University of Anbar

Engineering Mechanics: Statics
CHE211

Lecture # 01

Objectives of Lecture Note

- To provide an introduction to the basic quantities and idealizations of mechanics.
- To give a statement of Newton's Laws of Motion and Gravitation.
- To examine the standard procedures for performing numerical calculations.
- To present a general guide for solving problems.

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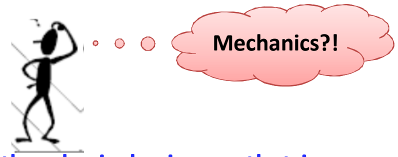
Concept: Mechanics



- ***Mechanics:*** is a branch of the physical sciences that is concerned with the state of rest or motion of bodies that are subjected to the action of ***force***.
- In general. this subject can be subdivided into three branches: ***rigid-body mechanics***, ***deformable-body*** and ***fluid mechanics***.

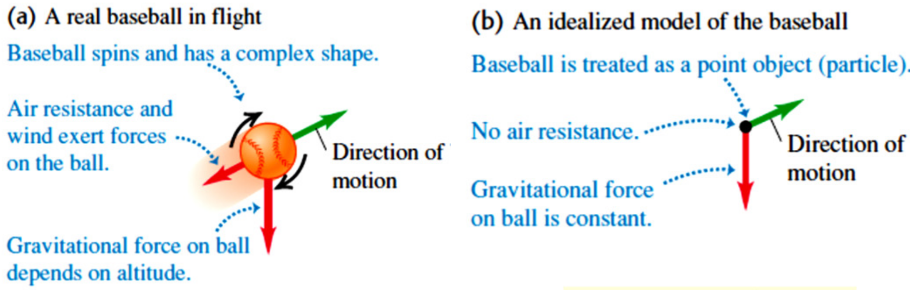
- **Rigid-Body Mechanics:**
- **1. Statics: Deals with effect of force on bodies which are not moving (at rest or move with constant velocity).**
- **2. Dynamics: Deals with force effect on moving bodies (accelerated motion of bodies).**

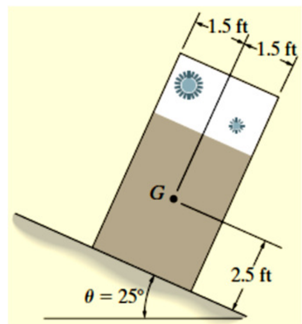
Idealization or Models

- **Models or idealizations are used in mechanics in order to simplify application of the theory. Here, we will consider three important idealizations;**

- **Particle: A particle has a mas, but a size that can be neglected. (negligible dimensions)**
- **Rigid Body: A rigid body can be considered as a combination of a large number of particles in which all the particles remain at a fixed distance from one another, both before and after applying load.**
- **Concentrated Force: A concentrated force represents the effect of a loading which is assumed to act at a point on a body.**

Idealized Models





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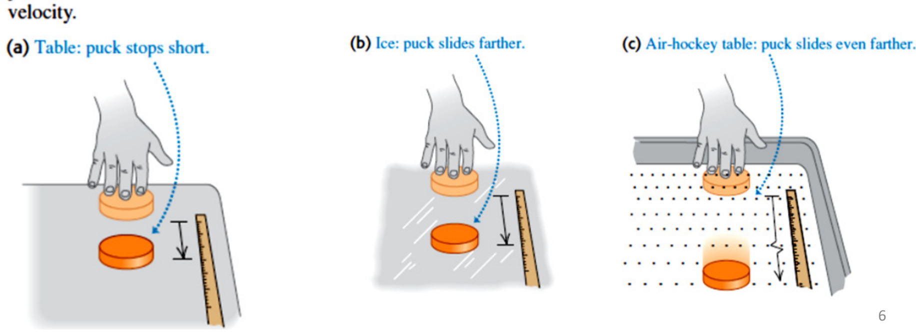
Newton's Three Laws of Motion?!

- **Newton's Three Laws of Motion:** Engineering mechanics is formulated on the basis of Newton's three laws of motion.
- **First Law (Law of Inertia):** A particle originally at rest, or moving in a straight line with ***constant velocity***, tends to remain in this state provided the particle is ***NOT*** subjected to an unbalanced force.

The slicker the surface, the farther a puck slides after being given an initial velocity. On an air-hockey table (c) the friction force is practically zero, so the puck continues with almost constant velocity.

Things want to stay in the position where they are!

Things want to keep doing what they are already doing!



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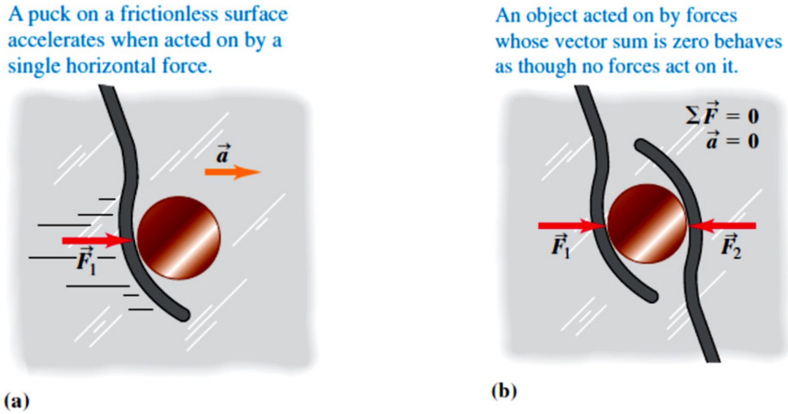
Inertia

- The tendency of an object to remain at rest, or to keep moving once it is set in motion, results from a property called ***inertia***.

Laziness=Inertia



A puck responding to a nonzero net force (a) and to two forces whose vector sum is zero (b).

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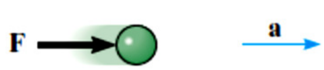
Newton's Three Laws of Motion?!

- **Second Law:** A particle acted upon by an **unbalanced force** **F** experiences an acceleration **a** that has the same direction as the force and magnitude directly proportional to the force. If **F** is applied to a particle of mass of ***m*** , this law may be expressed mathematically as;

$$\mathbf{F} = m\mathbf{a}$$



Accelerated motion

Equation of $\sum F = ma$ is valid only when the mass *m* is *constant*. It's easy to think of systems whose masses change, such as a leaking tank truck, a rocket ship, or a moving railroad car being loaded with coal. Such systems are better handled by using the concept of momentum.

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Newton's Three laws of Motion?!

Newton's third law

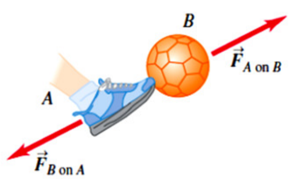
For two interacting objects *A* and *B*, the formal statement of Newton's third law is

$$\vec{F}_{A\text{ on }B} = -\vec{F}_{B\text{ on }A}.$$

Newton's own statement, translated from the Latin of the *Principia*, is

> To every action there is always opposed an equal reaction; or, the mutual actions of two objects upon each other are always equal, and directed to contrary parts.

If body *A* exerts a force $\vec{F}_{A\text{ on }B}$ on body *B*, then body *B* exerts a force $\vec{F}_{B\text{ on }A}$ on body *A* that is equal in magnitude and opposite in direction: $\vec{F}_{A\text{ on }B} = -\vec{F}_{B\text{ on }A}$.



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Newton's Law of Gravitational Attraction

- **Newton's Law of Gravitational Attraction:** Shortly after formulating his three laws of motion, Newton postulated a law governing the gravitational attraction between any two particles. Stated mathematically as;

$$F = G\frac{m_1 m_2}{r^2}$$

where

F = force of gravitation between the two particles

G = universal constant of gravitation; according to experimental evidence, $G = 66.73(10^{-12})\ \text{m}^3/(\text{kg}\cdot\text{s}^2)$

m_1, m_2 = mass of each of the two particles

r = distance between the two particles

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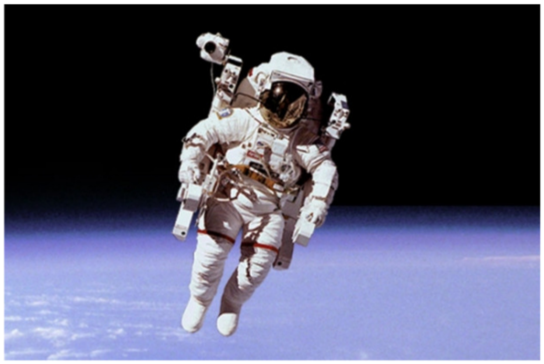
Gravitational Attraction of the Earth (Weight)

- The gravitational attraction of the earth on a body (its weight) exists whether the body is at rest or in motion.

$$W = mg$$

Weight of a body is *not* an absolute quantity!!



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Scalar Vs. Vector

- **Scalar:** A positive or negative physical quantity defined strictly by it's ***magnitude***. Examples of scalar quantities include length, mass, volume and time.
- **Vector:** Any physical quantity which is defined by both a ***magnitude*** and ***direction.*** Examples of vectors encountered in statics are force, velocity, position and moment.
- **Notation:**
  - **(In Print): A** (bold face) is vector and *A* (italicized) is scalar
  - **Handwriting:** $\vec{A}$

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Scalar Vs. Vector

- Some physical quantities, such as time, temperature, mass, density, and electric charge, can be described completely by a single number with a unit.
- Many other quantities, however, have a directional quality and cannot be described by a single number.
- A familiar example is velocity. To describe the motion of an airplane, we have to say not only how fast it is moving, but also in what direction. To fly from Baghdad to Basra, the plane has to head south, not north. Force is another example. When we push or pull on a body, we exert a force on it. To describe a force, we need to describe the direction in which it acts, as well as its magnitude, or "how hard" the force pushes or pulls.
- When a physical quantity is described by a ***single number***, we call it a ***scalar quantity***. In contrast, a ***vector quantity*** has both a ***magnitude*** (the "how much" or "how big" part) and a ***direction*** in space.

Scalar Vs. Vector

Vector

– Represented graphically as an arrow

– Length of arrow = Magnitude of Vector

– Angle between the reference axis and arrow's line of action = Direction of Vector

– Arrowhead = Sense of Vector

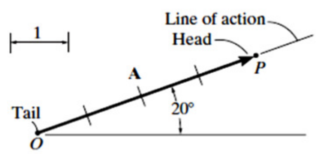
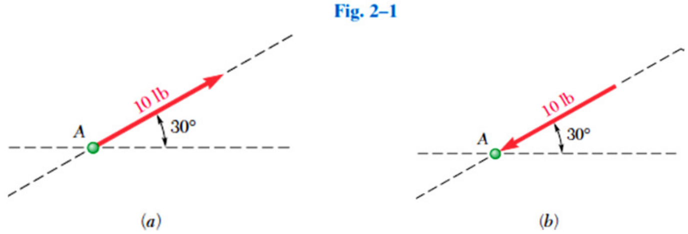


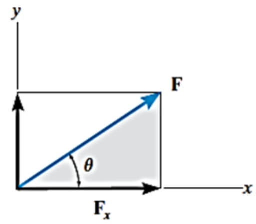
Fig. 2–1



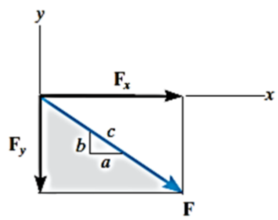
Addition of a System of Coplanar Forces

When a force is resolved into two components along the x and y axes, the components are then called *rectangular components*. For analytical work we can represent these components in one of two ways, using either scalar notation or Cartesian vector notation.

- **Scalar Notation**
- $\Sigma F_{RX} = \Sigma F_X$
- $\Sigma F_{RY} = \Sigma F_Y$
- $F_R = \sqrt{(F_{RX})^2 + (F_{RY})^2}$
- $\theta = \tan^{-1}\left|\frac{F_{RY}}{F_{RX}}\right|$



$$F_x = F\cos\theta$$
$$F_y = F\sin\theta$$



$$F_x = F\left(\frac{a}{c}\right)$$
$$F_y = -F\left(\frac{b}{c}\right)$$

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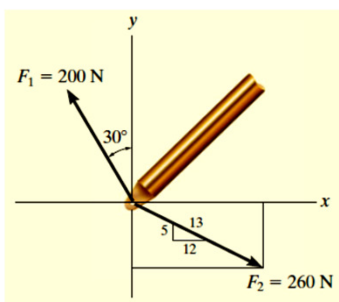
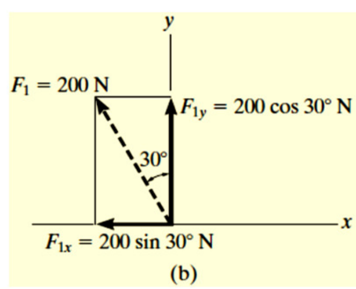
Example 1

Determine the x *and* y components of F_1 and F_2 acting on the boom shown in the Figure.

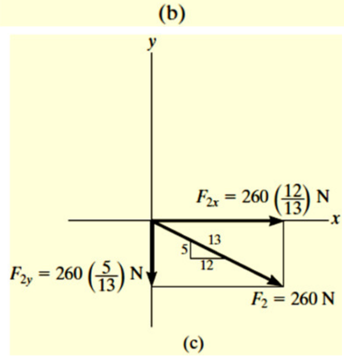




$$F_{1x} = -200 \sin 30^\circ \text{ N} = -100 \text{ N} = 100 \text{ N} \leftarrow$$
$$F_{1y} = 200 \cos 30^\circ \text{ N} = 173 \text{ N} = 173 \text{ N} \uparrow$$

$$\frac{F_{2x}}{260 \text{ N}} = \frac{12}{13} \qquad F_{2x} = 260 \text{ N}\left(\frac{12}{13}\right) = 240 \text{ N} \rightarrow$$

$$F_{2y} = 260 \text{ N}\left(\frac{5}{13}\right) = 100 \text{ N} \downarrow$$



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Example 2

The link in the Figure below is subjected to two forces F_1 and F_2. Determine the magnitude and direction of the resultant force .

Scalar Notation. First we resolve each force into its x and y components, Fig. 2–19b, then we sum these components algebraically.

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \quad (F_R)_x = 600 \cos 30° \text{ N} - 400 \sin 45° \text{ N} = 236.8 \text{ N} \rightarrow$$

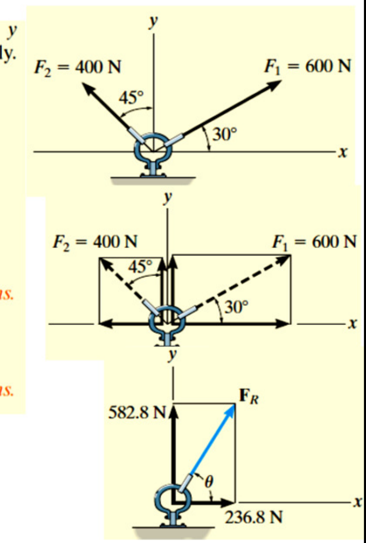
$$+\uparrow (F_R)_y = \Sigma F_y; \quad (F_R)_y = 600 \sin 30° \text{ N} + 400 \cos 45° \text{ N} = 582.8 \text{ N}\uparrow$$

The resultant force, shown in Fig. 2–18c, has a *magnitude* of

$$F_R = \sqrt{(236.8 \text{ N})^2 + (582.8 \text{ N})^2} = 629 \text{ N} \qquad \textit{Ans.}$$

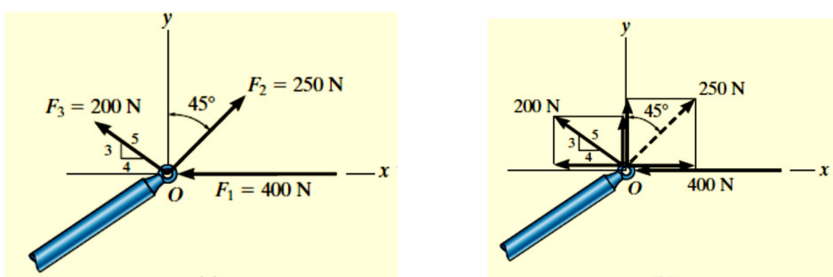
From the vector addition,

$$\theta = \tan^{-1}\left(\frac{582.8 \text{ N}}{236.8 \text{ N}}\right) = 67.9° \qquad \textit{Ans.}$$



Example 3

The end of the boom O in the figure is subjected to three concurrent and coplanar forces. Determine the magnitude and direction of the resultant force.

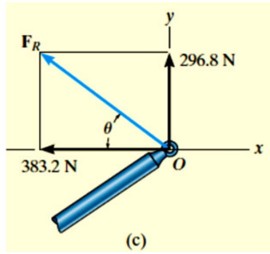


(a) (b)

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \quad (F_R)_x = -400 \text{ N} + 250 \sin 45° \text{ N} - 200\left(\tfrac{4}{5}\right) \text{ N} = -383.2 \text{ N} = 383.2 \text{ N} \leftarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \quad (F_R)_y = 250 \cos 45° \text{ N} + 200\left(\tfrac{3}{5}\right) \text{ N} = 296.8 \text{ N}\uparrow$$

Example 3 Cont..



(c)

The resultant force, shown in Fig. 2–20*c*, has a *magnitude* of

$$F_R = \sqrt{(-383.2\text{ N})^2 + (296.8\text{ N})^2}$$
$$= 485\text{ N}$$

From the vector addition in Fig. 2–20*c*, the direction angle θ is

$$\theta = \tan^{-1}\left(\frac{296.8}{383.2}\right) = 37.8^\circ$$

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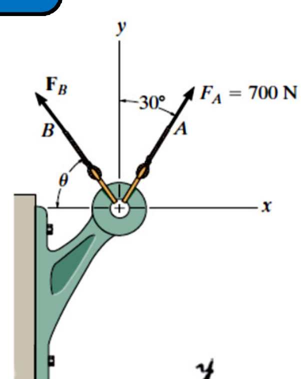
Example 4

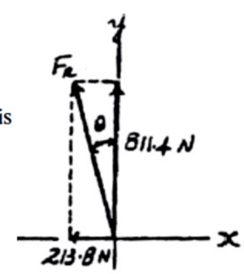
Determine the magnitude and orientation, measured counterclockwise from the positive $y\ axis$, of the resultant force acting on the bracket, if $F_B = 600\ N$ and $\theta = 20°$.



$$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = 700 \sin 30^\circ - 600 \cos 20^\circ$$
$$= -213.8\text{ N} = 213.8\text{ N} \leftarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = 700 \cos 30^\circ + 600 \sin 20^\circ$$
$$= 811.4\text{ N} \uparrow$$

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{213.8^2 + 811.4^2} = 839\text{ N}$$



The direction angle θ measured counterclockwise from the positive y axis

$$\theta = \tan^{-1}\frac{F_{R_x}}{F_{R_y}} = \tan^{-1}\left(\frac{213.8}{811.4}\right) = 14.8^\circ$$

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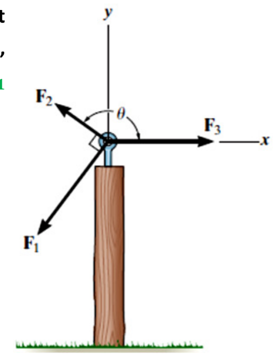
Example 5

The three concurrent forces acting on the post produce a resultant force $F_R = 0$. If $F_2 = \frac{1}{2}F_1$, and F_1 is to be $90°$ from F_2 as shown, determine the required magnitude of F_3 expressed in terms of F_1 and the angle θ.



$\sum F_{R\acute{x}} = 0; \quad F_3 \cos(\theta - 90°) = F_1 \qquad (1)$

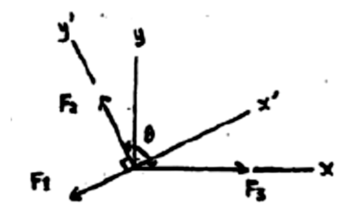
$\sum F_{R\acute{y}} = 0; \quad F_3 \sin(\theta - 90°) = F_2 \qquad (2)$

By dividing Eq. 2 by 1

$$tan(\theta - 90°) = \frac{F_2}{F_1} = \frac{1}{2}$$

$$\theta - 90° = 26.57°$$

$$\theta = 116.57° = 117°$$



Solving Eq. 1

$$F_3 = \frac{F_1}{cos(117° - 90°)}$$

$$F_3 = 1.12F_1$$

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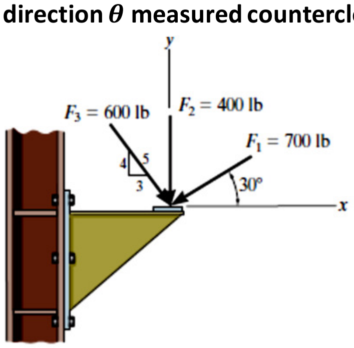
Try it Yourself GOOD LUCK ☺

Determine the magnitude of the resultant force acting on the corbel and its direction θ measured counterclockwise from the x $axis$.



$$\overset{+}{\rightarrow}(F_R)_x = \Sigma F_x;$$

$$(F_R)_x = -(700 \text{ lb}) \cos 30° + 0 + \left(\tfrac{3}{5}\right)(600 \text{ lb})$$

$$= -246.22 \text{ lb}$$

$$+\uparrow(F_R)_y = \Sigma F_y;$$

$$(F_R)_y = -(700 \text{ lb}) \sin 30° - 400 \text{ lb} - \left(\tfrac{4}{5}\right)(600 \text{ lb})$$

$$= -1230 \text{ lb}$$

$$F_R = \sqrt{(246.22 \text{ lb})^2 + (1230 \text{ lb})^2} = 1254 \text{ lb}$$

$$\phi = \tan^{-1}\left(\frac{1230 \text{ lb}}{246.22 \text{ lb}}\right) = 78.68°$$

$$\theta = 180° + \phi = 180° + 78.68° = 259°$$

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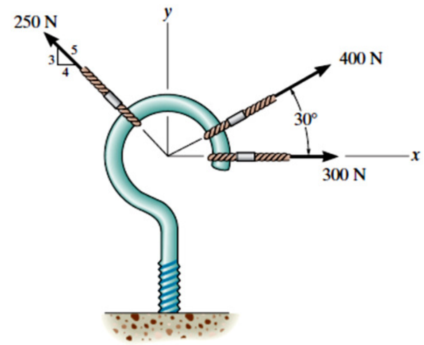
Try it Yourself GOOD LUCK ☺

Determine the magnitude and direction of the resultant force.



$$F_{Rx} = 300 + 400\cos 30^\circ - 250\left(\tfrac{4}{5}\right) = 446.4\text{ N}$$

$$F_{Ry} = 400\sin 30^\circ + 250\left(\tfrac{3}{5}\right) = 350\text{ N}$$

$$F_R = \sqrt{(446.4)^2 + 350^2} = 567\text{ N}$$ *Ans.*

$$\theta = \tan^{-1}\frac{350}{446.4} = 38.1^\circ$$ *Ans.*

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