University of Anbar

Engineering Mechanics: Statics
CHE211

Lecture # 02

Objectives of Lecture Note

- To show how to add forces and resolve them into components using scalar notation.
- To introduce the concept of the free-body diagram for a particle.
- To show how to solve particle equilibrium problem using the equations of equilibrium.
- To solve various particle equilibrium problems using the equations of equilibrium.

Condition for the Equilibrium of a Particle

- A particle is to be said in *equilibrium* if it remains at rest if originally at rest, or has a constant velocity if originally in motion.

- A system is said to be in "static equilibrium" if the following conditions are met:

$$\sum \boldsymbol{F} = \mathbf{0}$$
$$\sum \boldsymbol{F_x} = \mathbf{0}$$
$$\sum \boldsymbol{F_y} = \mathbf{0}$$

Note: These conditions are applied to a ***particle***, for rigid bodies the sum of the torques/moments also need to equal zero

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Condition for the Equilibrium of a Particle

- Particle at *equilibrium* if
  - At rest
  - Moving at a constant velocity

- Newton's first law of motion

$$\sum \boldsymbol{F} = \mathbf{0}$$

where $\sum F$ is the vector sum of all the forces acting on the particle

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Condition for the Equilibrium of a Particle

- Newton's second law of motion

$$\sum \boldsymbol{F} = m\boldsymbol{a}$$

- When the force fulfill Newton's first law of motion,

$$m\boldsymbol{a} = 0$$
$$\boldsymbol{a} = 0$$

therefore, the particle is moving in constant velocity or at rest

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The Free-Body Diagram

- **To apply the equation of equilibrium we must account for all the known and unknown forces ($\sum \boldsymbol{F}$) which act on the particle.**
- **The best way to do this is to think of the particle as isolated and "free" from its surroundings.**
- **A drawing that shows the particle with all the forces that act on it is called a *free body diagram (FBD).***

The free-body diagram is the most important single step in the solution of problems in mechanics

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The Free-Body Diagram

Procedure for Drawing a FBD

1. **Decide which system to isolate.**

2. **Draw outlined shape**

   -Isolate particle from its surroundings

3. **Show all the forces**

   -Indicate all the forces

   -Active forces: set the particle in motion

   -Reactive forces: result of constraints and supports that tend to prevent motion.

4. **Identify each forces**

   -Known forces should be labeled with proper magnitude and direction

   -Letters are used to represent magnitude and directions of unknown forces

The Free-Body Diagram

Procedure for Drawing a FBD

1. **Decide which system to isolate.**
2. **Draw outlined shape**
3. **Show all the forces**
4. **Identify each forces**

- **The bucket is held in equilibrium by the cable**
- **Force in the cable = *weight of the bucket***
- **Isolate the bucket for FBD**
- **Two forces acting on the bucket, weight *W* and force *T* of the cable**
- **Resultant of forces = 0**

$$W = T$$

Types of Connections

Basically, there are two types of connections often encountered in particle equilibrium problems.

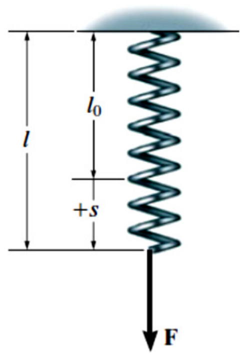
i. Springs

ii. Cables and Pulley

Springs:

- **Linear elastic spring:** **change in length is directly proportional to the force acting on it**
- ***Spring constant or stiffness k*****:**
  defines the elasticity of the spring
- ***Magnitude of force*** **when spring is elongated or compressed**

$$F = ks$$



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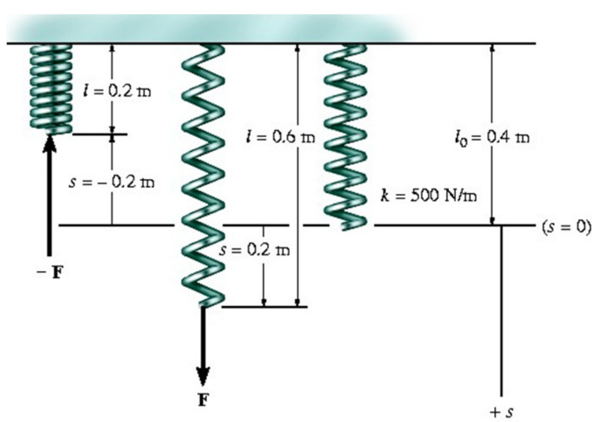
Springs

- s is determined from the difference in spring's deformed length l and its undeformed length l_o

$$s = l - l_o$$

- If s is positive, **F** "pull" onto the spring
- If s is negative, **F** "push" onto the spring



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Springs

Example

Given $l_o = 0.4m$ and $k = 500\ N/m$

To stretch it until $l = 0.6m$,

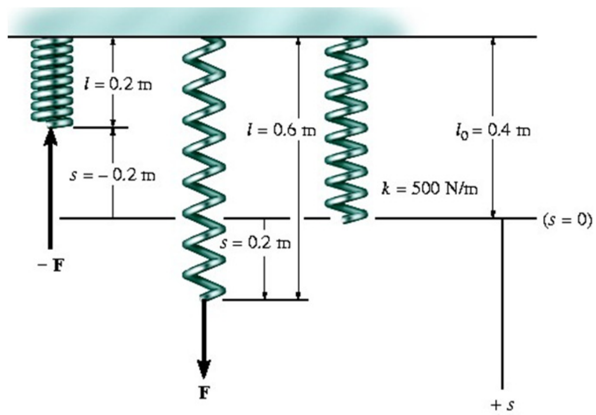
A force $F = ks = (500N/m)(0.6m - 0.4m) = 100N$ is needed

To compress it until $l = 0.2m$,

A force, $F = ks$

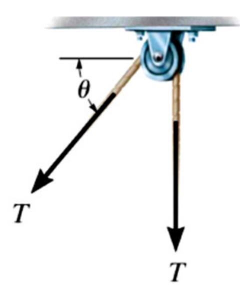
=(500N/m)(0.2m – 0.4m)

= -100N is needed



Cables and Pulley

- Cables (or cords) are assumed to have negligible weight and they cannot stretch
- A cable only support tension or pulling force
- Tension always acts in the direction of the cable
- Tension force in a continuous cable must have a constant magnitude for equilibrium
- For any angle θ, the cable is subjected to a constant tension T throughout its length



Cable is in tension

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Procedure for Drawing a Free-Body Diagram

Since we must account for *all the forces acting on the particle* when applying the equations of equilibrium, the importance of first drawing a free-body diagram cannot be overemphasized. To construct a free-body diagram, the following three steps are necessary.

Draw Outlined Shape.

Imagine the particle to be *isolated* or cut "free" from its surroundings by drawing its outlined shape.

Show All Forces.

Indicate on this sketch *all* the forces that act *on the particle*. These forces can be *active forces*, which tend to set the particle in motion, or they can be *reactive forces* which are the result of the constraints or supports that tend to prevent motion. To account for all these forces, it may be helpful to trace around the particle's boundary, carefully noting each force acting on it.

Identify Each Force.

The forces that are *known* should be labeled with their proper magnitudes and directions. Letters are used to represent the magnitudes and directions of forces that are unknown.

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Examples to Draw Free Body Diagram

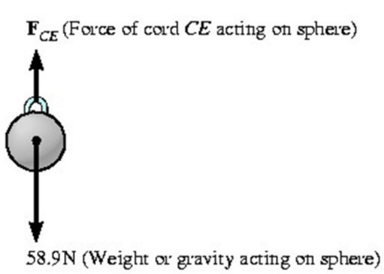
- **Example:**

The sphere has a mass of $6\,kg$ and is supported. Draw a *free-body diagram* of the sphere, the cord CE and the knot at C.

Solution

Sphere:

- **Two forces acting, weight and the force on cord CE.**
- **Weight of 6kg (9.81m/s^2) = 58.9N**



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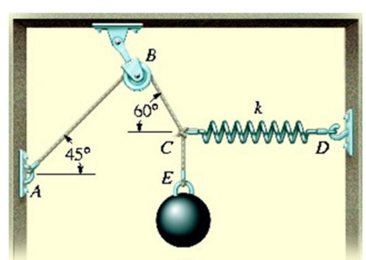

(a)

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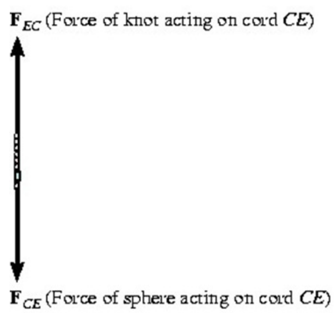
Examples to Draw Free Body Diagram

Solution

Cord CE:

- Two forces acting, force of the sphere and force of the knot
- Newton's Third Law: F_{CE} is equal but opposite
- F_{CE} and F_{EC} pull the cord in tension
- For equilibrium, $F_{CE} = F_{EC}$



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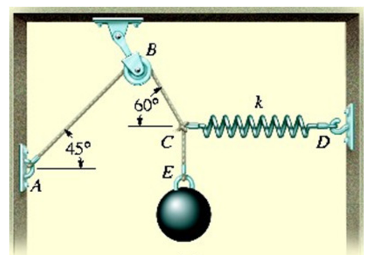

(a)

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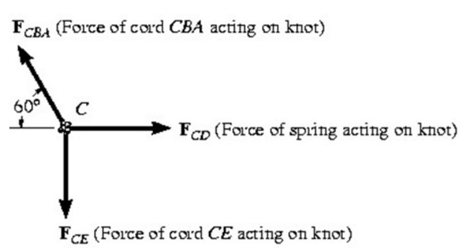
Examples to Draw Free Body Diagram

Solution

Knot C:

- Three forces acting, force by cord CBA, cord CE and spring CD
- Important to know that the weight of the sphere does not act directly on the knot but subjected to by the cord CE



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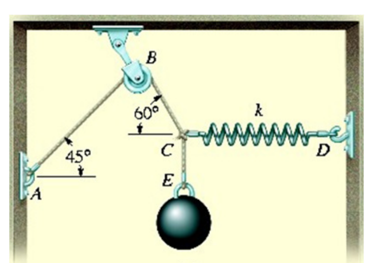

(a)

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A General Rule in a Class

Problem solutions which do not include a FBD will receive substantially reduced credit. You will never get full credit for a problem solution if you don't draw a FBD

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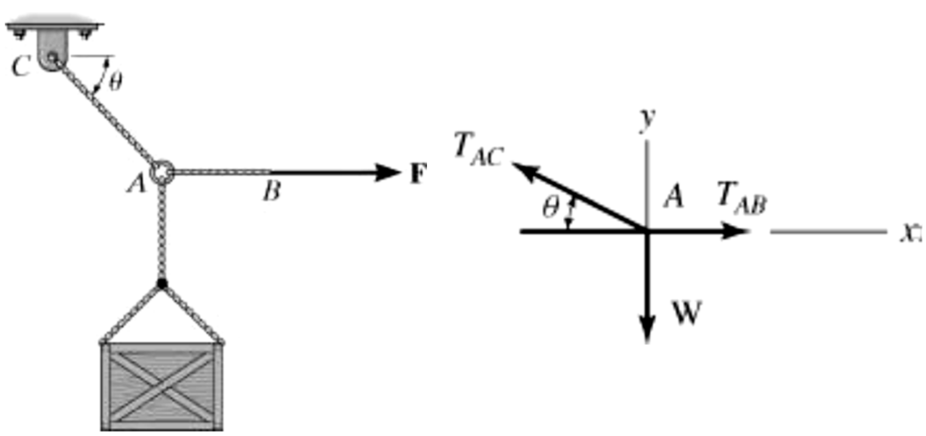
Examples to Draw Free Body Diagram



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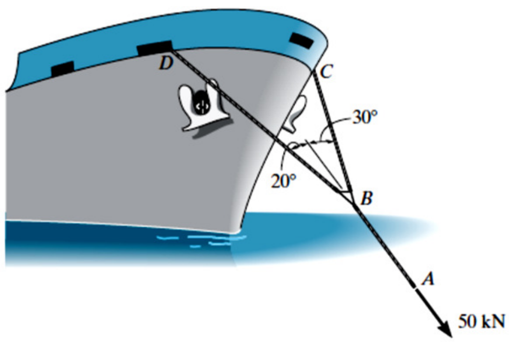
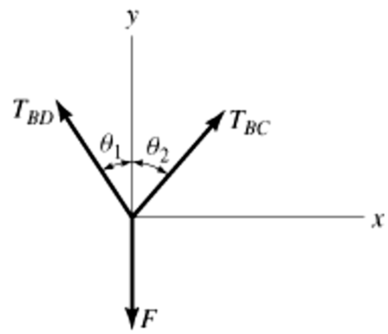
Examples to Draw Free Body Diagram





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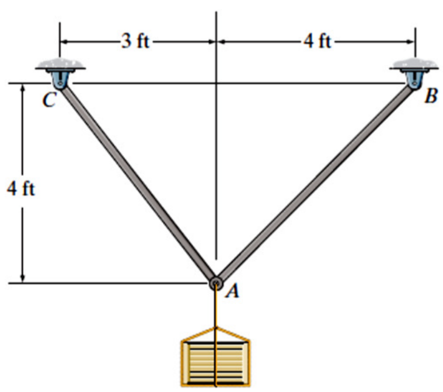
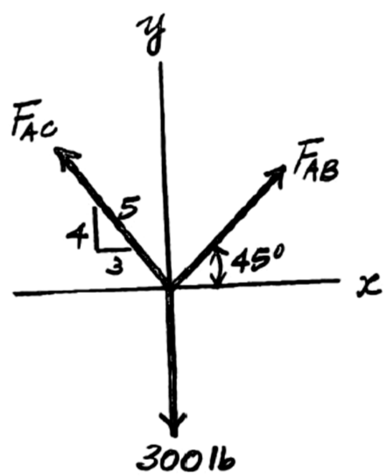
Examples to Draw Free Body Diagram





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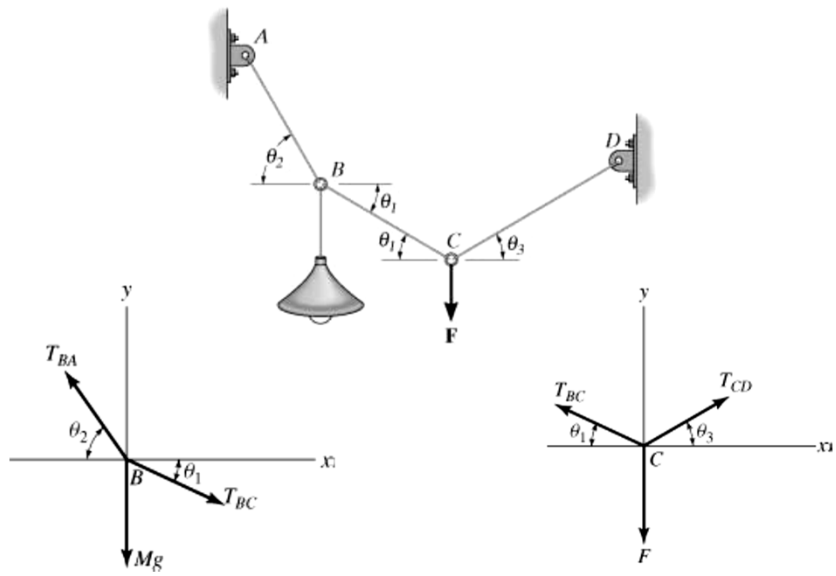
Examples to Draw Free Body Diagram



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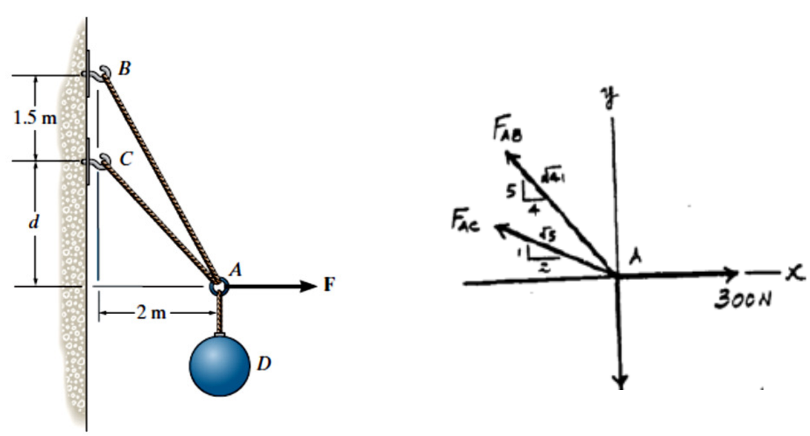
Examples to Draw Free Body Diagram



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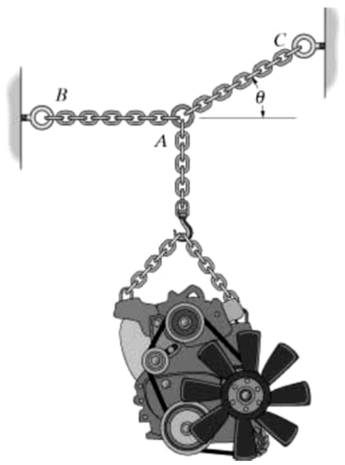
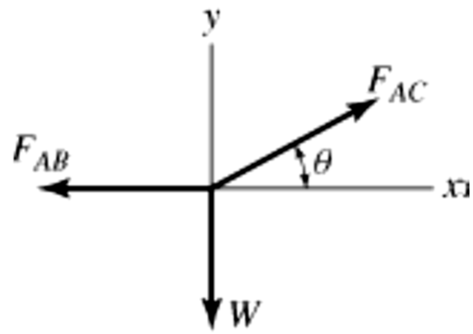
Examples to Draw Free Body Diagram





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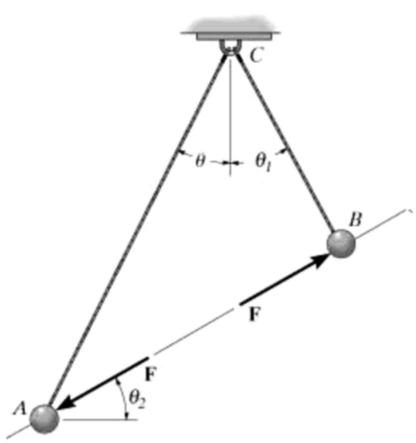
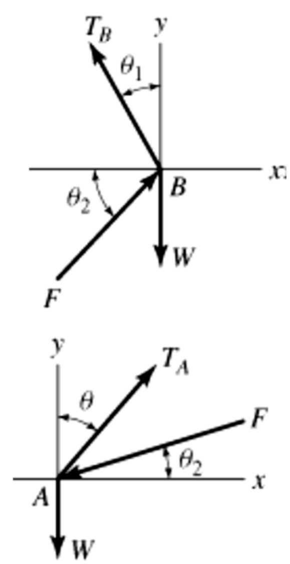
Examples to Draw Free Body Diagram





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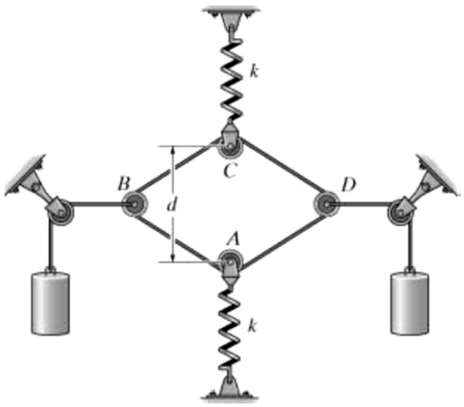
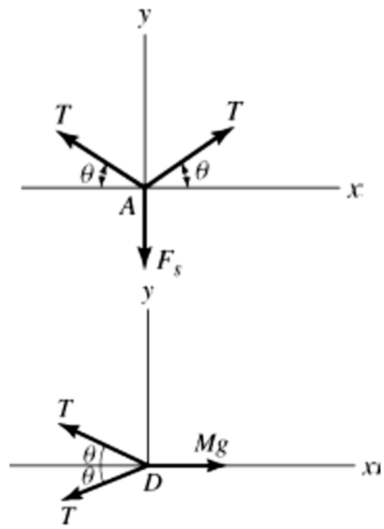
Examples to Draw Free Body Diagram





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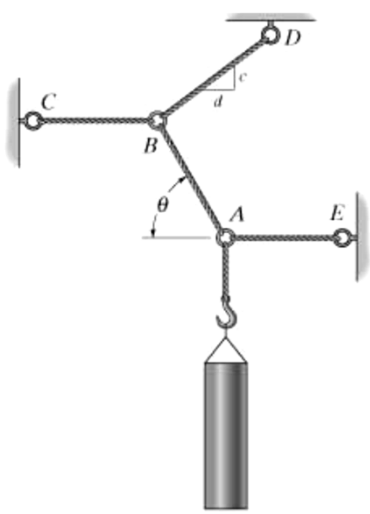
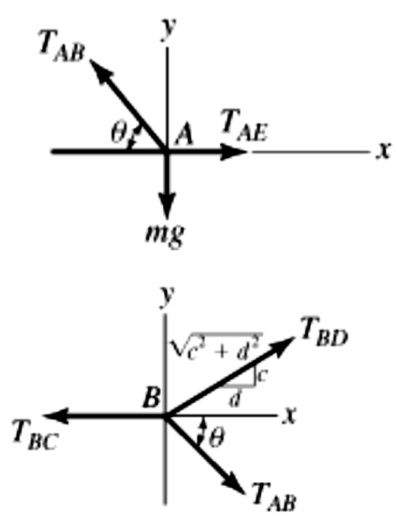
Examples to Draw Free Body Diagram





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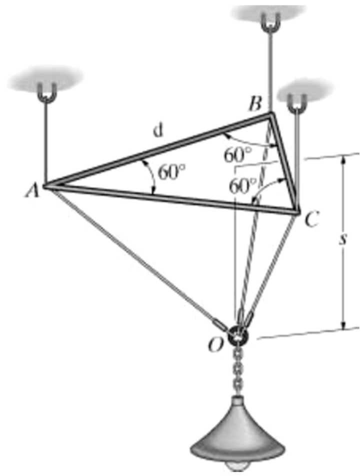
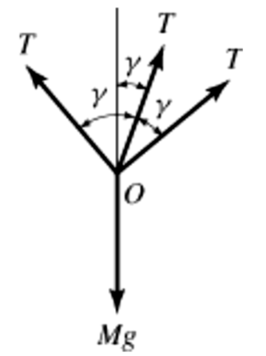
Examples to Draw Free Body Diagram





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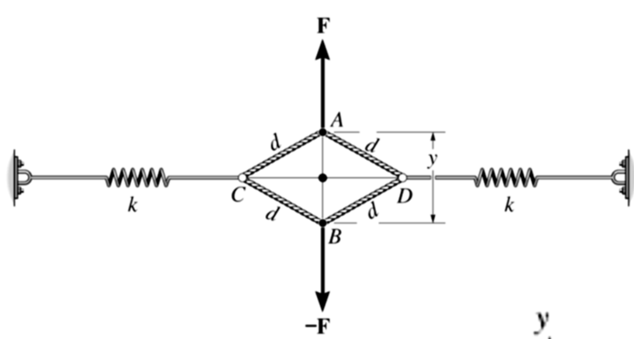
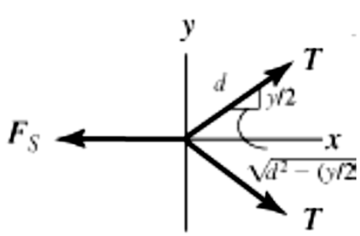
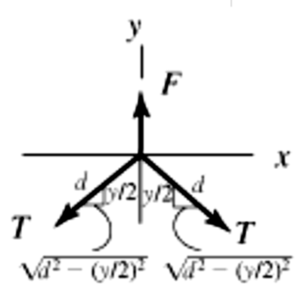
Examples to Draw Free Body Diagram







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Reading Quiz

1. When a particle is in equilibrium, the sum of forces acting on it equals (______). Choose the most appropriate answer;

a) A constant

b) A positive number

c) Zero

d) A negative number

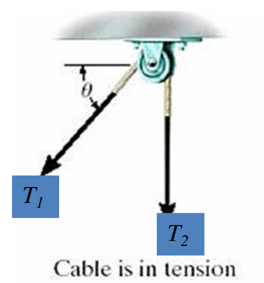
2. For a frictionless pulley and cable, tensions in the cables are related as

a) $T_1 > T_2$

b) $T_1 = T_2$

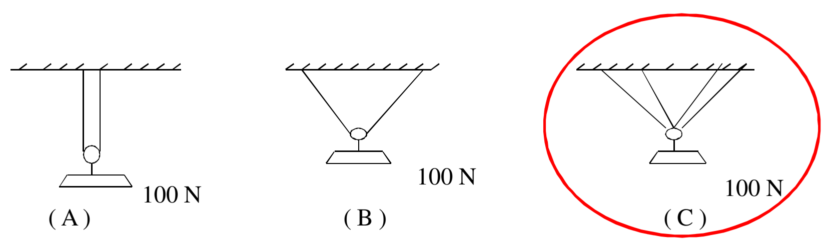
c) $T_1 < T_2$

d) $T_1 = T_2 \sin\theta$



Reading Quiz

3. Assuming you know the geometry of the ropes, you cannot determine the forces in the cables in which system below?



4. Why?

a) The weight is too heavy.

b) The cables are too thin.

c) There are more unknowns than equations.

d) There are too few cables for a 100 N weight.

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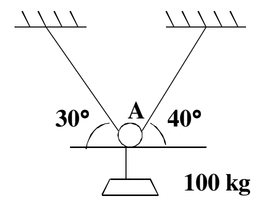
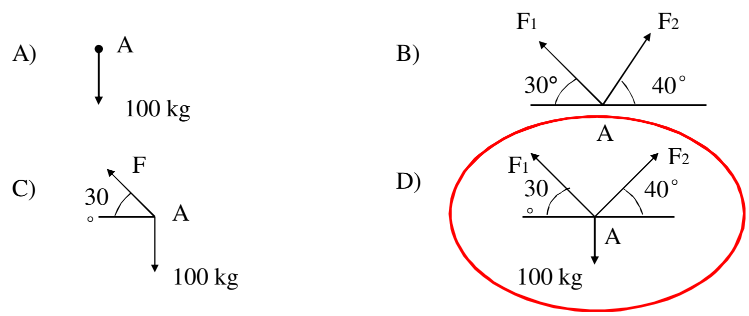
Reading Quiz

5. Select the correct FBD of particle A.



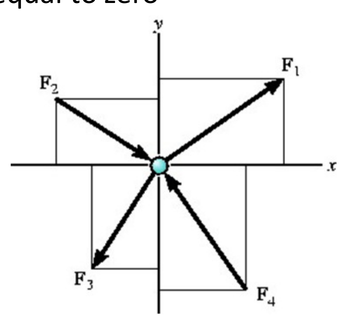


Coplanar Force System

- If a particle is subjected to coplanar forces in the $x - y$ plane
- Then, resolve into $\boldsymbol{i}$ and $\boldsymbol{j}$ components for equilibrium

$$\sum \boldsymbol{F}_x = 0$$
$$\sum \boldsymbol{F}_y = 0$$

- Scalar equations of equilibrium require that the algebraic sum of the x and y components to equal to zero



Coplanar Force System

Scalar Notation

- **Sense of direction = an algebraic sign that corresponds to the arrowhead direction of the component along each axis.**
- **For unknown magnitude, assume arrowhead sense of the force.**
- **Since magnitude of the force is always positive, if the scalar is negative, the force is acting in the opposite direction.**

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad +F + 10\text{ N} = 0$$

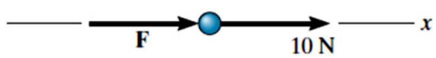


Fig. 3–5

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Procedure for Analysis

Coplanar force equilibrium problems for a particle can be solved using the following procedure.

Free-Body Diagram.

- Establish the x, y axes in any suitable orientation.
- Label all the known and unknown force magnitudes and directions on the diagram.
- The sense of a force having an unknown magnitude can be assumed.

Equations of Equilibrium.

- Apply the equations of equilibrium, $\Sigma F_x = 0$ and $\Sigma F_y = 0$.
- Components are positive if they are directed along a positive axis, and negative if they are directed along a negative axis.
- If more than two unknowns exist and the problem involves a spring, apply $F = ks$ to relate the spring force to the deformation s of the spring.
- Since the magnitude of a force is always a positive quantity, then if the solution for a force yields a negative result, this indicates that its sense is the reverse of that shown on the free-body diagram.

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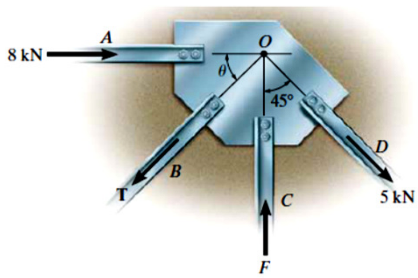
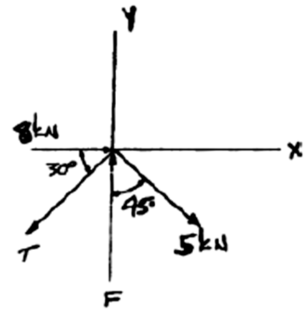
Example 1

- The members of a truss are connected to the gusset plate. If the forces are concurrent at point O, determine the magnitudes of F and T for equilibrium. Take $\theta = 30°$.





$$\rightarrow + \sum F_x = 0\,; \quad -T\cos 30^{\circ} + 8 + 5\sin 45^{\circ} = 0$$

$$T = 13.3\ KN$$

$$\uparrow + \sum F_y = 0\,; \quad F - 13.3\sin 30^{\circ} - 5\cos 45^{\circ} = 0$$

$$F = 10.2\ KN$$

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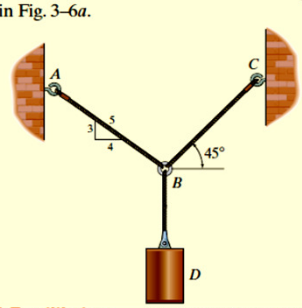
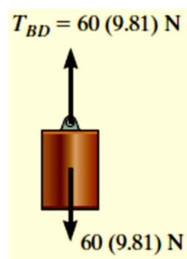
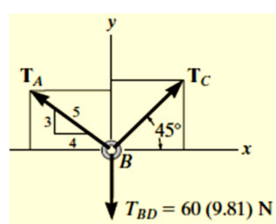
Example 2

Determine the tension in cables BA and BC necessary to support the 60-kg cylinder in Fig. 3–6a.







Equations of Equilibrium. Applying the equations of equilibrium along the x and y axes, we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad T_C \cos 45° - \left(\tfrac{4}{5}\right)T_A = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad T_C \sin 45° + \left(\tfrac{3}{5}\right)T_A - 60(9.81)\text{ N} = 0 \qquad (2)$$

Equation (1) can be written as $T_A = 0.8839T_C$. Substituting this into Eq. (2) yields

$$T_C \sin 45° + \left(\tfrac{3}{5}\right)(0.8839T_C) - 60(9.81)\text{ N} = 0$$

so that

$$T_C = 475.66\text{ N} = 476\text{ N} \qquad \textit{Ans.}$$

Substituting this result into either Eq. (1) or Eq. (2), we get

$$T_A = 420\text{ N} \qquad \textit{Ans.}$$

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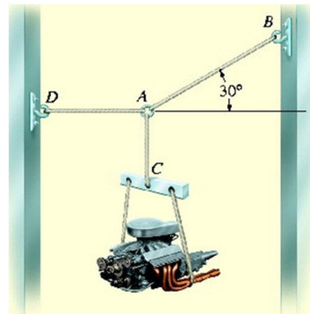
Example 3

Determine the tension in cables AB and AD for equilibrium of the 250 kg engine.



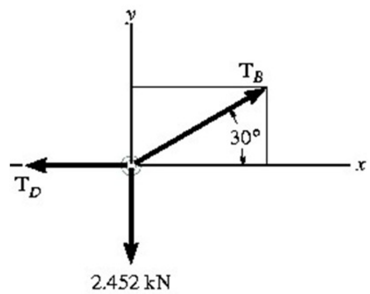
Solution

+→ $\sum F_x = 0;\ T_B cos30° - T_D = 0$

+↑ $\sum F_y = 0;\ T_B sin30° - 2.452kN = 0$

Solving,

$$T_B = 4.90kN$$
$$T_D = 4.25kN$$



Note: Neglect the weights of the cables since they are small compared to the weight of the engine.

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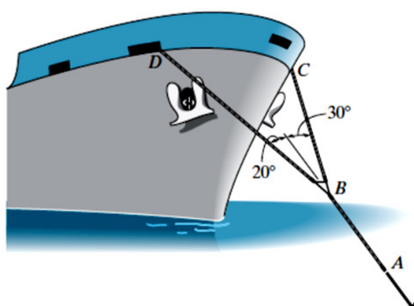
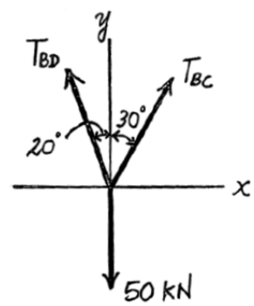
Example 4

The towing pendant *AB* is subjected to the force of 50 kN exerted by a tugboat. Determine the force in each of the bridles, *BC* and *BD*, if the ship is moving forward with constant velocity.





$$\rightarrow + \sum F_x = 0\ ;\quad T_{BC} sin30^{\circ} - T_{BD} sin20^{\circ} = 0$$

$$\uparrow + \sum F_y = 0\ ;\quad T_{BC} cos30^{\circ} + T_{BD} cos20^{\circ} - 50 = 0$$

$$T_{BC} = 22.3\ KN$$

$$T_{BD} = 32.6\ KN$$

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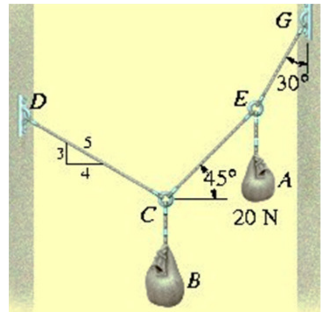
Example 5

If the sack at *A* has a weight of $20N$ (≈ 2kg), determine the weight of the sack at *B* and the force in each cord needed to hold the system in the equilibrium position shown.

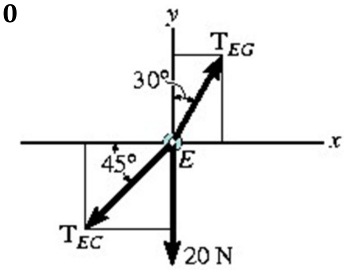


Solution

FBD at Point E

$+\rightarrow \sum F_x = 0;\ T_{EG} sin30° - T_{EC} cos45° = 0$

$+\uparrow \sum F_y = 0;\ T_{EG} cos30° - T_{EC} sin45° - 20N = 0$



Solving,

$$T_{EC} = 38.6\ kN$$
$$T_{EG} = 54.6\ kN$$

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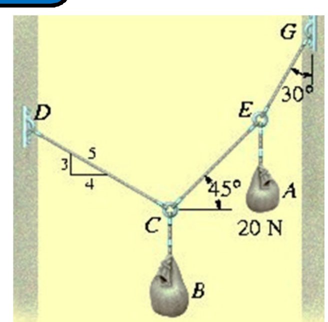
Example 5 Cont..

Solution

FBD at Point C

- Three forces acting, forces by cable CD and EC (known) and weight of sack B on cable CB

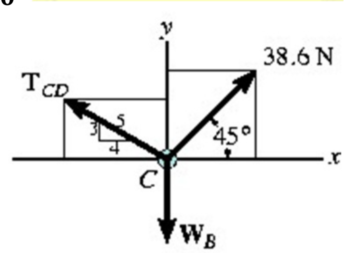


$+\rightarrow \sum F_x = 0;\quad 38.6\ cos\,45° - \left(\frac{4}{5}\right) T_{CD} = 0$

$+\uparrow \sum F_y = 0;\quad \left(\frac{3}{5}\right) T_{CD} + 38.6 \sin 45° N - W_B = 0$

Solving,

$$T_{CD} = 34.1\ kN$$
$$W_B = 47.8\ kN$$



(c)

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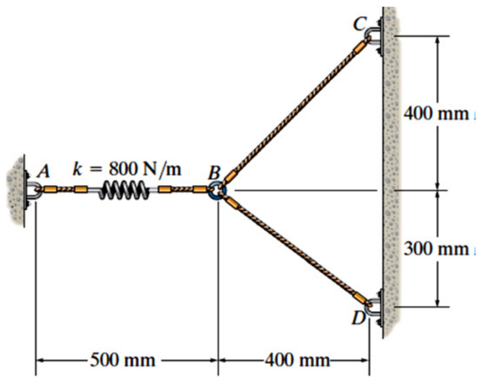
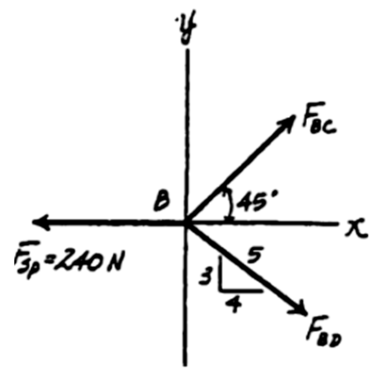
Example 6

The spring has a stiffness of $k = 800\ N/m$ and an unstretched length of $200\ mm$. Determine the force in cables *BC* and *BD* when the spring is held in the position shown.





The force in the spring:

The spring stretches $S = l - l_{\circ} = 0.5 - 0.2 = 0.3m$

$F_{sp} = ks = 800(0.3) = 240\ N$

Example 6 Cont..

The spring has a stiffness of $k = 800\ N/m$ and an unstretched length of $200\ mm$. Determine the force in cables *BC* and *BD* when the spring is held in the position shown.

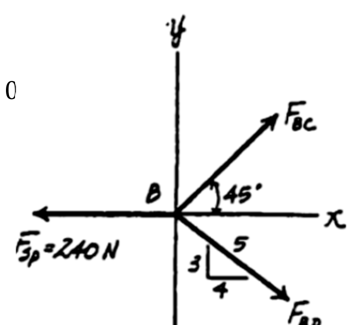
Equations of Equilibrium:

$\rightarrow + \sum F_x = 0\,;\quad F_{BC} cos45° + F_{BD}\left(\frac{4}{5}\right) - 240 = 0$

$0.7071F_{BC} + 0.8F_{BD} = 240$

$\uparrow + \sum F_y = 0\,;\quad F_{BC} sin45^{\circ} - F_{BD}\left(\frac{3}{5}\right) = 0$

$F_{BC} = 0.8485F_{BD}$



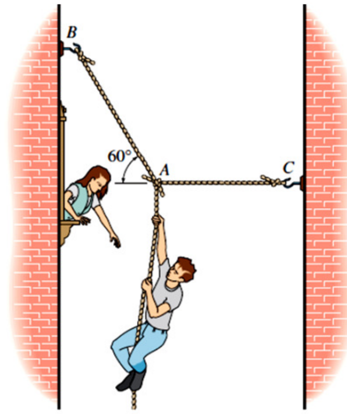
Solving Equations.

$F_{BD} = 171\ N$

$F_{BC} = 145\ N$

Example 7

Romeo tries to reach Juliet by climbing with constant velocity up a rope which is knotted at point A. Any of the three segments of the rope can sustain a maximum force of $2\ kN$ before it breaks. Determine if Romeo, who has a mass of $65\ kg$, can climb the rope, and if so, can he along with Juliet, who has a mass of $60\ kg$, climb down with constant velocity?



Case 1: Romeo Only

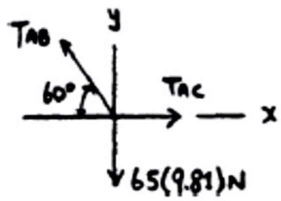
$$+\uparrow \Sigma F_y = 0; \qquad T_{AB} \sin 60° - 65(9.81) = 0$$

$$T_{AB} = 736.29 \text{ N} < 2000 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad T_{AC} - 736.29 \cos 60° = 0$$

$$T_{AC} = 368.15 \text{ N} < 2000 \text{ N}$$

Yes, Romeo can climb up the rope.



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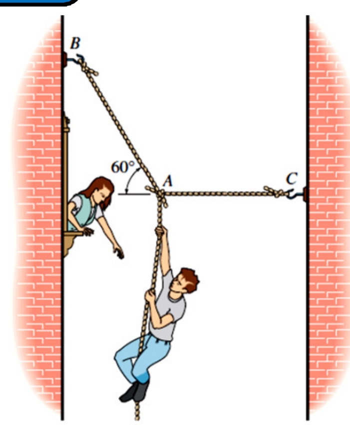
Example 7 Cont..

Romeo tries to reach Juliet by climbing with constant velocity up a rope which is knotted at point A. Any of the three segments of the rope can sustain a maximum force of $2\ kN$ before it breaks. Determine if Romeo, who has a mass of $65\ kg$, can climb the rope, and if so, can he along with Juliet, who has a mass of $60\ kg$, climb down with constant velocity?



Case 2: Romeo along with Juliet

$$+\uparrow \Sigma F_y = 0; \qquad T_{AB} \sin 60° - 125(9.81) = 0$$

$$T_{AB} = 1415.95 \text{ N} < 2000 \text{ N}$$

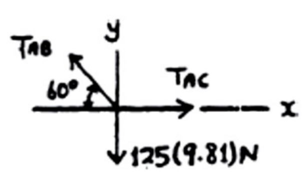
$$\xrightarrow{+} \Sigma F_x = 0; \qquad T_{AC} - 1415.95 \cos 60° = 0$$

$$T_{AC} = 708 \text{ N} < 2000 \text{ N}$$

Also, for the vertical segment,

$$T = 125(9.81) = 1226 \text{ N} < 2000 \text{ N}$$

Yes, Romeo and Juliet can climb down.



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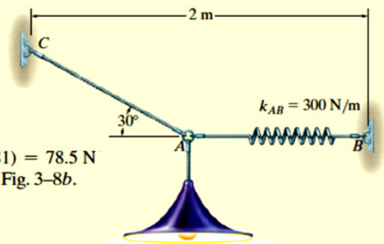
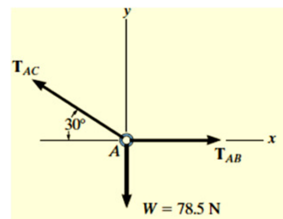
Example 8

Determine the required length of cord AC in Fig. 3–8*a* so that the 8-kg lamp can be suspended in the position shown. The *undeformed* length of spring AB is $l'_{AB} = 0.4$ m, and the spring has a stiffness of $k_{AB} = 300$ N/m.



Free-Body Diagram. The lamp has a weight $W = 8(9.81) = 78.5$ N and so the free-body diagram of the ring at A is shown in Fig. 3–8*b*.



Equations of Equilibrium. Using the *x*, *y* axes,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad T_{AB} - T_{AC}\cos 30^\circ = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad T_{AC}\sin 30^\circ - 78.5\text{ N} = 0$$

Solving, we obtain

$$T_{AC} = 157.0\text{ N}$$
$$T_{AB} = 135.9\text{ N}$$

The stretch of spring AB is therefore

$$T_{AB} = k_{AB}s_{AB}; \qquad 135.9\text{ N} = 300\text{ N/m}(s_{AB})$$
$$s_{AB} = 0.453\text{ m}$$

so the stretched length is

$$l_{AB} = l'_{AB} + s_{AB}$$
$$l_{AB} = 0.4\text{ m} + 0.453\text{ m} = 0.853\text{ m}$$

The horizontal distance from C to B, Fig. 3–8*a*, requires

$$2\text{ m} = l_{AC}\cos 30^\circ + 0.853\text{ m}$$
$$l_{AC} = 1.32\text{ m} \qquad \textit{Ans.}$$

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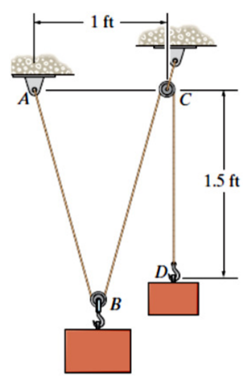
Example 9

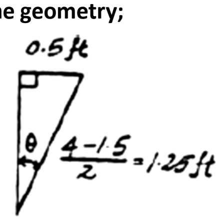
A "scale" is constructed with a 4-ft-long cord and the 10-lb block *D*. The cord is fixed to a pin at *A* and passes over two *small* pulleys at *B* and *C*. Determine the weight of the suspended block at *B* if the system is in equilibrium.



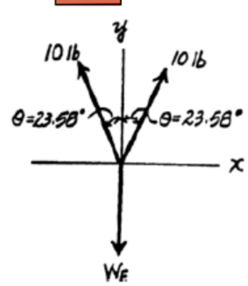
Solution:

Free Body diagram: The tension force in the cord is the same throughout the cord, that is 10 lb. from the geometry;

$$\theta = \sin^{-1}\left(\frac{0.5}{1.25}\right) = 23.58^\circ$$



Equation of Equilibrium:

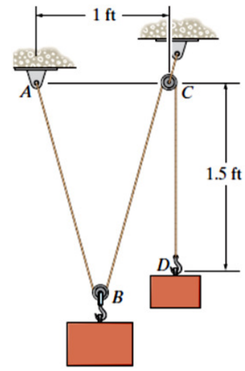


$$\uparrow +\sum F_y = 0; \qquad 2(10)cos23.58^\circ - W = 0$$

$$W = 18.3\ lb$$

Try it Yourself ☺

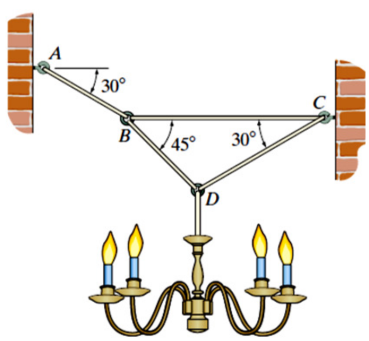
A "scale" is constructed with a 4-ft-long cord and the 18.3-lb block B. The cord is fixed to a pin at *A* and passes over two *small* pulleys at *B* and *C*. Determine the weight of the suspended block at *D* if the system is in equilibrium.

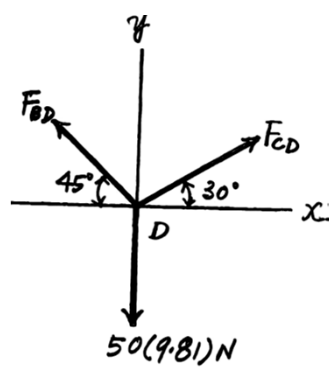


Solution:

Example 10

Determine the tension developed in each wire used to support the 50-kg chandelier.





$$\rightarrow + \sum F_x = 0 ; \quad F_{CD}\cos 30^\circ - F_{BD}\cos 45^\circ = 0$$

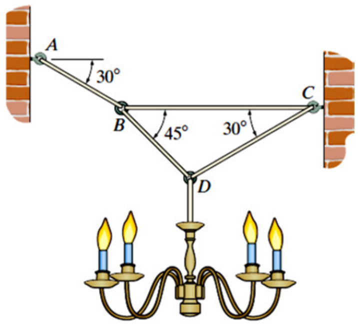
$$\uparrow + \sum F_y = 0 ; \quad F_{CD}\sin 30^\circ + F_{BD}\sin 45^\circ - 50(9.81) = 0$$

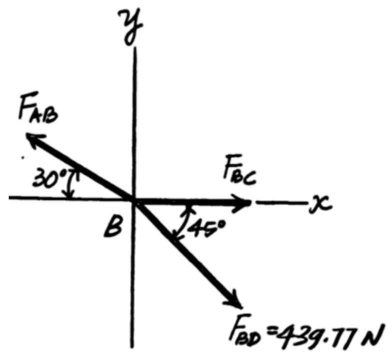
$$F_{CD} = 359\ N$$

$$F_{BD} = 439.77\ N$$

Example 10 Cont..

Determine the tension developed in each wire used to support the 50-kg chandelier.





$$\uparrow + \sum F_y = 0; \quad F_{AB} sin30^\circ - 439.77 sin45^\circ = 0$$

$$F_{AB} = 621.93\ N = 622\ N$$

$$\rightarrow + \sum F_x = 0; \quad F_{BC} + 439.77 cos45^\circ - 621.93 cos30^\circ = 0$$

$$F_{BC} = 228\ N$$

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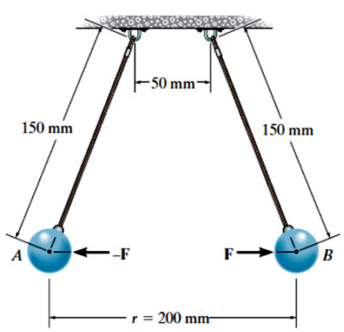
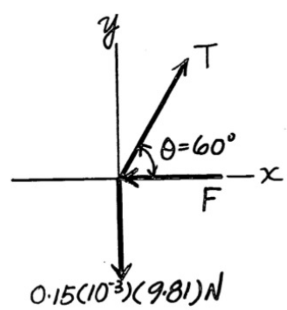
Example 11

Two electrically charged pith balls, each having a mass of $0.15g$, are suspended from light threads of equal length. Determine the magnitude of the horizontal repulsive force, F, acting on each ball if the measured distance between them is $r = 200\ mm$.

- **Solution:**





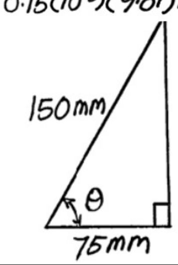
$$\cos\theta = \frac{75}{150}; \ \theta = 60^\circ$$

$$\uparrow + \sum F_y = 0; \quad T\ sin\ 60^\circ - 0.15(10)^{-3}(9.81) = 0$$

$$T = 1.699(10)^{-3}\ N$$

$$\rightarrow + \sum F_x = 0; \quad 1.699(10)^{-3} cos60 - F = 0$$

$$F = 0.85(10)^{-3}\ N$$
$$= 0.850\ mN$$



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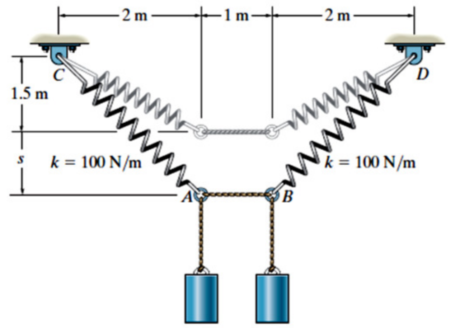
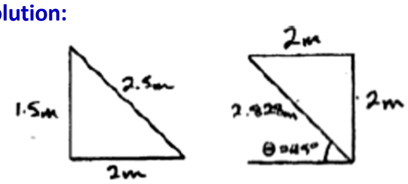
Example 12

Determine the mass of each of the two cylinders if they cause a sag of $S = 0.5m$ when suspended from the rings at *A* and *B*. Note that $S = 0$ when the cylinders are removed.



Solution:

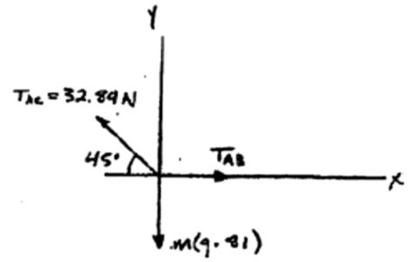


$F = KS$

$T_{AC} = 100\frac{N}{m}(2.828 - 2.5) = 32.84\ N$

$+\uparrow \sum F_y = 0;\quad 32.84 \sin 45^{\circ} - m(9.81) = 0$

$m = 2.37\ kg$



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