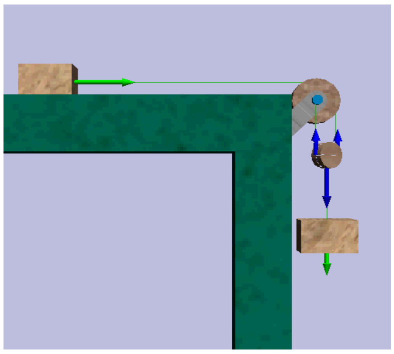
For this equation to be satisfied, the respective i, j, k components on the left side must equal the corresponding components on the right side. Consequently, we may write the following three scalar equations:



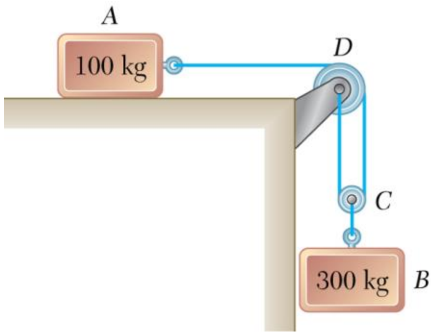
$$\Sigma F_x = ma_x$$
$$\Sigma F_y = ma_y$$
$$\Sigma F_z = ma_z$$

In particular, if the particle is constrained to move only in the x–y plane, then the first two of these equations are used to specify the motion.

Example 1: The two blocks shown start from rest. The horizontal plane and the pulley are frictionless, and the pulley is assumed to be of negligible mass. Determine the acceleration of each block and the tension in the cord.



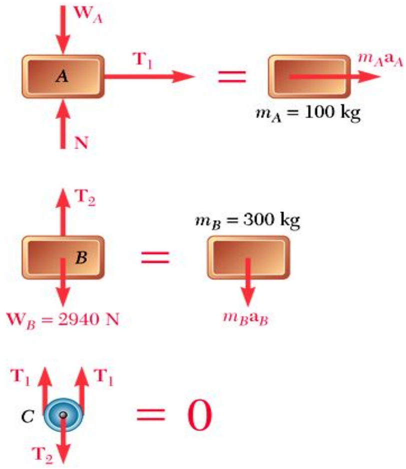
SOLUTION:



Kinematic relationship: If A moves x_A to the right, B moves down 0.5 x_A:

$$x_B = \tfrac{1}{2} x_A \qquad a_B = \tfrac{1}{2} a_A$$

Draw free body diagrams & apply Newton’s law:



$$\sum F_x = m_A a_A \quad \Longrightarrow \quad T_1 = (100) a_A$$

$$\sum F_y = m_B a_B \quad \Longrightarrow \quad m_B g - T_2 = m_B a_B$$

$$300 \times 9.81 - T_2 = (300) a_B$$

$$T_2 = 2940\text{-}(300) a_B$$

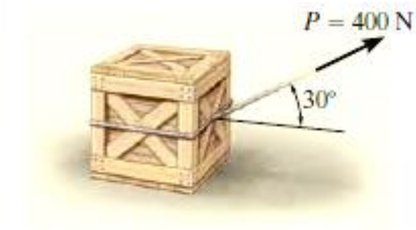
$$\sum F_y = m_C a_C \quad \Longrightarrow \quad T_2 - 2T_1 = 0$$

$$2940\text{-}(300) a_B - 2T_1 = 0 \qquad 2940\text{-}(300) a_B - 200 a_A = 0$$

$$2940\text{-}(300) a_B - 2 \times 200 a_B = 0$$

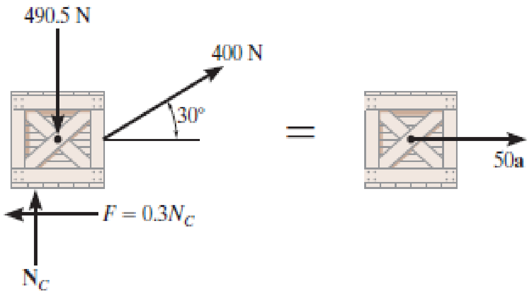
$$a_B = 4.2\, m/s^2 \qquad a_A = 8.4\, m/s^2 \qquad T_1 = 840\, N \qquad T_2 = 1680\, N$$

Example 2: The 50-kg crate shown in Fig. rests on a horizontal surface for which the coefficient of kinetic friction is $\mu_k = 0.3$. If the crate is subjected to a 400-N towing force as shown, determine the velocity of the crate in 3 s starting from rest.



SOLUTION:

$W = mg = 50$ kg (9.81 m/s²) = 490.5 N.



$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad 400 \cos 30° - 0.3N_C = 50a$$

$$+\uparrow \Sigma F_y = ma_y; \qquad N_C - 490.5 + 400 \sin 30° = 0$$

$$N_C = 290.5 \text{ N}$$

$$a = 5.185 \text{ m/s}^2$$

$$v = v_0 + a_c t = 0 + 5.185(3)$$

$$= 15.6 \text{ m/s} \rightarrow$$

Example 3: A 10-kg projectile is fired vertically upward from the ground, with an initial velocity of 50 m/s. Determine the maximum height to which it will travel if (*a*) atmospheric resistance is neglected; and (*b*) atmospheric resistance is measured as $F_D = (0.01v^2)$ N, where v is the speed of the projectile at any instant, measured in m/s.

SOLUTION:

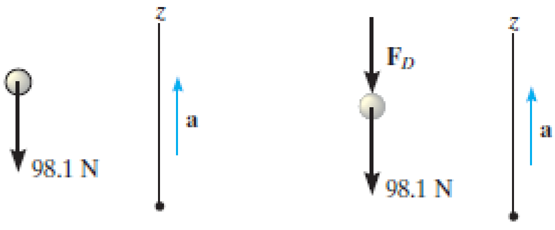
$W = mg = 10(9.81) = 98.1$ N

$$+\uparrow \Sigma F_z = ma_z; \qquad -98.1 = 10a, \qquad a = -9.81 \text{ m/s}^2$$

Kinematics. Initially, $z_0 = 0$ and $v_0 = 50$ m/s, and at the maximum height $z = h$, $v = 0$. Since the acceleration is *constant*, then

$(+\uparrow)$
$$v^2 = v_0^2 + 2a_c(z - z_0)$$
$$0 = (50)^2 + 2(-9.81)(h - 0)$$
$$h = 127 \text{ m} \qquad \textit{Ans.}$$

$F_D = (0.01v^2)$



$+\uparrow \Sigma F_z = ma_z;\quad -0.01v^2 - 98.1 = 10a,\quad a = -(0.001v^2 + 9.81)$

Kinematics. Here the acceleration is *not constant* since F_D depends on the velocity. Since $a = f(v)$, we can relate a to position using

$(+\uparrow)\ a\,dz = v\,dv;\qquad -(0.001v^2 + 9.81)\,dz = v\,dv$

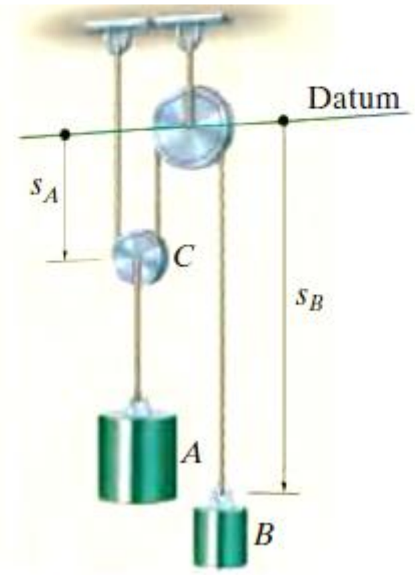
Separating the variables and integrating, realizing that initially $z_0 = 0$, $v_0 = 50$ m/s (positive upward), and at $z = h$, $v = 0$, we have

$$\int_0^h dz = -\int_{50\text{ m/s}}^{0} \frac{v\,dv}{0.001v^2 + 9.81} = -500\ln(v^2 + 9810)\Big|_{50\text{ m/s}}^{0}$$

$$h = 114 \text{ m} \qquad \textit{Ans.}$$

NOTE: The answer indicates a lower elevation than that obtained in part (*a*) due to atmospheric resistance or drag.

Example 4: The 100-kg block *A* shown in Fig. is released from rest. If the masses of the pulleys and the cord are neglected, determine the velocity of the 20-kg block *B* in 2 s.

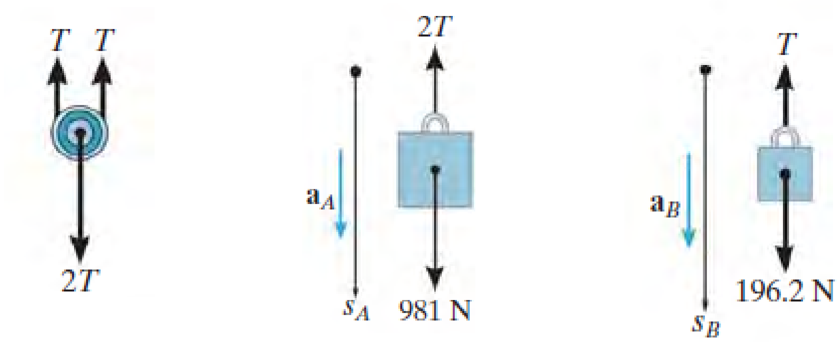


SOLUTION:

Notice from free body diagram that for *A* to remain stationary:
T = 1/2 * 9.81 *100= 490.5 N,

whereas for *B* to remain static:

T = 9.81 *20= 196.2 N.



Equations of Motion. Block A,

$+\downarrow \Sigma F_y = ma_y;$ $\quad 981 - 2T = 100a_A$

Block B,

$+\downarrow \Sigma F_y = ma_y;$ $\quad 196.2 - T = 20a_B$

$$2s_A + s_B = l$$

where l is constant and represents the total vertical length of cord. Differentiating this expression twice with respect to time yields

$$2a_A = -a_B$$

$$T = 327.0 \text{ N}$$
$$a_A = 3.27 \text{ m/s}^2$$
$$a_B = -6.54 \text{ m/s}^2$$

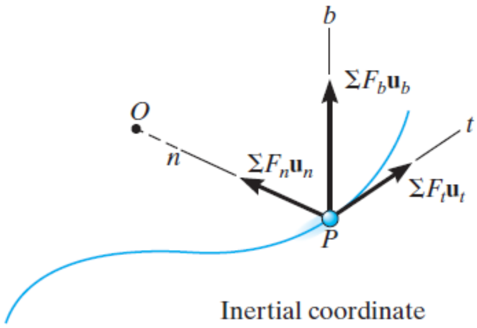
Hence when block A accelerates *downward*, block B accelerates *upward* as expected. Since a_B is constant, the velocity of block B in 2 s is thus

$(+\downarrow)$
$$v = v_0 + a_B t$$
$$= 0 + (-6.54)(2)$$
$$= -13.1 \text{ m/s} \qquad \textit{Ans.}$$

The negative sign indicates that block B is moving upward.

2-7 Equations of Motion: Normal and Tangential Coordinates

When a particle moves along a curved path which is known, the equation of motion for the particle may be written in the tangential, normal, and binomial directions. Note that there is no motion of the particle in the binomial direction, since the particle is constrained to move along the path. We have



Inertial coordinate system

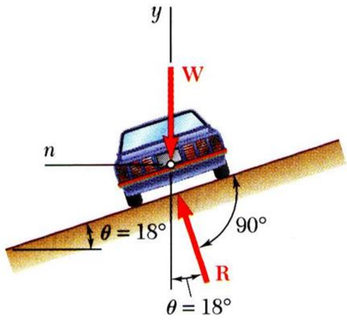
$$\Sigma \mathbf{F} = m\mathbf{a}$$
$$\Sigma F_t \mathbf{u}_t + \Sigma F_n \mathbf{u}_n + \Sigma F_b \mathbf{u}_b = m\mathbf{a}_t$$

This equation is satisfied provided

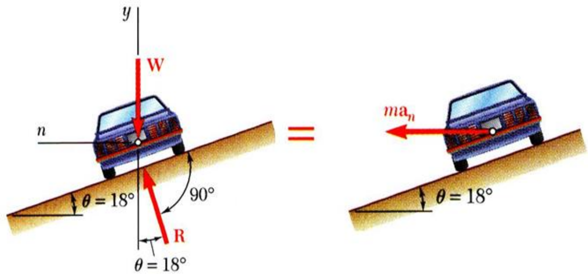
$$\Sigma F_t = m a_t$$
$$\Sigma F_n = m a_n$$

Recall that $a_t (= dv/dt)$ represents the time rate of change in the magnitude of velocity. So if $\Sigma \mathbf{F}_t$ acts in the direction of motion, the particle's speed will increase, whereas if it acts in the opposite direction, the particle will slow down. Likewise, $a_n (= v^2/\rho)$ represents the time rate of change in the velocity's direction. It is caused by $\Sigma \mathbf{F}_n$, which *always* acts in the positive n direction, i.e., toward the path's center of curvature. For this reason it is often referred to as the *centripetal force*.

Example 5: Determine the rated speed of a highway curve of radius r = 400 ft banked through an angle θ = 18°. The rated speed of a banked highway curve is the speed at which a car should travel if no lateral friction force is to be exerted at its wheels.



SOLUTION:



Resolve the equation of motion for the car into vertical and normal components:

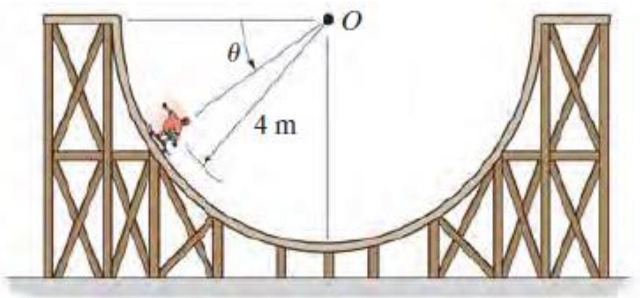
$$\sum F_y = 0 : R\cos\theta - W = 0 ....R = \frac{W}{\cos\theta}$$

$$R\sin\theta = \frac{W}{g} a_n .... \frac{W}{\cos\theta}\sin\theta = \frac{W}{g}\frac{v^2}{\rho}$$

Solve for the vehicle speed:

$$v^2 = g\rho\tan\theta = \left(32.2\,\text{ft/s}^2\right)\left(400\,\text{ft}\right)\tan 18° = 64.7\,\text{ft/s} = 44.1\,\text{mi/h}$$

Example 6: The 60-kg skateboarder in Fig. coasts down the circular track. If he starts from rest when $\theta = 0^o$, determine the magnitude of the normal reaction the track exerts on him when $\theta = 60^o$. Neglect his size for the calculation.



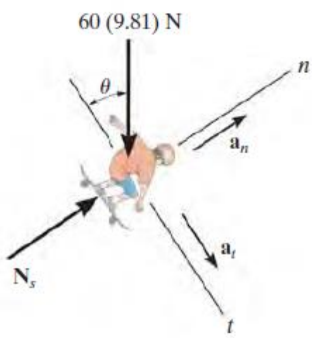
SOLUTION:

Equations of Motion.

$$+\nearrow \Sigma F_n = ma_n; \quad N_s - [60(9.81)\text{N}]\sin\theta = (60\text{ kg})\left(\frac{v^2}{4\text{ m}}\right)$$

$$+\searrow \Sigma F_t = ma_t; \quad [60(9.81)\text{N}]\cos\theta = (60\text{ kg})\,a_t$$

$$a_t = 9.81\cos\theta$$



Since a_t is expressed in terms of θ, the equation $v\,dv = a_t\,ds$ must be used to determine the speed of the skateboarder when $\theta = 60^o$. Using the geometric relation $s = \theta\, r$, where $ds = r\, d\theta = (4 \text{ m})\, d\theta$, and the initial condition $v = 0$ at $\theta = 0^o$, we have,

$$v\,dv = a_t\,ds$$

$$\int_0^v v\,dv = \int_0^{60^\circ} 9.81 \cos\theta(4\,d\theta)$$

$$\left.\frac{v^2}{2}\right|_0^v = 39.24 \sin\theta \Big|_0^{60^\circ}$$

$$\frac{v^2}{2} - 0 = 39.24(\sin 60^\circ - 0)$$

$$v^2 = 67.97 \text{ m}^2/\text{s}^2$$

Substituting this result and $\theta = 60^o$ into Eq. of N_s, yields

$$N_s = 1529.23 \text{ N} = 1.53 \text{ kN}$$

2-8 Equations of Motion: Cylindrical Coordinates

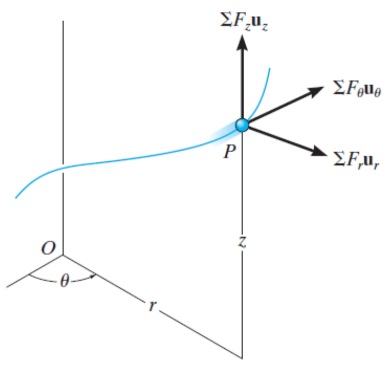
When all the forces acting on a particle are resolved into cylindrical components, i.e., along the unit-vector directions $\mathbf{u}_r$, $\mathbf{u}_\theta$, $\mathbf{u}_z$, the equation of motion can be expressed as

$$\Sigma\mathbf{F} = m\mathbf{a}$$

$$\Sigma F_r\mathbf{u}_r + \Sigma F_\theta\mathbf{u}_\theta + \Sigma F_z\mathbf{u}_z = ma_r\mathbf{u}_r + ma_\theta\mathbf{u}_\theta + ma_z\mathbf{u}_z$$

To satisfy this equation, we require

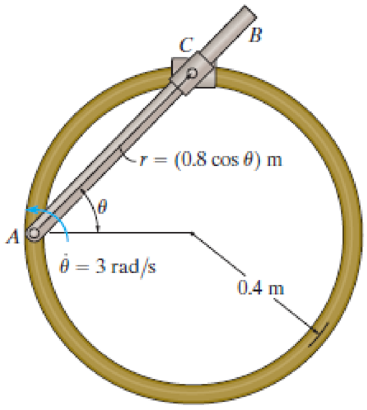
$$\Sigma F_r = ma_r$$
$$\Sigma F_\theta = ma_\theta$$
$$\Sigma F_z = ma_z$$



Inertial coordinate system

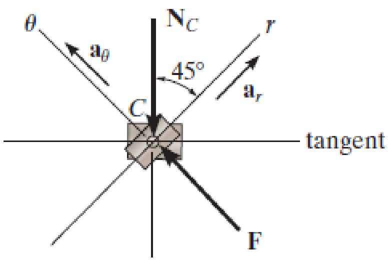
Example 7: The smooth 0.5-kg double-collar in Fig. can freely slide on arm *AB* and the circular guide rod.

If the arm rotates with a constant angular velocity of $\dot{\theta}$ = 3 rad/s, determine the force the arm exerts on the ollar at the instant θ= 45°. Motion is in the horizontal plane.



SOLUTION:

Free-Body Diagram. The normal reaction $\boldsymbol{N}_C$ of the circular guide rod and the force ***F*** of arm *AB* act on the collar in the plane of motion. Note that ***F*** acts perpendicular to the axis of arm *AB*, that is, in the direction of the u axis, while $\boldsymbol{N}_C$ acts perpendicular to the tangent of the circular path at θ= 45°. The four unknowns are N_C, F, a_r, a_θ.



Equations of Motion.

$$+\nearrow\Sigma F_r = ma_r: \qquad -N_C \cos 45° = (0.5 \text{ kg})\, a_r \qquad (1)$$

$$+\nwarrow\Sigma F_\theta = ma_\theta: \qquad F - N_C \sin 45° = (0.5 \text{ kg})\, a_\theta \qquad (2)$$

Kinematics. Using the chain rule, the first and second time derivatives of *r* when θ= 45°, $\dot{\theta}$ = 3 rad/s, θ = 0, are

$$r = 0.8 \cos\theta = 0.8\cos 45° = 0.5657 \text{ m}$$

$$\dot{r} = -0.8 \sin\theta\,\dot{\theta} = -0.8\sin 45°(3) = -1.6971 \text{ m/s}$$

$$\ddot{r} = -0.8\left[\sin\theta\,\ddot{\theta} + \cos\theta\,\dot{\theta}^2\right]$$

$$= -0.8[\sin 45°(0) + \cos 45°(3^2)] = -5.091 \text{ m/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -5.091 \text{ m/s}^2 - (0.5657 \text{ m})(3 \text{ rad/s})^2 = -10.18 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (0.5657 \text{ m})(0) + 2(-1.6971 \text{ m/s})(3 \text{ rad/s})$$

$$= -10.18 \text{ m/s}^2$$

Substituting these results into Eqs. (1) and (2) and solving, we get

$$N_C = 7.20 \text{ N}$$

$$F = 0 \qquad \textit{Ans.}$$