University of Anbar

Engineering Mechanics: Statics
CHE 211

Lecture # 03

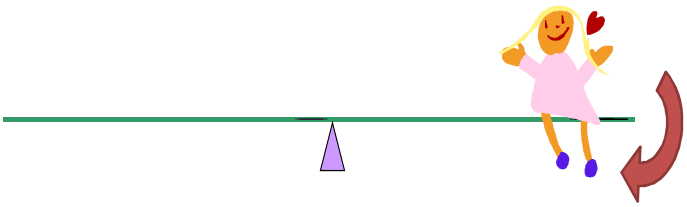
Objectives of Lecture Note

- Understand and define moment.
- Determine moments of a force in 2-D case.
- To provide a method of finding the moment of a force about specified axis.
- To determine the moment of a couple.

2

What is Moment?!

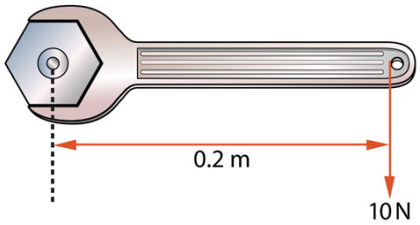
- A force can cause many things to move or stop. When a force causes an object to turn, this turning effect is called torque, but most often it is called the moment of a force or simply the moment.
- Example: A person sitting on a seesaw.



3

Everyday Examples

1. Turning a door knob
2. Opening a door
3. Scissors
4. Turning a steering wheel
5. Crane at the construction sites lifting objects
6. Electrical fan
7. Spanner and Nut

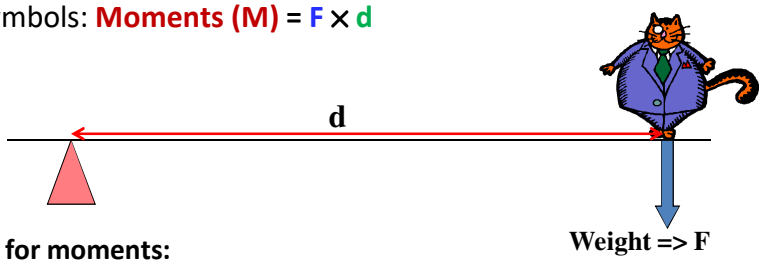


Moment of a Force - Scalar Formulation

The tendency of a force to rotate a rigid body about any defined axis is called the *Moment* of the force about the axis.

Moment = **Force**×**PERPENDICULAR distance** between force and pivot

In symbols: **Moments (M)** = **F** × **d**



Unit for moments:

(N·m) in the SI system

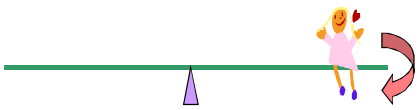
(lb. ft) or (lb. in) in the US Customary system

5

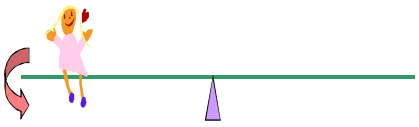
Types of Moments

There are 2 types of moments:

•Clockwise moment

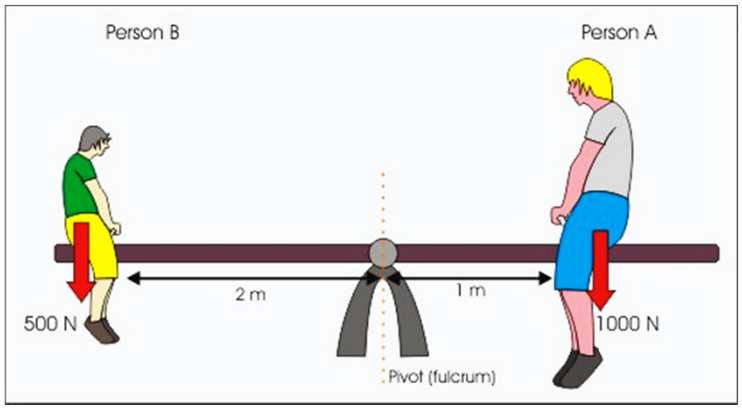


•Counterclockwise
(or Anticlockwise) moment



Moment caused by a Force

Is it possible to balance objects of different weights?

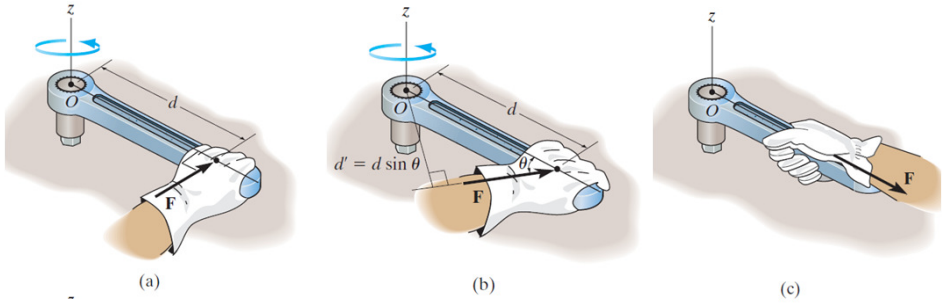


7

Moment of a Force - Scalar Formulation

Torque or Moment is the tendency for an object to rotate about a point when a force is applied (not in the line of action)

$$M_a > M_b > M_c = 0$$

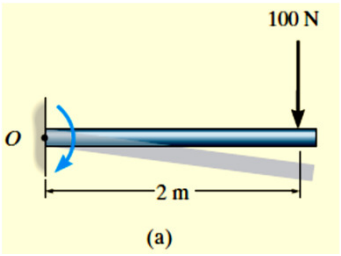


Example 1

For each case illustrated in Fig. 4-4. determine the moment of the force about point *O*.

$M_o = Fd$

Fig. 4.4 a $M_o = (100\text{N})(2\text{m}) = 200\ \text{N.m}$



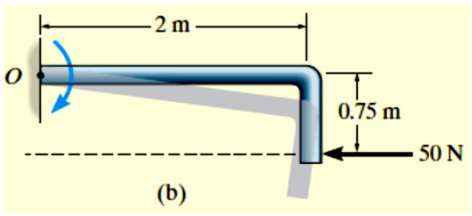


Fig. 4.4 b $M_o = (50\text{N})(0.75\text{m}) = 37.5\ \text{N.m}$

9

Example 1 Cont..

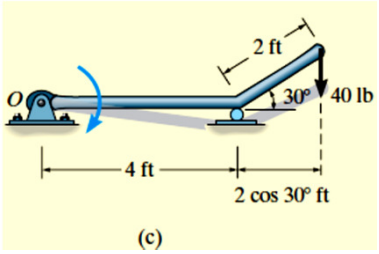


Fig. 4.4 c $M_o = (40\text{lb})(4\text{ft} + 2\cos 30^\circ\text{ft}) = 229\text{lb.ft}$

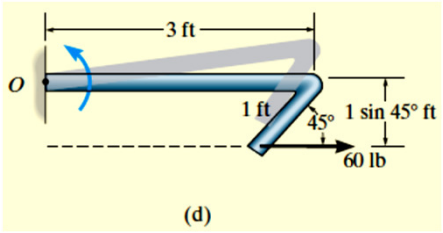


Fig. 4.4 d $M_o = (60\text{lb})(1\sin 45^\circ\text{ ft}) = 42.4\text{lb.ft}$

10

Example 1 Cont..

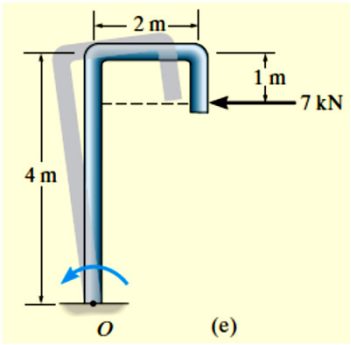
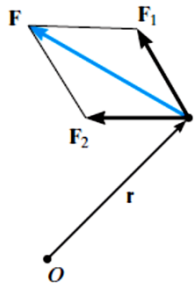


Fig. 4.4 e $M_o = (7\text{KN})(4\text{m} - 1\text{m}) = 21.0\ \text{KN.m}$

11

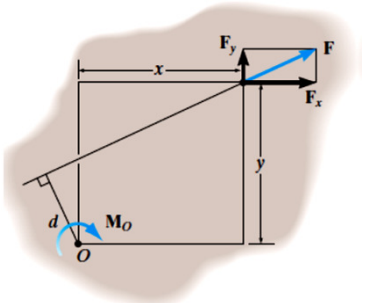
Principle of Moments



- The moment of a force about a point is equal to the sum of moments of the components of the force about that point

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2) = \mathbf{r} \times \mathbf{F}_1 + \mathbf{r} \times \mathbf{F}_2$$

- For two dimensional space, the moment is :

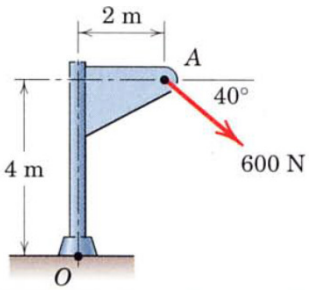


$$M_O = F_x y - F_y x$$

12

Example 2

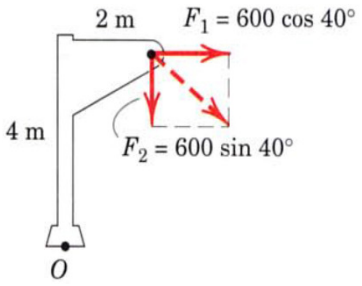
Replace the force by its rectangular components at A.



$F_1 = 600cos40^\circ = 460\ N$

$F_2 = 600sin40^\circ = 386\ N$

$M_o = 460(4) + 386(2) = 2610\ N.m$

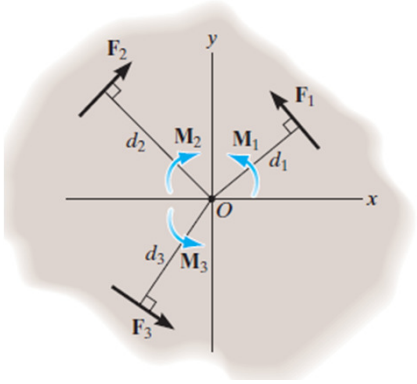


Resultant of Moment

- **Resultant of Moment**

$$\circlearrowleft + (M_R)_o = \Sigma Fd;$$

- **Clockwise is Negative**
- **Counterclockwise is Positive**



$$\circlearrowleft + (M_R)_o = \Sigma Fd; \qquad (M_R)_o = F_1d_1 - F_2d_2 + F_3d_3$$

14

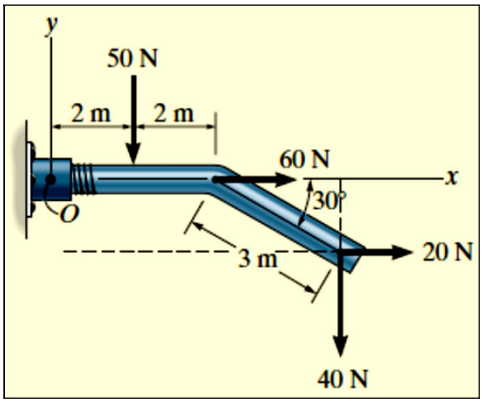
Example 3

Determine the resultant moment of the four forces acting on the rod shown in the Figure about point O.

$$\zeta + (M_R)_o = \Sigma Fd;$$

$$(M_R)_O = -50\text{N}(2\text{m}) + 60\text{N}(0) + 20\text{N}(3\sin 30^\circ\, m) - 40N(4m + 3cos30^\circ\, m)$$

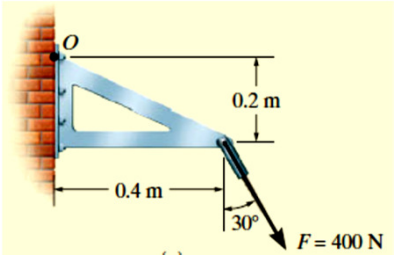
$$(\mathbf{M_R})_\mathbf{O} = -334\ \mathbf{N.m}$$

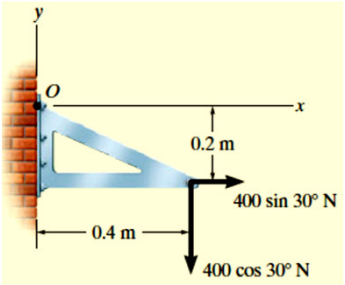


15

Example 4

Force F acts at the end of the angle bracket in the Figure. Determine the moment of the force about point O .





$$\zeta + M_O = 400 \sin 30^\circ\ \text{N}(0.2\ \text{m}) - 400 \cos 30^\circ\ \text{N}(0.4\ \text{m})$$
$$= -98.6\ \text{N} \cdot \text{m} = 98.6\ \text{N} \cdot \text{m}\ \circlearrowright$$

16

Example 5

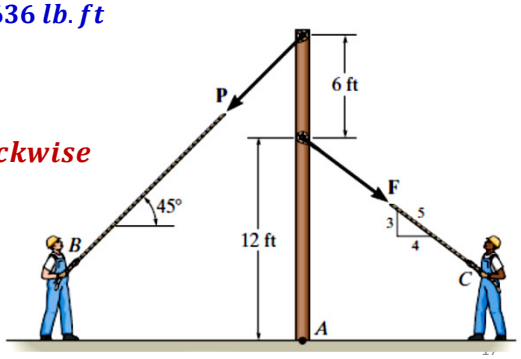
Two men exert forces of $F = 80lb$ and $P = 50lb$ on the ropes. Determine the moment of each force about *A*. Which way will the pole rotate, clockwise or counterclockwise?

$$(\mathbf{M_A})_C = 80\left(\frac{4}{5}\right)(12ft) = 768\ lb.ft$$

$$(\mathbf{M_A})_B = 50(cos45°)(18) = 636\ lb.ft$$

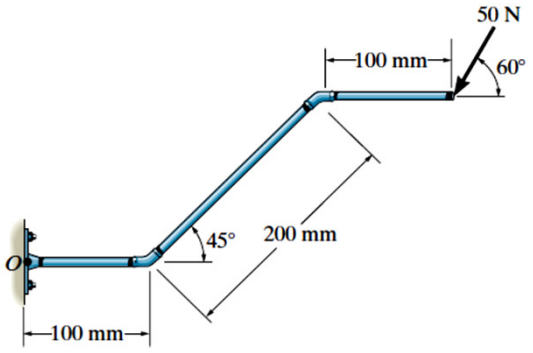
$$Since\ (M_A)_C > (M_A)_B$$

Then it is rotating Clockwise



Attention Quiz

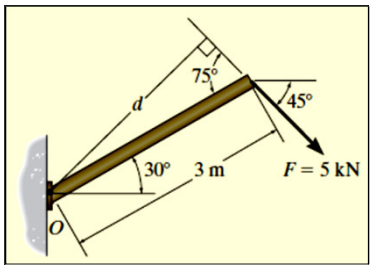
Determine the moment of the force about point *O*. Neglect the thickness of the member.

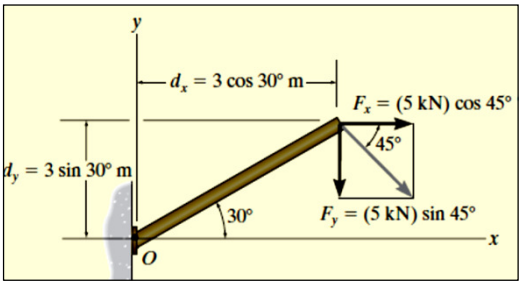


18

Example 6

- **Determine the moment of the force in the Figure. about point *O* .**





Considering conterclockwise moments as positive, and applying the principle of moments, we have

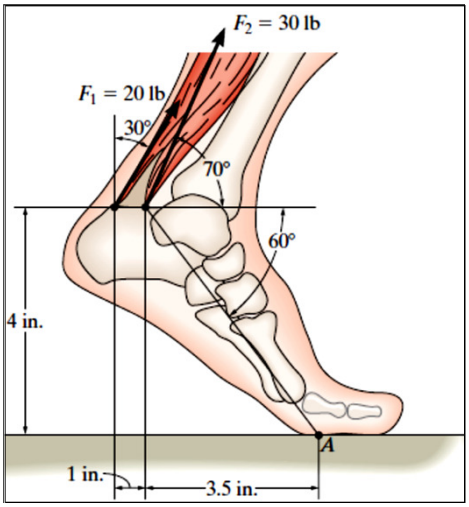
$$\zeta + M_O = -F_x d_y - F_y d_x$$
$$= -(5\cos 45°\ \text{kN})(3\sin 30°\ \text{m}) - (5\sin 45°\ \text{kN})(3\cos 30°\ \text{m})$$
$$= -14.5\ \text{kN}\cdot\text{m} = 14.5\ \text{kN}\cdot\text{m}\ \zeta \qquad \textit{Ans.}$$

19

Example 7

- **The foot segment is subjected to the pull of the two plantar flexor muscles. Determine the moment of each force about the point of contact *A* on the ground.**

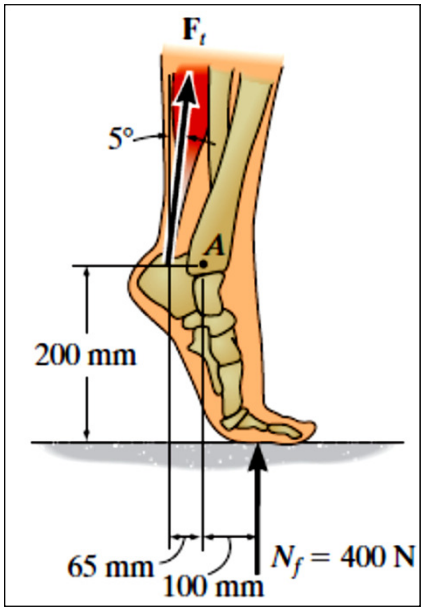
$$(M_A)_1 = 20cos30(4.5) + 20sin30(4) = 118\ lb.in$$
$$(M_A)_2 = 30cos70(4) + 30sin70(3.5) = 140\ lb.in$$



20

Example 8

The Achilles tendon force F_t is mobilized when one tries to stand on his/her toes. As this is done, each of his/her feet is subjected to a reactive force of N_f = 400 N. *If the resultant moment produced by forces F_t and N_f about the ankle joint A is required to be ZERO*, determine the magnitude of F_t .



$$(M_R)_A = \sum Fd$$

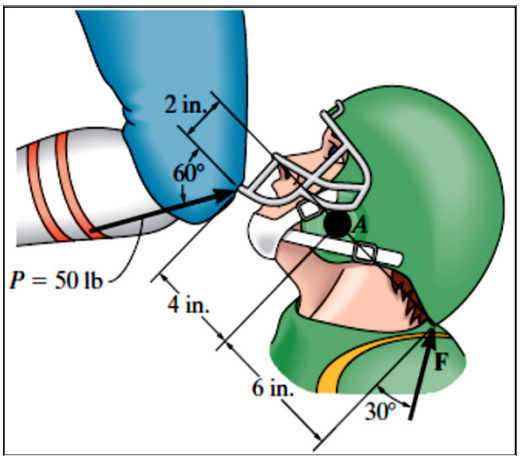
$$0 = 400\,(0.1) - F_t \cos 5^\circ (0.065)$$

$$F_t = 618 \text{ N}$$

21

Example 9

Serious neck injuries can occur when a football player is struck in the face guard of his helmet in the manner shown, giving rise to a guillotine mechanism. (a) Determine the moment of the knee force $P = 50\ lb$ about point *A*. (b) What would be the magnitude of the neck force F so that it gives the counterbalancing moment about *A*?



(a)

$$M_A = 50\sin 60^\circ (4) - 50\cos 60^\circ (2)$$
$$= 123.2 \text{ lb.in}$$

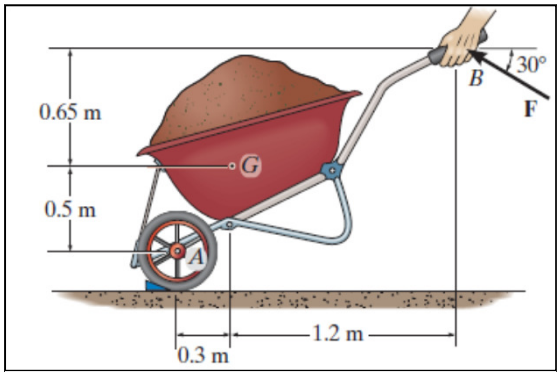
(b)

$$123.2 = F\cos 30^\circ (6)$$

$$F = 23.7 \text{ lb}$$

Example 10

- In order to hold the wheelbarrow in the position shown, force *F* must produce a counterclockwise moment of *200 N.m* about the axle at A. Determine the required magnitude of force *F*.



$$M_A = 200 = F\sin 30^\circ (1.5) + F\cos 30^\circ (1.15)$$

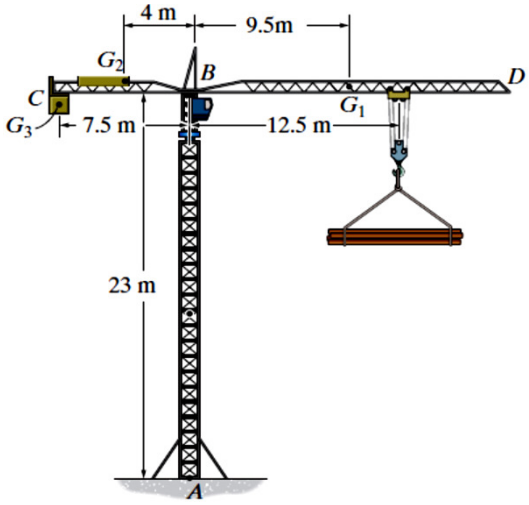
$$F = 115\ \text{N}$$

23

Example 11

The tower crane is used to hoist the $2\,Mg$ load upward at constant velocity. The $1.5\,Mg$ jib BD, $0.5\,Mg$ jib BC, and $6\,Mg$ counterweight C have centers of mass at G_1, G_2, and G_3, respectively. Determine the resultant moment produced by the load and the weights of the tower crane jibs about point A and about point B.

Answer: Since the moment arms of the weights and the load measured to points *A* and *B* are the same, the resultant moments produced by the load and the weight about points *A* and *B* are the same.

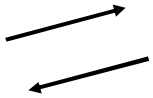


$$\zeta + (M_R)_A = (M_R)_B = \Sigma Fd; \qquad (M_R)_A = (M_R)_B = 6000(9.81)(7.5) + 500(9.81)(4) - 1500(9.81)(9.5) - 2000(9.81)(12.5) = 76\,027.5\ \text{N}\cdot\text{m} = 76.0\ \text{kN}\cdot\text{m}\ (\textit{Counterclockwise})$$

24

Couples

- **Definition**
  - **Two equal, non-collinear, parallel forces of opposite senses are called a couple.**



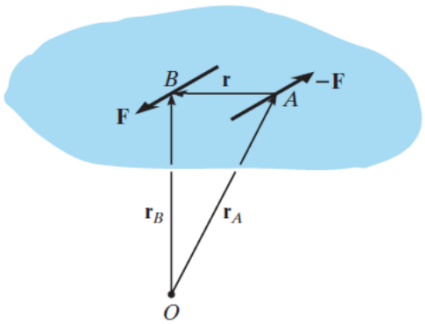
- **Effects**
  - **Rotation only, no translation**

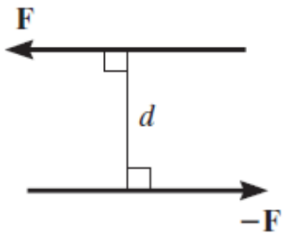
25

Moment of a Couple

- **Moment of a Couple: two parallel forces with the same magnitude acting in opposite directions, separated by a perpendicular distance**

$$M = Fd$$





26

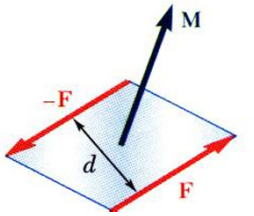
Moment of a Couple

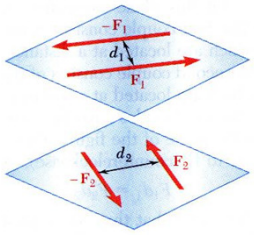
Two couples will have equal moments if

$$F_1 d_1 = F_2 d_2$$

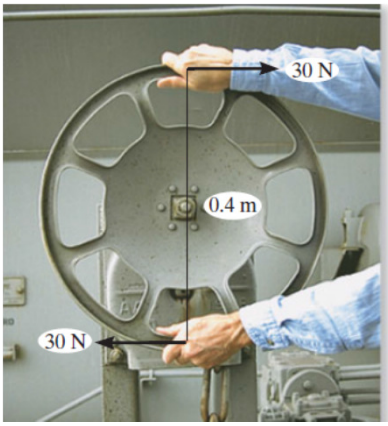
- The moment of a couple is a free vector, because it does not depend of the reference point.

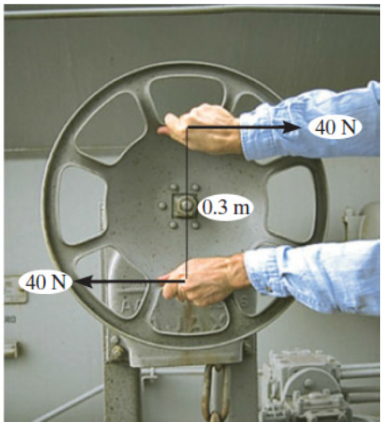
- The two couples have the same sense or the tendency to cause rotation in the same direction.





Equivalent Couples





A torque or moment of 12 N·m is required to rotate the wheel. Which one of the two grips of the wheel above will require less force to rotate the wheel?

28

Applications of Moment of a Couple

Steering wheels on vehicles have been made smaller than older vehicles because power steering does not require the driver to apply a large couple moment to the rim of the wheel.

29

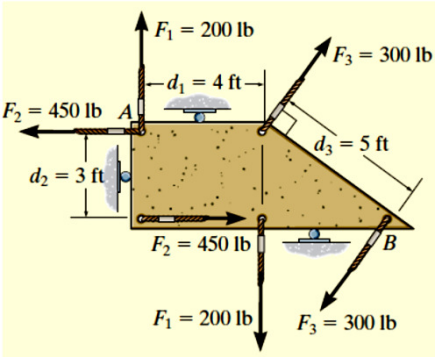
Important Points

- A couple moment is produced by two noncollinear forces that are equal in magnitude but opposite in direction. Its effect is to produce pure rotation, or tendency for rotation in a specified direction.
- A couple moment is a free vector, and as a result it causes the same rotational effect on a body regardless of where the couple moment is applied to the body.
- The moment of the two couple forces can be determined about *any point*. For convenience, this point is often chosen on the line of action of one of the forces in order to eliminate the moment of this force about the point.
- In three dimensions the couple moment is often determined using the vector formulation, $\mathbf{M} = \mathbf{r} \times \mathbf{F}$, where $\mathbf{r}$ is directed from *any point* on the line of action of one of the forces to *any point* on the line of action of the other force $\mathbf{F}$.
- A resultant couple moment is simply the vector sum of all the couple moments of the system.

30

Example 1

Determine the resultant couple moment of the three couples acting on the plate in the Fig.



SOLUTION

As shown the perpendicular distances between each pair of couple forces are d_1 = 4 ft, d_2 = 3 ft, and d_3 = 5 ft. Considering counterclockwise couple moments as positive, we have

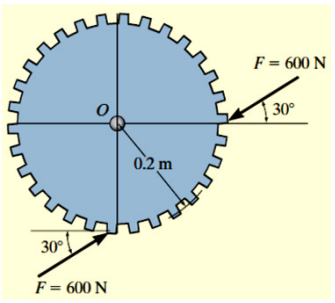
$$CC + M_R = \sum M; \quad M_R = -F_1 d_1 + F_2 d_2 - F_3 d_3$$
$$= -(200\ \text{lb})(4\ \text{ft}) + (450\ \text{lb})(3\ \text{ft}) - (300\ \text{lb})(5\ \text{ft})$$
$$= -950\ \text{lb.ft}$$

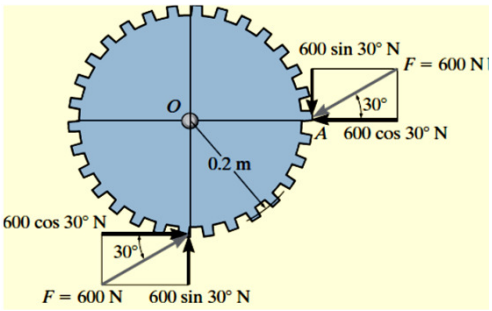
The negative sign indicates that M_R has a clockwise rotational sense

31

Example 2

Determine the magnitude and direction of the couple moment acting on the gear in the Figure.





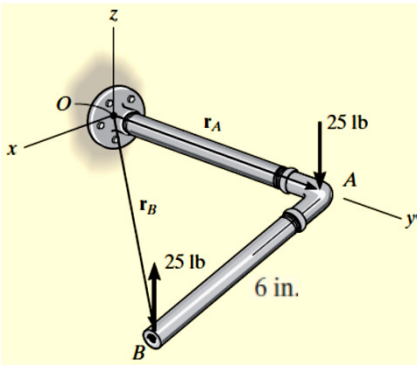
If we consider counterclockwise moments as positive, we have

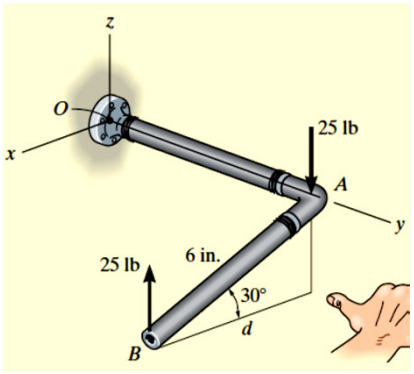
$$\zeta + M = \Sigma M_O;\ M = (600 \cos 30°\ \text{N})(0.2\ \text{m}) - (600 \sin 30°\ \text{N})(0.2\ \text{m})$$
$$= 43.9\ \text{N} \cdot \text{m}\ \circlearrowleft$$

32

Example 3

Determine the couple moment acting on the pipe shown in the Fig. Segment *AB* is directed 30° below the *x*–*y* plane.





$$\mathbf{M = Fd;} \quad \mathbf{M = 25\,(6\cos 30^{\circ}) = 129.9\ lb.in}$$

Example 4

The floor causes a couple moment of $M_A = 40N.m$ and $M_B = 30N.m$ on the brushes of the polishing machine.

(a) Determine the magnitude of the couple forces that must be developed by the operator on the handles so that the resultant couple moment on the polisher is zero.

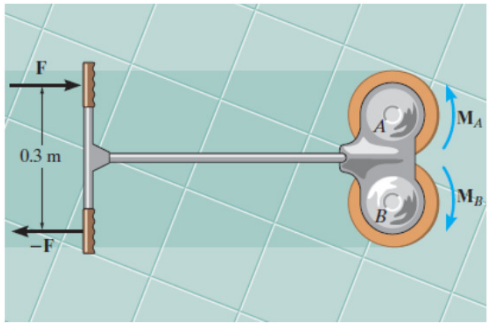
(b) What is the magnitude of these forces if the brush at *B* suddenly stops so that $M_B = 0$?

$$\zeta + M_R = 40 - 30 - F'(0.3) = 0$$

$$F' = 33.3\ \text{N} \quad \text{Ans}$$

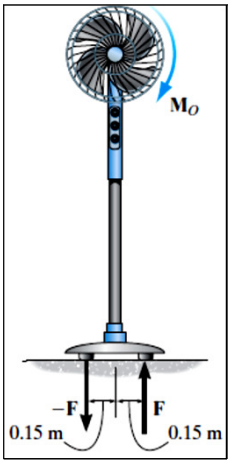
$$\zeta + M_R = 40 - F(0.3) = 0$$

$$F = 133\ \text{N} \quad \text{Ans}$$



Example 5

The frictional effects of the air on the blades of the standing fan creates a couple moment of M_0= 6 N.m on the blades. Determine the magnitude of the couple forces at the base of the fan *so that the resultant couple moment on the fan is zero.*



Solution:

Couple Moment: The couple moment of F produces a counterclockwise moment of M_C=F (0.15+0.15) = 0.3F. Since the resultant couple moment about the axis perpendicular to the page is required to be zero,

$$+(M_C)_R = \Sigma M; \qquad 0 = 0.3F - 6 \qquad F = 20\,\text{N}$$

35

Example 6

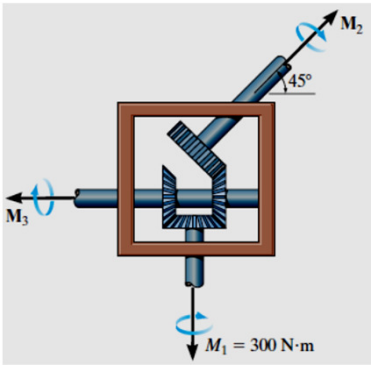
Determine the required magnitude of the couple moments M_2 and M_3 *so that the resultant couple moment is zero*.

Solution:

Since the couple moment is the free vector, it can act at any point without altering its effect. Thus, the couple moments M_1, M_2 and M_3 can be simplified. Since the resultant of M_1, M_2 and M_3 is required to be zero

$$(M_R)_y = \Sigma M_y; \qquad 0 = M_2 \sin 45° - 300$$
$$M_2 = 424.26\ \text{N}\cdot\text{m} = 424\ \text{N}\cdot\text{m}$$

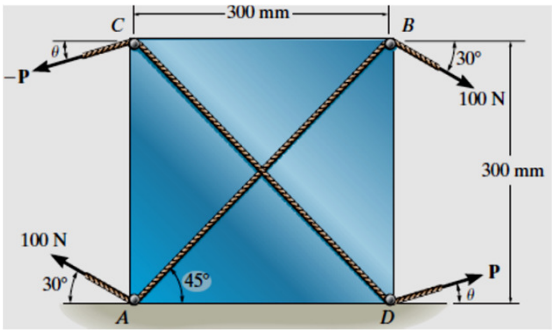
$$(M_R)_x = \Sigma M_x; \qquad 0 = 424.26 \cos 45° - M_3$$
$$M_3 = 300\ \text{N}\cdot\text{m}$$



36

Example 7

The cord passing over the two small pegs *A* and *B* of the square board is subjected to a tension of $100\ N$. Determine the required tension *P* acting on the cord that passes over pegs *C* and *D* so that the resultant couple produced by the two couples is $15\ N.m$ acting clockwise. Take $\theta = 15°$.



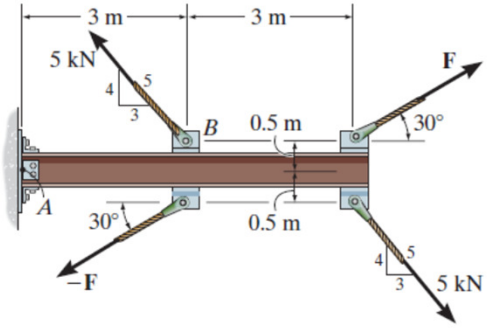
$$(+M_R) = 100\cos 30^\circ\,(0.3) + 100\sin 30^\circ\,(0.3) - P\sin 15^\circ(0.3) - P\cos 15^\circ(0.3) = 15$$

$$P = 70.7\ \text{N}$$

37

Example 8

Two couples act on the cantilever beam. If $F = 6\ kN$, determine the resultant couple moment.



$$(+(M_C)_1 = 6\sin 30°(3) - 6\cos 30°(0.5 + 0.5) = 3.804\ \text{kN}\cdot\text{m}$$

$$(+(M_C)_2 = 5\left(\frac{3}{5}\right)(0.5 + 0.5) - 5\left(\frac{4}{5}\right)(3) = -9\ \text{kN}\cdot\text{m}$$

Thus, the resultant couple moment can be determined from

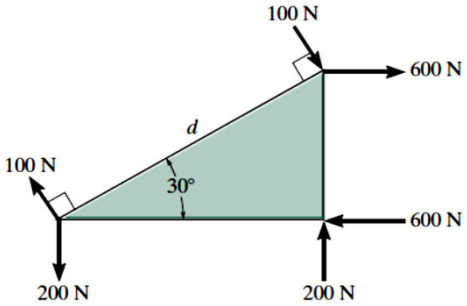
$$(M_C)_R = (M_C)_1 + (M_C)_2$$

$$= 3.804 - 9 = -5.196\ \text{kN}\cdot\text{m} = 5.20\ \text{kN}\cdot\text{m}\ \text{(clockwise)}$$

38

Example 9

The ends of the triangular plate are subjected to three couples. Determine the plate dimension d so that the resultant couple is $350\ N.m$ clockwise.



$$\zeta + M_R = \Sigma M_A; \qquad -350 = 200(d\cos 30°) - 600(d\sin 30°) - 100d$$

$$d = 1.54\ \text{m}$$

39

Reading Quiz

1. In statics, a couple is defined as _____ separated by a perpendicular distance.

A. Two forces in the same direction.

B. Two forces of equal magnitude.

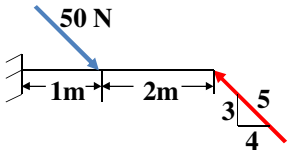
C. Two forces of equal magnitude acting in the same direction.

D. Two forces of equal magnitude acting in opposite directions.

2. A couple is applied to the beam as shown. Its moment equals _____ N·m.

A) 50 **B) 60**

C) 80 **D) 100**




THE
END
thank you all!
41