University of Anbar

Engineering Mechanics: Statics
CHE 211

Lecture # 04

Objectives of Lecture Note

- To simplify the forces and couples systems.

Simplification of a Force and Couple System

Sometimes it is convenient to reduce a system of forces and couple moments acting on a body to a simpler form by replacing it with an *equivalent system*, consisting of a single resultant force acting at a specific point and a resultant couple moment.

The point IS on the line of action of the force

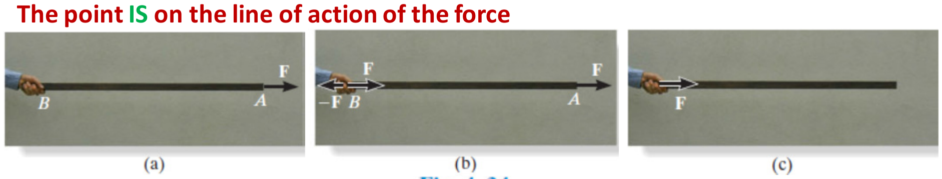

Fig. 4–34

The point IS NOT on the line of action of the force

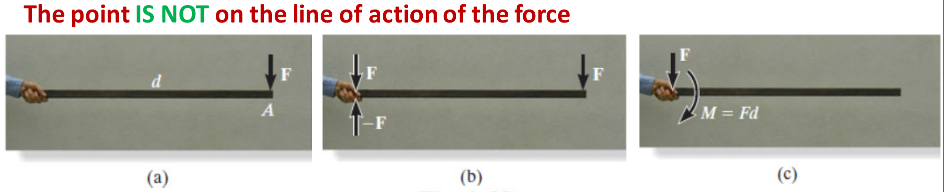

Fig. 4–35

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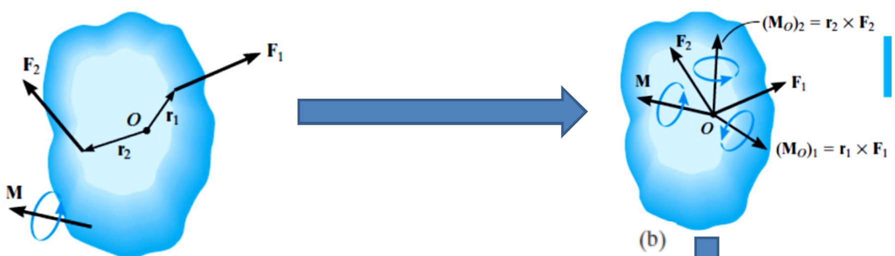
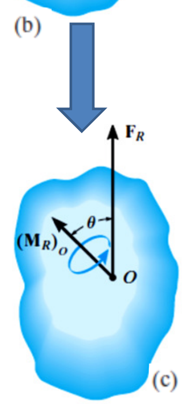
Resultants of a Force and Couple System



$$\mathbf{F}_R = \Sigma \mathbf{F}$$
$$(\mathbf{M}_R)_O = \Sigma \mathbf{M}_O + \Sigma \mathbf{M}$$

$$(F_R)_x = \Sigma F_x$$
$$(F_R)_y = \Sigma F_y$$
$$(M_R)_O = \Sigma M_O + \Sigma M$$



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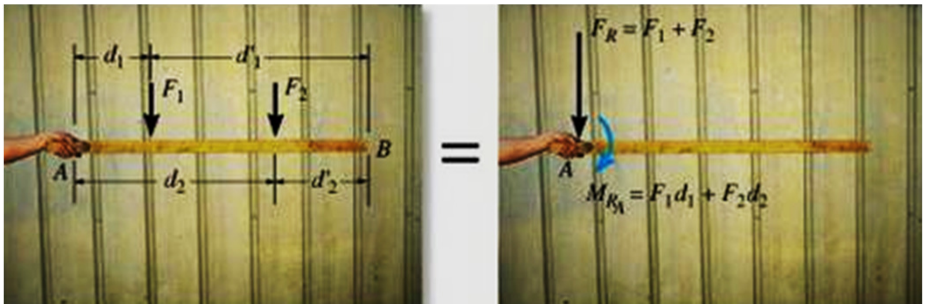
Resultants of a Force and Couple System



Resultant Force $\mathbf{F_R = F_1 + F_2}$

Couple Moment $\mathbf{(M_R)_A = F_1 d_1 + F_2 d_2}$

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Procedure for Analysis

The following points should be kept in mind when simplifying a force and couple moment system to an equivalent resultant force and couple system.

- Establish the coordinate axes with the origin located at point O and the axes having a selected orientation.

Force Summation.

- If the force system is *coplanar*, resolve each force into its x and y components. If a component is directed along the positive x or y axis, it represents a positive scalar; whereas if it is directed along the negative x or y axis, it is a negative scalar.
- In three dimensions, represent each force as a Cartesian vector before summing the forces.

Moment Summation.

- When determining the moments of a *coplanar* force system about point O, it is generally advantageous to use the principle of moments, i.e., determine the moments of the components of each force, rather than the moment of the force itself.
- In three dimensions use the vector cross product to determine the moment of each force about point O. Here the position vectors extend from O to any point on the line of action of each force.

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Example 1

Replace the forces acting on the brace by an equivalent resultant force and couple moment acting at point A.

$$+\rightarrow F_{Rx} = \Sigma F_x;$$

$$F_{Rx} = -100N - 400\cos 45^\circ N$$

$$= -382.8N = 382.8N \leftarrow$$

$$+\uparrow F_{Ry} = \Sigma F_y;$$

$$F_{Ry} = -600N - 400\sin 45^\circ N$$

$$= -882.8N = 882.8N \downarrow$$

$$F_R = \sqrt{(F_{Rx})^2 + (F_{Ry})^2} = \sqrt{(382.8)^2 + (882.8)^2} = 962N$$

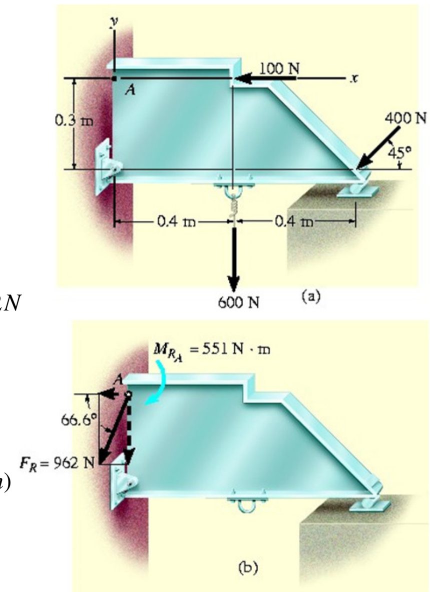
$$\theta = \tan^{-1}\left(\frac{F_{Ry}}{F_{Rx}}\right) = \tan^{-1}\left(\frac{882.8}{382.8}\right) = 66.6^\circ$$

$$CCW + M_{RA} = \Sigma M_A;$$

$$M_{RA} = 100N(0) - 600N(0.4m) - (400\sin 45^\circ N)(0.8m)$$

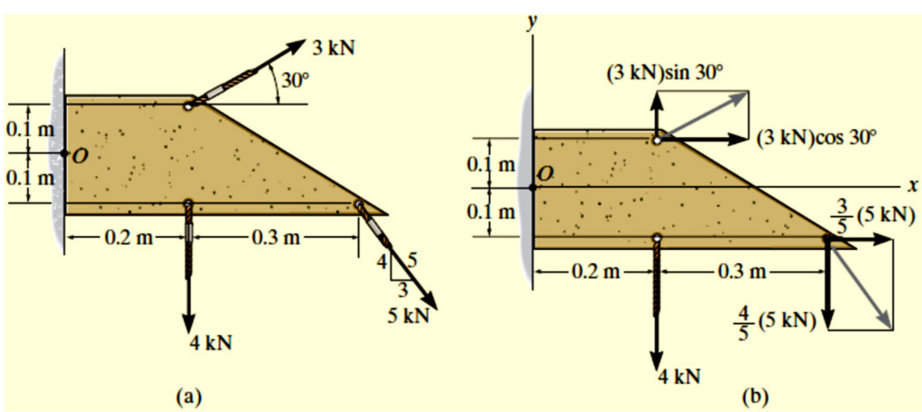
$$-(400\cos 45^\circ N)(0.3m)$$

$$= -551N.m = 551N.m(CW)$$



Example 2

Replace the force and couple system shown in the Fig. by an equivalent resultant force and couple moment acting at point *O*.



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Example 2 Cont..

Force Summation. The 3 kN and 5 kN forces are resolved into their x and y components as shown in Fig. 4–37b. We have

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = (3 \text{ kN})\cos 30^\circ + \left(\tfrac{3}{5}\right)(5 \text{ kN}) = 5.598 \text{ kN} \rightarrow$$

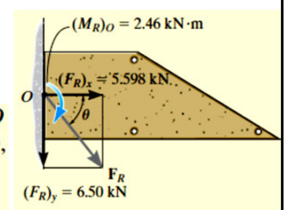
$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = (3 \text{ kN})\sin 30^\circ - \left(\tfrac{4}{5}\right)(5 \text{ kN}) - 4 \text{ kN} = -6.50 \text{ kN} = 6.50 \text{ kN}\downarrow$$

Using the Pythagorean theorem, Fig. 4–37c, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{(5.598 \text{ kN})^2 + (6.50 \text{ kN})^2} = 8.58 \text{ kN} \qquad \textit{Ans.}$$

Its direction θ is

$$\theta = \tan^{-1}\left(\frac{(F_R)_y}{(F_R)_x}\right) = \tan^{-1}\left(\frac{6.50 \text{ kN}}{5.598 \text{ kN}}\right) = 49.3^\circ$$



(c)

Moment Summation. The moments of 3 kN and 5 kN about point O will be determined using their x and y components. Referring to Fig. 4–37b, we have

$$\circlearrowleft + (M_R)_O = \Sigma M_O;$$

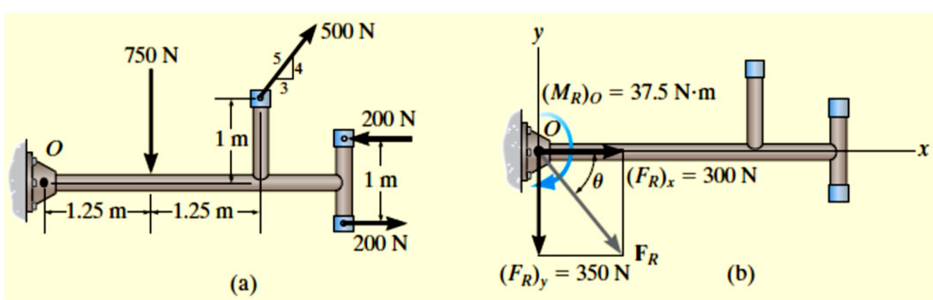
$$(M_R)_O = (3 \text{ kN})\sin 30^\circ(0.2 \text{ m}) - (3 \text{ kN})\cos 30^\circ(0.1 \text{ m}) + \left(\tfrac{3}{5}\right)(5 \text{ kN})(0.1 \text{ m}) - \left(\tfrac{4}{5}\right)(5 \text{ kN})(0.5 \text{ m}) - (4 \text{ kN})(0.2 \text{ m})$$

$$= -2.46 \text{ kN}\cdot\text{m} = 2.46 \text{ kN}\cdot\text{m} \circlearrowright \qquad \textit{Ans.}$$

This clockwise moment is shown in Fig. 4–37c.

Example 3

Replace the force and couple system acting on the member in the Fig. by an equivalent resultant force and couple moment acting at point *O* .



Example 3 Cont..

Force Summation. Since the couple forces of 200 N are equal but opposite, they produce a zero resultant force, and so it is not necessary to consider them in the force summation. The 500-N force is resolved into its *x* and *y* components, thus,

$$\xrightarrow{+} (F_R)_x = \Sigma F_x;\quad (F_R)_x = \left(\tfrac{3}{5}\right)(500\text{ N}) = 300\text{ N} \rightarrow$$

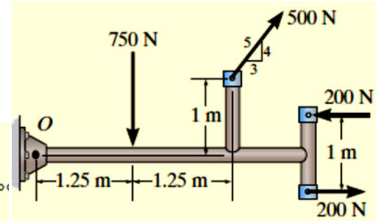
$$+\uparrow (F_R)_y = \Sigma F_y;\quad (F_R)_y = (500\text{ N})\left(\tfrac{4}{5}\right) - 750\text{ N} = -350\text{ N} = 350\text{ N}\downarrow$$

From Fig. 4–15*b*, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{(300\text{ N})^2 + (350\text{ N})^2} = 461\text{ N}$$

And the angle θ is

$$\theta = \tan^{-1}\left(\frac{(F_R)_y}{(F_R)_x}\right) = \tan^{-1}\left(\frac{350\text{ N}}{300\text{ N}}\right) = 49.4°$$

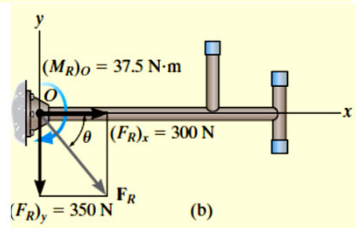


Moment Summation. Since the couple moment is a free vector, it can act at any point on the member. Referring to Fig.*a* , we have

$$\circlearrowleft + (M_R)_O = \Sigma M_O + \Sigma M$$

$$(M_R)_O = (500\text{ N})\left(\tfrac{4}{5}\right)(2.5\text{ m}) - (500\text{ N})\left(\tfrac{3}{5}\right)(1\text{ m}) - (750\text{ N})(1.25\text{ m}) + 200\text{ N}\cdot\text{m}$$

$$= -37.5\text{ N}\cdot\text{m} = 37.5\text{ N}\cdot\text{m} \circlearrowright$$



Example 4

Replace the force system acting on the truss by a resultant force and couple moment at point *C*.

$$\xrightarrow{+} \Sigma(F_R)_x = \Sigma F_x;\quad (F_R)_x = 500\left(\frac{4}{5}\right) = 400\text{ lb} \rightarrow$$

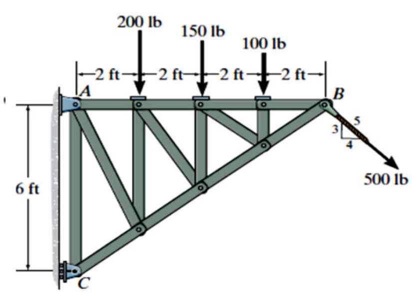
$$+\uparrow (F_R)_y = \Sigma F_y;\quad (F_R)_y = -200 - 150 - 100 - 500\left(\frac{3}{5}\right) = -750\text{ lb} = 750\text{ lb}\downarrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{400^2 + 750^2} = 850\text{ lb}$$

The angle θ of $\mathbf{F}_R$ is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{750}{400}\right] = 61.93° = 61.9°$$



Equivalent Couple Moment: Summing the moment of the forces and force components, Fig. *a*, algebraically about point *C*,

$$\circlearrowleft + (M_R)_C = \Sigma M_C;\quad (M_R)_C = -200(2) - 150(4) - 100(6) - 500\left(\frac{3}{5}\right)(8) - 500\left(\frac{4}{5}\right)(6)$$

$$= -6400\text{ lb}\cdot\text{ft} = 6.40\text{ kip}\cdot\text{ft (clockwise)}\qquad \textbf{Ans.}$$

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Example 5

Replace the force and couple moment system acting on the overhang beam by a resultant force and couple moment at point *A*.

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \quad (F_R)_x = 26\left(\frac{5}{13}\right) - 30\sin 30° = -5\text{ kN} = 5\text{ kN} \leftarrow$$

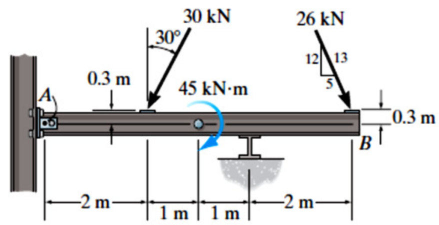
$$+\uparrow (F_R)_y = \Sigma F_y; \quad (F_R)_y = -26\left(\frac{12}{13}\right) - 30\cos 30° = -49.98\text{ kN} = 49.98\text{ kN} \downarrow.$$

The magnitude of the resultant force $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2} = \sqrt{5^2 + 49.98^2} = 50.23\text{kN} = 50.2\text{ kN}$$

The angle θ of $\mathbf{F}_R$ is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{49.98}{5}\right] = 84.29° = 84.3°$$



$$\circlearrowleft +(M_R)_A = \Sigma M_A; \quad (M_R)_A = 30\sin 30°(0.3) - 30\cos 30°(2) - 26\left(\frac{5}{13}\right)(0.3) - 26\left(\frac{12}{13}\right)(6) - 45$$

$$= -239.46\text{ kN}\cdot\text{m} = 239\text{ kN}\cdot\text{m (clockwise)}$$

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Example 6

Replace the two forces by an equivalent resultant force and couple moment at point *O*. Set $F = 20\ lb$.

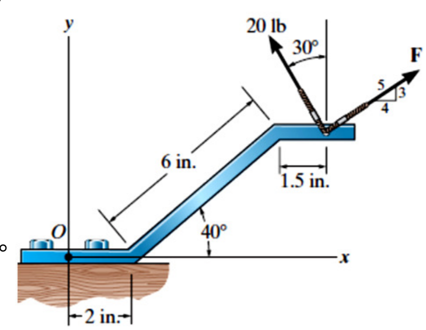
$$(F_R)_x = \sum F_X; \quad (F_R)_x = 20\left(\frac{4}{5}\right) - 20sin30° = 6\ lb$$

$$(F_R)_y = \sum F_Y; \quad (F_R)_y = 20cos30° + 20\left(\frac{3}{5}\right) = 29.32\ lb$$

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

$$F_R = \sqrt{(6)^2 + (29.32)^2} = 29.9\text{ lb}$$

$$\theta = tan^{-1} = \frac{F_{Ry}}{F_{Rx}} = tan^{-1} = \left(\frac{29.32}{6}\right) = 78.4°$$



$$CCW + M_{Ro} = \sum M_o; \quad M_{Ro} = 20sin30°(6sin40°) + 20cos30°(3.5 + 6cos40°)$$

$$-\frac{4}{5}(20)(6sin40°) + \frac{3}{5}(20)(3.5 + 6cos40°) = 214\ lb.in$$

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Further Simplification of a Force and Couple System

- In the previous section, we developed a way to reduce a force and couple moment system acting on a rigid body into an equivalent resultant force $\boldsymbol{F}_R$ acting at a specific ***point O*** and a resultant couple moment $(\boldsymbol{M}_R)_{\boldsymbol{O}}$.

- The force system can be further reduced to an equivalent single resultant force provided the lines of action of $\boldsymbol{F}_R$ and $(\boldsymbol{M}_R)_{\boldsymbol{O}}$ are ***perpendicular*** to each other. Because of this condition, only concurrent, coplanar, and parallel force systems can be further simplified.

Further Simplification of a Force and Couple System

- **Concurrent Force System: Since a *concurrent force system* is one in which the lines of action of all the forces intersect at a common point *O* , Fig. 4–40 *a* , then the force system produces no moment about this point. As a result, the equivalent system can be represented by a single resultant force $\boldsymbol{F}_R = \boldsymbol{F}$ acting at *O* , Fig. 4–40 *b* .**

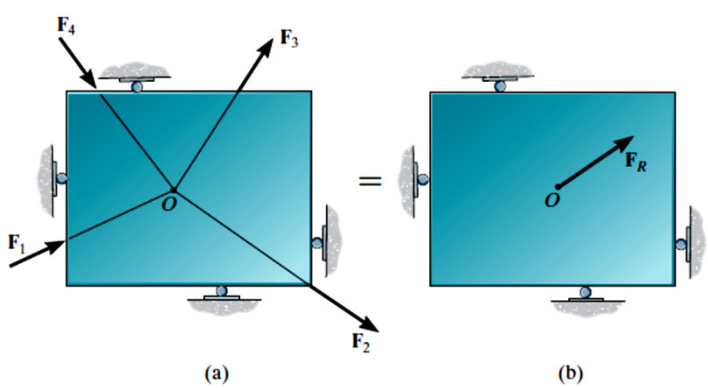
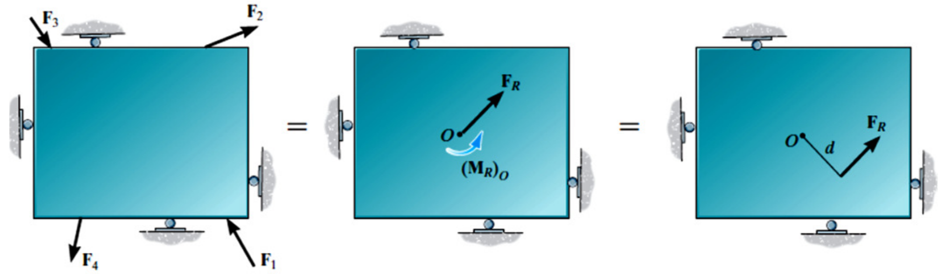


Fig. 4–40

Further Simplification of a Force and Couple System

Coplanar Force System: In the case of a ***coplanar force system***, the lines of action of all the forces lie in the same plane, Fig. 4–41*a*, and so the resultant force $F_R = \sum F$ of this system also lies in this plane. Furthermore, the moment of each of the forces about any ***point O*** is directed perpendicular to this plane. Thus, the resultant moment $(M_R)_O$ and resultant force F_R will be ***mutually perpendicular***, Fig. 4–41*b*. The resultant moment can be replaced by moving the resultant force F_R a perpendicular or moment arm distance ***d*** away from ***point O*** such that F_R produces the *same moment* ($(M_R)_O$ about ***point O***, Fig. 4–41*c*. This distance ***d*** can be determined from the scalar equation $(M_R)_O = F_R d = \sum M_O$, or $d = (M_R)_O / F_R$.

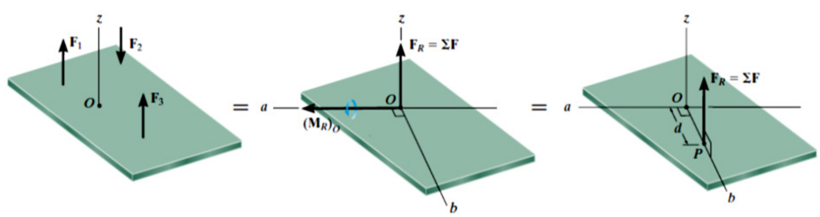


(a) (b) (c)

Fig. 4–41

Further Simplification of a Force and Couple System

- **Parallel Force System**

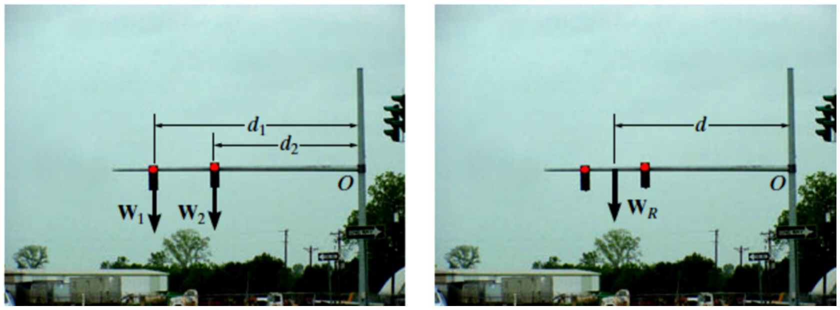


(a) (b) (c)

Fig. 4–42

Further Simplification of a Force and Couple System

- **Parallel Force System**



Here the weights of the traffic lights are replaced by their resultant force $W_R = W_1 + W_2$ which acts at a distance $d = (W_1 d_1 + W_2 d_2)/W_R$ from O. Both systems are equivalent.

Procedure for Analysis

The technique used to reduce a coplanar or parallel force system to a single resultant force follows a similar procedure outlined in the previous section.

- Establish the x, y, z, axes and locate the resultant force $\mathbf{F}_R$ an arbitrary distance away from the origin of the coordinates.

Force Summation.

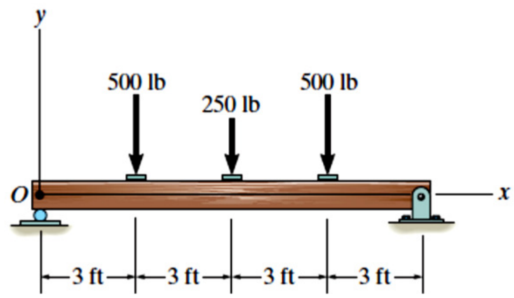
- The resultant force is equal to the sum of all the forces in the system.
- For a coplanar force system, resolve each force into its x and y components. Positive components are directed along the positive x and y axes, and negative components are directed along the negative x and y axes.

Moment Summation.

- The moment of the resultant force about point O is equal to the sum of all the couple moments in the system plus the moments of all the forces in the system about O.
- This moment condition is used to find the location of the resultant force from point O.

Example 1

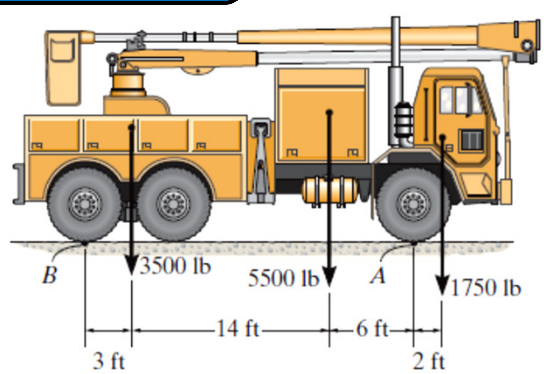
Replace the loading system by an equivalent resultant force and specify where the resultant's line of action intersects the beam measured from O .



$$+\downarrow F_R = \Sigma F_y; \quad F_R = 500 + 250 + 500$$
$$= 1250 \text{ lb}$$
$$\curvearrowleft + F_R x = \Sigma M_O;$$
$$1250(x) = 500(3) + 250(6) + 500(9)$$
$$x = 6 \text{ ft}$$

Example 2

The weights of the various components of the truck are shown. Replace this system of forces by an equivalent resultant force and specify its location measured from point *A*.



Equivalent Forces:

$$F_R = \sum F_y$$

$$F_R = -1750 - 5500 - 3500 = -10750 = 10750 \text{ lb} \downarrow$$

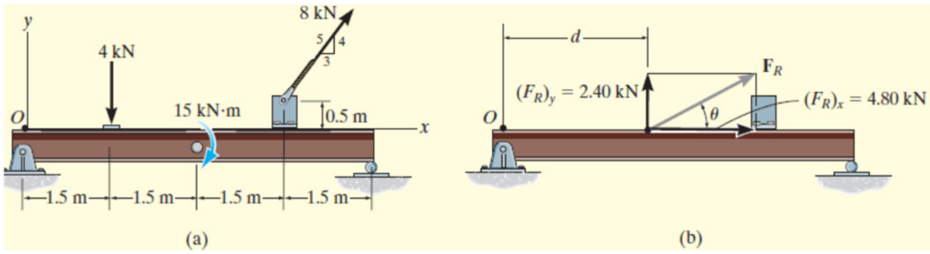
Location of Resultant Force From Point A:

$$\curvearrowleft + (M_R)_A = \sum M_A$$

$$10750(d) = 3500(20) + 5500(6) - 1750(2)$$
$$d = 9.26\ ft$$

Example 3

Replace the force and couple moment system acting on the beam in the Fig. by an equivalent resultant force, and find where its line of action intersects the beam, measured from point *O* .



$$(F_R)_x = \sum F_x; \quad (F_R)_x = 8KN\left(\tfrac{3}{5}\right) = 4.80\ KN \rightarrow$$

$$(F_R)_y = \sum F_y; \quad (F_R)_y = -4\ KN + 8KN\left(\tfrac{4}{5}\right) = 2.40\ KN\ \uparrow$$

$$F_R = \sqrt{(4.80KN)^2 + (2.40KN)^2} = 5.37\text{ KN}, \qquad \theta = \tan^{-1}\left(\frac{2.40}{4.8}\right) = 26.6°$$

$$\circlearrowleft + (M_R)_O = \Sigma M_O; \quad 2.40\text{ kN}(d) = -(4\text{ kN})(1.5\text{ m}) - 15\text{ kN}\cdot\text{m}$$

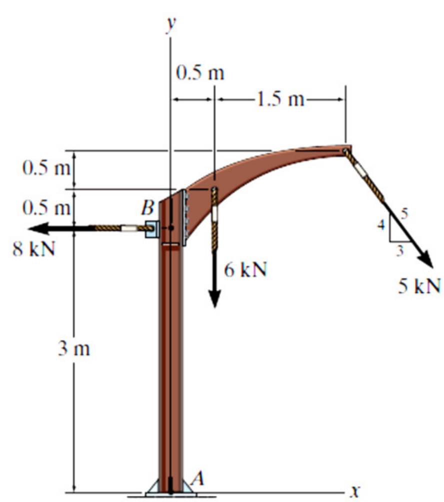
$$-\left[8\text{ kN}\left(\tfrac{3}{5}\right)\right](0.5\text{ m}) + \left[8\text{ kN}\left(\tfrac{4}{5}\right)\right](4.5\text{ m})$$

$$d = 2.25\text{ m} \qquad \textit{Ans.}$$

Example 4

Replace the loading system by an equivalent resultant force and specify where the resultant's line of action intersects the member *AB* measured from *A* .

Solution:



$$\overset{+}{\rightarrow}(F_R)_x = \Sigma F_x;$$

$$(F_R)_x = \left(\tfrac{3}{5}\right)5\text{ kN} - 8\text{ kN}$$

$$= -5\text{ kN} = 5\text{ kN} \leftarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y;$$

$$(F_R)_y = -6\text{ kN} - \left(\tfrac{4}{5}\right)5\text{ kN}$$

$$= -10\text{ kN} = 10\text{ kN}\downarrow$$

$$F_R = \sqrt{5^2 + 10^2} = 11.2\text{ kN}$$

$$\theta = \tan^{-1}\left(\frac{10\text{ kN}}{5\text{ kN}}\right) = 63.4°$$

$$\circlearrowleft + (M_R)_A = \Sigma M_A;$$

$$5\text{ kN}(d) = 8\text{ kN}(3\text{ m}) - 6\text{ kN}(0.5\text{ m})$$

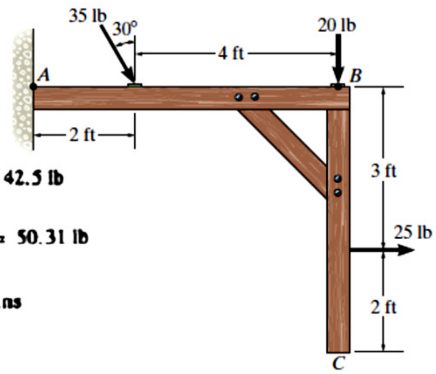
$$-\left[\left(\tfrac{4}{5}\right)5\text{ kN}\right](2\text{ m})$$

$$-\left[\left(\tfrac{3}{5}\right)5\text{ kN}\right](4\text{ m})$$

$$d = 0.2\text{ m}$$

Example 5

Replace the force system acting on the frame by **an equivalent resultant force** and **specify** where the resultant's line of action intersects member ***AB***, measured from point ***A***.



$\xrightarrow{+} F_{Rx} = \Sigma F_x: \quad F_{Rx} = 35 \sin 30° + 25 = 42.5 \text{ lb}$

$+\downarrow F_{Ry} = \Sigma F_y: \quad F_{Ry} = 35 \cos 30° + 20 = 50.31 \text{ lb}$

$F_R = \sqrt{(42.5)^2 + (50.31)^2} = 65.9 \text{ lb}$ **Ans**

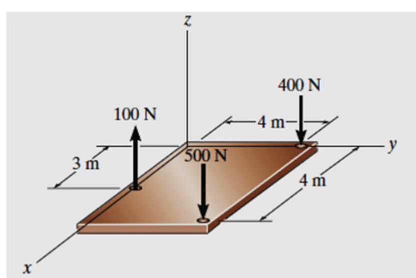
$\theta = \tan^{-1}\left(\frac{50.31}{42.5}\right) = 49.8°$ **Ans**

$\circlearrowright + M_{RA} = \Sigma M_A: \quad 50.31\,(d) = 35 \cos 30°\,(2) + 20\,(6) - 25\,(3)$

$d = 2.10 \text{ ft}$ **Ans**

Example 6

Replace the loading shown by an equivalent single resultant force and **specify** the x and y ***coordinates*** of its line of action.



$+\downarrow F_R = \Sigma F_z; \quad F_R = 400 + 500 - 100$

$= 800 \text{ N}$

$M_{Rx} = \Sigma M_x; \; -800y = -400(4) - 500(4)$

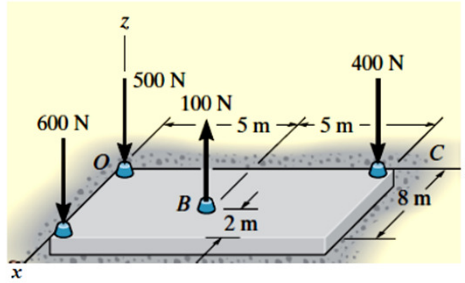
$y = 4.50 \text{ m}$

$M_{Ry} = \Sigma M_y; \quad 800x = 500(4) - 100(3)$

$x = 2.125 \text{ m}$

Example 7

Replace the loading shown by an equivalent single resultant force and specify the x and y coordinates of its line of action.



Force Summation. From Fig. 4–46a, the resultant force is

$+\uparrow F_R = \Sigma F;$ $\quad F_R = -600\text{ N} + 100\text{ N} - 400\text{ N} - 500\text{ N}$

$= -1400\text{ N} = 1400\text{ N}\downarrow$

$(M_R)_x = \Sigma M_x;$

$-(1400\text{ N})y = 600\text{ N}(0) + 100\text{ N}(5\text{ m}) - 400\text{ N}(10\text{ m}) + 500\text{ N}(0)$

$-1400y = -3500 \qquad y = 2.50\text{ m}$

$(M_R)_y = \Sigma M_y;$

$(1400\text{ N})x = 600\text{ N}(8\text{ m}) - 100\text{ N}(6\text{ m}) + 400\text{ N}(0) + 500\text{ N}(0)$

$1400x = 4200$

$x = 3\text{ m}$

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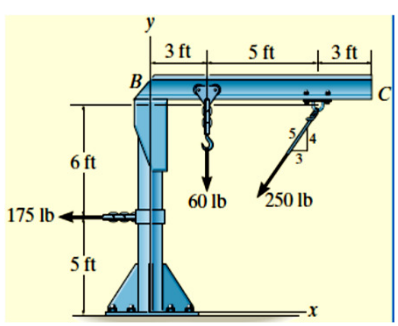
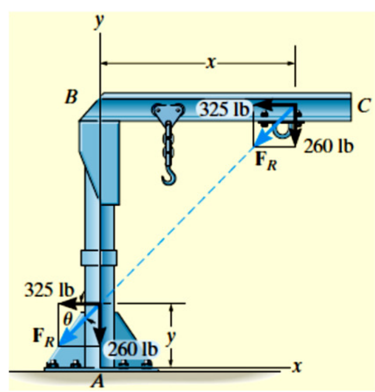
Example 8

The jib crane shown in the Figure is subjected to three coplanar forces. Replace this loading by an equivalent resultant force and specify where the resultant's line of action intersects the column *AB* and boom *BC* .





$(F_R)_x = \sum F_x;\ (F_R)_x = -250\left(\frac{3}{5}\right) - 175 = -325 = 325 \leftarrow$

$(F_R)_y = \sum F_y;\ (F_R)_y = -\ 250\left(\frac{4}{5}\right) - 60 = -260lb = 260\downarrow$

$F_R = \sqrt{(325)^2 + (260)^2} = 416\text{ lb},\qquad \theta = \tan^{-1}\left(\frac{260}{325}\right) = 38.7°$

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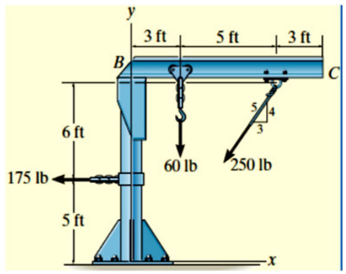
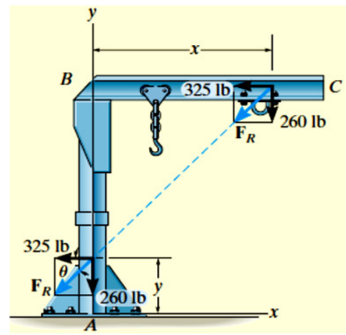
Example 8 Cont..





Moments will be summed about point *A*. Assuming the line of action of F_R *intersects AB* at a distance *y* from *A*, we have

$$\circlearrowleft + (M_R)_A = \Sigma M_A; \qquad 325\text{ lb }(y) + 260\text{ lb }(0)$$
$$= 175\text{ lb }(5\text{ ft}) - 60\text{ lb }(3\text{ ft}) + 250\text{ lb}\left(\tfrac{3}{5}\right)(11\text{ ft}) - 250\text{ lb}\left(\tfrac{4}{5}\right)(8\text{ ft})$$
$$y = 2.29\text{ ft}$$

By the principle of transmissibility, F_R can be placed at a distance x where it intersects BC, Fig. 4–45 b. In this case we have

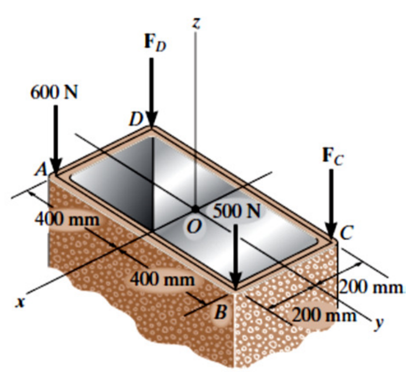
$$\circlearrowleft + (M_R)_A = \Sigma M_A; \qquad 325\text{ lb }(11\text{ ft}) - 260\text{ lb }(x)$$
$$= 175\text{ lb }(5\text{ ft}) - 60\text{ lb }(3\text{ ft}) + 250\text{ lb}\left(\tfrac{3}{5}\right)(11\text{ ft}) - 250\text{ lb}\left(\tfrac{4}{5}\right)(8\text{ ft})$$
$$x = 10.9\text{ ft}$$

Example 9

The tube supports the four parallel forces. Determine the magnitudes of forces F_C and F_D acting at *C* and *D* so that the equivalent resultant force of the force system acts through the midpoint *O* of the tube.

Since the resultant force passes through point *O*, the resultant moment components about *x* and *y* axes are both zero.

$$\Sigma M_x = 0; \qquad F_D(0.4) + 600(0.4) - F_C(0.4) - 500(0.4) = 0$$
$$F_C - F_D = 100 \qquad (1)$$
$$\Sigma M_y = 0; \qquad 500(0.2) + 600(0.2) - F_C(0.2) - F_D(0.2) = 0$$
$$F_C + F_D = 1100 \qquad (2)$$

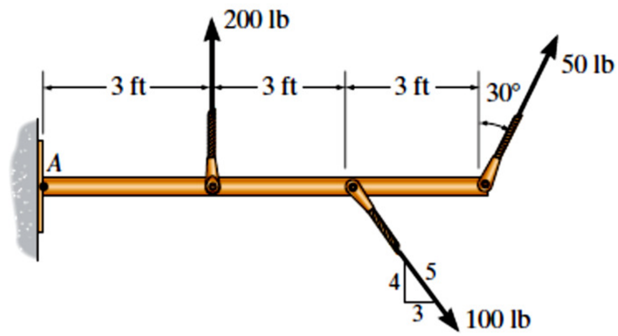


Solving Eqs. (1) and (2) yields:

$F_C = 600$ N $\qquad F_D = 500$ N $\qquad$ **Ans**

Try it Yourself ☺

Replace the loading system by an equivalent resultant force and specify where the resultant's line of action intersects the member measured from *A* .



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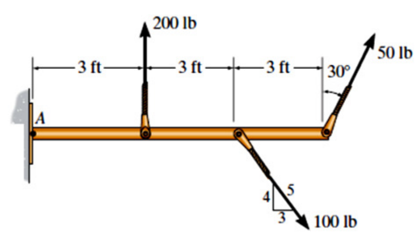
Solution

Replace the loading system by an equivalent resultant force and specify where the resultant's line of action intersects the member measured from *A* .



$$\xrightarrow{+} (F_R)_x = \Sigma F_x;$$
$$(F_R)_x = 100\left(\tfrac{3}{5}\right) + 50 \sin 30^\circ = 85 \text{ lb} \rightarrow$$
$$+\uparrow (F_R)_y = \Sigma F_y;$$
$$(F_R)_y = 200 + 50 \cos 30^\circ - 100\left(\tfrac{4}{5}\right)$$
$$= 163.30 \text{ lb}\uparrow$$
$$F_R = \sqrt{85^2 + 163.30^2} = 184 \text{ lb}$$
$$\theta = \tan^{-1}\left(\tfrac{163.30}{85}\right) = 62.5^\circ \measuredangle \qquad \textit{Ans.}$$
$$\circlearrowleft + (M_R)_A = \Sigma M_A;$$
$$163.30(d) = 200(3) - 100\left(\tfrac{4}{5}\right)(6) + 50 \cos 30^\circ(9)$$
$$d = 3.12 \text{ ft} \qquad \textit{Ans.}$$

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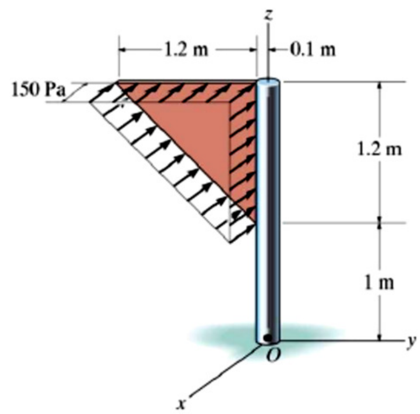
Reduce a Simple Distributed Loading



- The uniform wind pressure is acting on a triangular sign (shown in light brown).
- To be able to design the joint between the sign and the sign post, we need to determine a single equivalent resultant force and its location.

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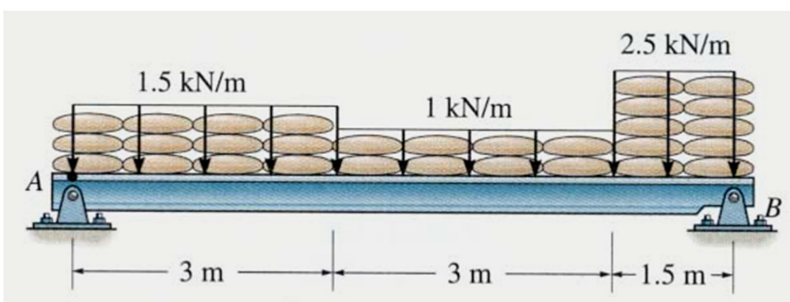
Reduce a Simple Distributed Loading



The sandbags on the beam create a distributed load.

How can we determine a single equivalent resultant force and its location?

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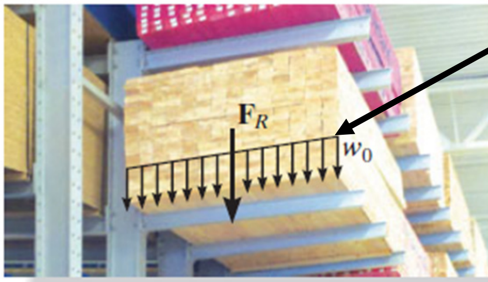
Reduce a Simple Distributed Loading



A distributed load on the beam exists due to the weight of the lumber.

Is it possible to reduce this force system to a single force that will have the same external effect? If yes, how?

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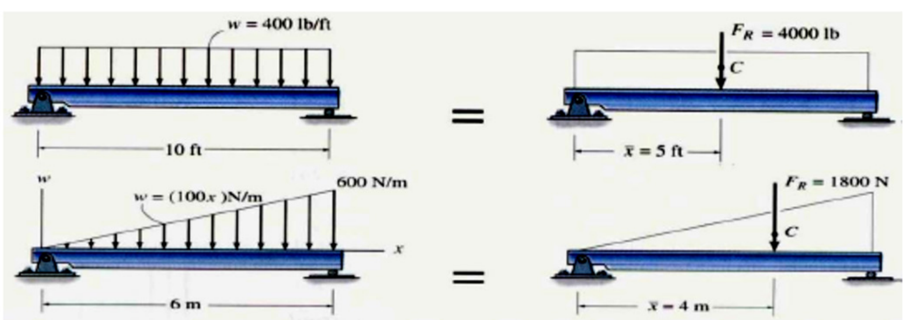
Example 1



The rectangular load: F_R = 400 × 10 = 4,000 lb and = 5 ft.

The triangular loading:
F_R = (0.5) (600) (6) = 1,800 N and = 6 – (1/3) 6 = 4 m.
Please note that the centroid in a right triangle is at a distance one third the width of the triangle as measured from its base.

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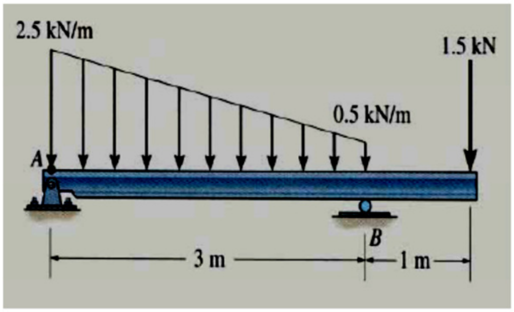
Example 2



Given: The loading on the beam as shown.

Find: The equivalent force and its location from point A.

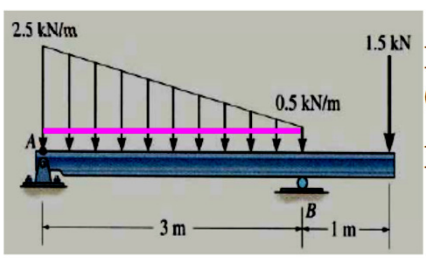
Plan:

1) Consider the trapezoidal loading as two separate loads (one rectangular and one triangular).

2) Find F_R and for each of these two distributed loads.

3) Determine the overall F_R and for the three point loadings.

Example 2 Cont..



For the rectangular loading of height 0.5 kN/m and width 3 m,

F_{R1} = 0.5 kN/m × 3 m = 1.5 kN

= 1.5 m from A

For the triangular loading of height 2 kN/m and width 3 m,

F_{R2} = (0.5) (2 kN/m) (3 m) = 3 kN

and its line of action is at = 1 m from A

For the combined loading of the three forces,

F_R = 1.5 kN + 3 kN + 1.5 kN = 6 kN

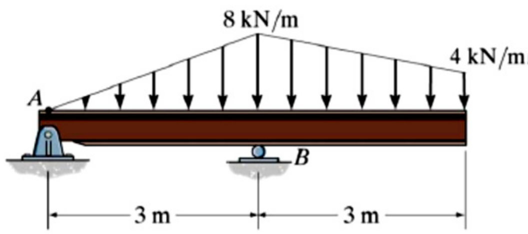
+ M_{RA} = (1.5) (1.5) + 3 (1) + (1.5) 4 = 11.25 kN • m

Now, M = 11.25 kN • m

Hence, $\bar{x}$ = (11.25) / (6) = 1.88 m from A.

Example 3

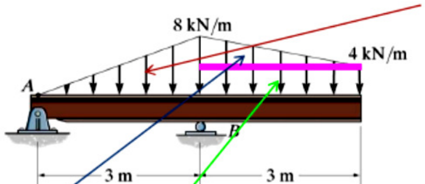
Find the equivalent force to replace the loading and identify its location from point A.



Plan:

1. The distributed loading can be divided into three parts. (one rectangular loading and two triangular loadings).
2. Find F_R and its location for each of these three distributed loads.
3. Determine the overall F_R of the three point loadings and its location.

Example 3 Cont..



For the left triangular loading of height 8 kN/m and width 3 m,

F_{R1} = (0.5) 8 kN/m × 3 m = 12 kN

x_1 = (2/3)(3m) = 2 m from A

For the top right triangular loading of height 4 kN/m and width 3 m,

F_{R2} = (0.5) (4 kN/m) (3 m) = 6 kN

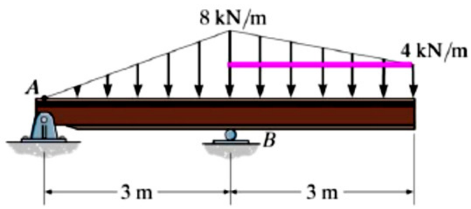
and its line of action is at = (1/3)(3m) + 3 = 4 m from A

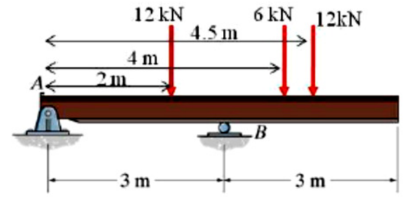
For the rectangular loading of height 4 kN/m and width 3 m,

F_{R3} = (4 kN/m) (3 m) = 12 kN

and its line of action is at = (1/2)(3m) + 3 = 4.5 m from A

Example 3 Cont..





For the combined loading of the three forces, add them.

F_R = 12 kN + 6 kN + 12 kN = 30 kN

$+\circlearrowleft M_{RA}$ = (2) (12) + 4 (6) + (4.5) 12 = 102 kN • m

Now, (F_R x) has to equal M_{RA} = 102 kN • m

So solve for x to find the equivalent force's location.

Hence, x = (102 kN • m) / (30 kN) = 3.4 m from A.

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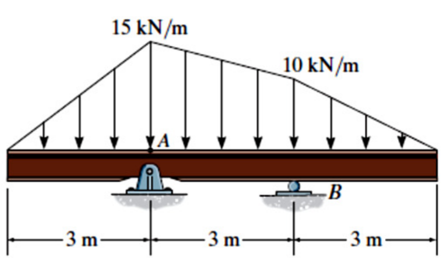
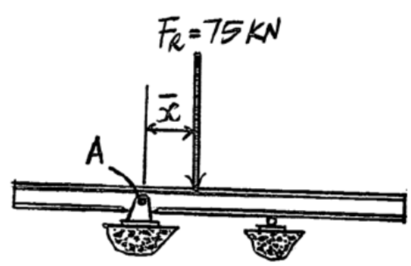
Example 4

Replace the distributed loading with an equivalent resultant force. and specify its location on the beam measured from point *A* .





$+\downarrow F_R = \Sigma F_y;$ $\quad F_R = \frac{1}{2}(15)(3) + \frac{1}{2}(5)(3) + 10(3) + \frac{1}{2}(10)(3) = 75\text{ kN} \downarrow$

$\circlearrowleft +(M_R)_A = \Sigma M_A;\quad -75(\bar{x}) = \frac{1}{2}(15)(3)(1) - \frac{1}{2}(5)(3)(1) - 10(3)(1.5) - \frac{1}{2}(10)(3)(4)$

$\bar{x} = 1.20\text{ m}$

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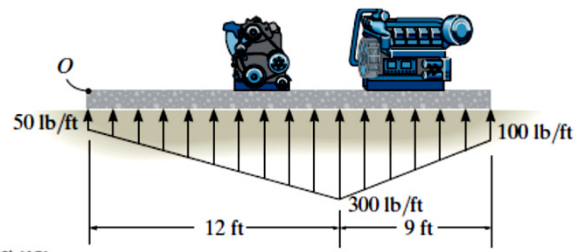
Example 5

The distribution of soil loading on the bottom of a building slab is shown. Replace this loading by an equivalent resultant force and specify its location, measured from point *O*.



$$+\uparrow F_R = \Sigma F_y;\quad F_R = 50(12) + \tfrac{1}{2}(250)(12)$$

$$+ \tfrac{1}{2}(200)(9) + 100(9)$$

$$= 3900 \text{ lb} = 3.90 \text{ kip} \uparrow$$

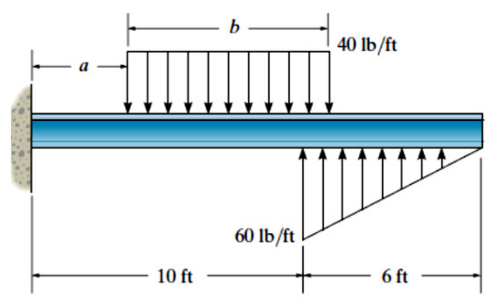
$$\circlearrowleft + M_{R_O} = \Sigma M_O;\quad 3900(d) = 50(12)(6) + \tfrac{1}{2}(250)(12)(8)$$

$$+ \tfrac{1}{2}(200)(9)(15) + 100(9)(16.5)$$

$$d = 11.3 \text{ ft}$$

Example 6

The beam is subjected to the distributed loading. Determine the length *b* of the uniform load and its position *a* on the beam such that the *resultant force and couple moment acting on the beam are zero.*



- **Solution:**

Replace $F_R = 0$

$\uparrow + F_R = \sum F_y$

$0 = 0.5(60)(6) - (40)(b)$

$b = 4.5\ ft$

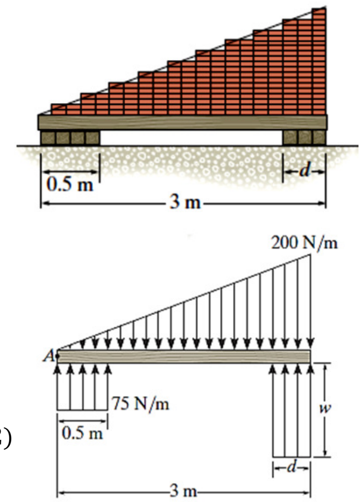
Replace $M_{R_A} = 0.$ *Using the result* $b = 4.50\ ft$, *we have*

$M_{R_A} = \sum M_A;\quad 0 = (0.5)(60)(6)(12) - 40(4.5)\left(a + \frac{4.5}{2}\right)$

$a = 9.75\ ft$

Example 7

The bricks on top of the beam and the supports at the bottom create the distributed loading shown in the second figure. Determine the required intensity *w* and dimension *d* of the right support so that the resultant force and couple moment about point *A* of the system are both zero.



- **Solution:**

$Require\ F_R = 0$

$\uparrow + F_R = \sum F_y;$

$0 = (75)(0.5) + wd - 0.5(200)(3)$

$wd = 262.5$

$d = \dfrac{262.5}{w}$

$Require\ M_{R_A} = 0$

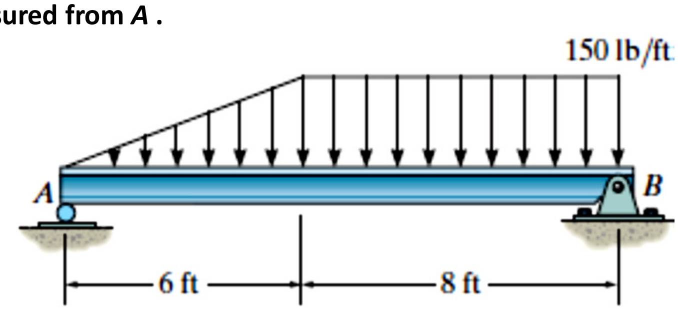
$M_{R_A} = \sum M_A$

$0 = (75)(0.5)(0.25) + wd\left(3 - \frac{d}{2}\right) - 0.5(200)(3)(2)$

$d = 1.50\ m \qquad w = 175\ N/m$

Try it Yourself ☺

Determine the resultant force and specify where it acts on the beam measured from *A* .



F_R	d from A
(a) 1700 lb	(a) 8.4 ft
(b) 1650 lb	(b) 8.36 ft
(c) 1600 lb	(c) 8.2 ft
(d) 1550 lb	(d) 8.15 ft

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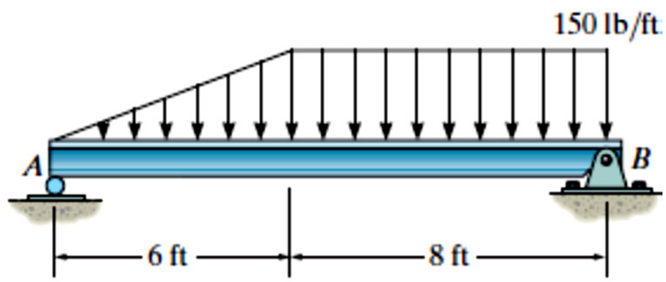
Solution

Determine the resultant force and specify where it acts on the beam measured from *A* .



$$F_R = \tfrac{1}{2}(6)(150) + 8(150) = 1650 \text{ lb}$$

$$\circlearrowleft +M_{A_R} = \Sigma M_A;$$

$$1650d = \left[\tfrac{1}{2}(6)(150)\right](4) + [8(150)](10)$$

$$d = 8.36 \text{ ft}$$

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