University of Anbar

Engineering Mechanics: Statics
CHE 211

Lecture #05

Objectives of Lecture Note

- To discuss the concept of the center of gravity, center of mass, and the centroid.
- To show how to determine the centroid for a system of discrete particles and a body of arbitrary shape.

Composite Bodies

- A *composite body* consists of a series of connected "simpler" shaped bodies, which may be rectangular, triangular, semicircular, etc.
- A *composite body* can often be sectioned or divided into its composite parts and, provided the ***weight*** and location of the center of gravity of each of these parts are known.

$$\bar{x} = \frac{\Sigma \tilde{x} W}{\Sigma W} \quad \bar{y} = \frac{\Sigma \tilde{y} W}{\Sigma W} \quad \bar{z} = \frac{\Sigma \tilde{z} W}{\Sigma W} \qquad (9\text{–}6)$$

Here

$\bar{x}, \bar{y}, \bar{z}$ represent the coordinates of the center of gravity G of the composite body.

$\tilde{x}, \tilde{y}, \tilde{z}$ represent the coordinates of the center of gravity of each composite part of the body.

ΣW is the sum of the weights of all the composite parts of the body, or simply the total weight of the body.

The centroid for composite lines, areas, and volumes can be found using relations analogous to Eqs. 9–6 ; however, the *W* are replaced by *L*, *A*, and *V* , respectively.

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Procedure for Analysis

The location of the center of gravity of a body or the centroid of a composite geometrical object represented by a line, area, or volume can be determined using the following procedure.

Composite Parts.

- Using a sketch, divide the body or object into a finite number of composite parts that have simpler shapes.
- If a composite body has a *hole*, or a geometric region having no material, then consider the composite body without the hole and consider the hole as an *additional* composite part having *negative* weight or size.

Moment Arms.

- Establish the coordinate axes on the sketch and determine the coordinates $\tilde{x}, \tilde{y}, \tilde{z}$ of the center of gravity or centroid of each part.

Summations.

- Determine $\bar{x}, \bar{y}, \bar{z}$ by applying the center of gravity equations, Eqs. 9–6, or the analogous centroid equations.
- If an object is *symmetrical* about an axis, the centroid of the object lies on this axis.

If desired, the calculations can be arranged in tabular form, as indicated in the following three examples.

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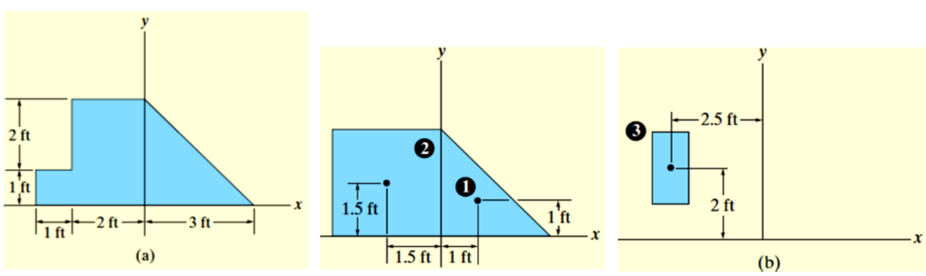
Composite Bodies

The center of gravity of this water tank can be determined by dividing it into composite parts and applying Eqs. 9–6 .

Example 1

Locate the centroid of the plate area shown in Fig. 9-17*a*.

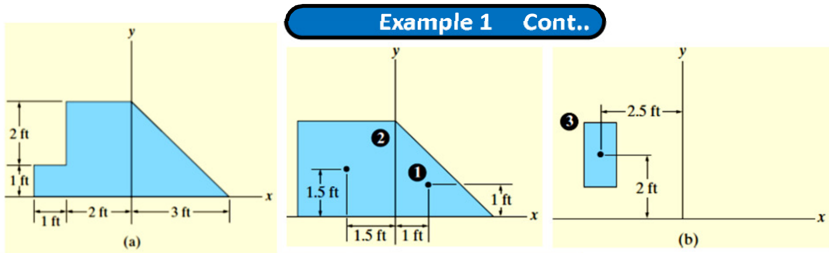


SOLUTION

Composite Parts. The plate is divided into three segments as shown in Fig. 9–17*b*. Here the area of the small rectangle 3 is considered "negative" since it must be subtracted from the larger one 2.

Moment Arms. The centroid of each segment is located as indicated in the figure. Note that the $\tilde{x}$ coordinates of 2 and 3 are *negative*.

Example 1 Cont..



Summations. Taking the data from Fig. 9–17*b*, the calculations are tabulated as follows:

Segment	A (ft^2)	$\tilde{x}$ (ft)	$\tilde{y}$ (ft)	$\tilde{x}A$ (ft^3)	$\tilde{y}A$ (ft^3)
1	$\frac{1}{2}(3)(3) = 4.5$	1	1	4.5	4.5
2	$(3)(3) = 9$	−1.5	1.5	−13.5	13.5
3	$-(2)(1) = -2$	−2.5	2	5	−4
	$\Sigma A = 11.5$			$\Sigma\tilde{x}A = -4$	$\Sigma\tilde{y}A = 14$

Thus,

$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{-4}{11.5} = -0.348 \text{ ft} \qquad \textit{Ans.}$$

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{14}{11.5} = 1.22 \text{ ft} \qquad \textit{Ans.}$$

NOTE: If these results are plotted in Fig. 9–17*a*, the location of point *C* seems reasonable.

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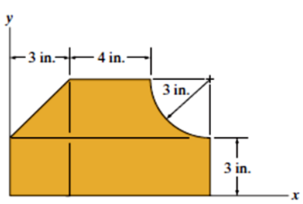
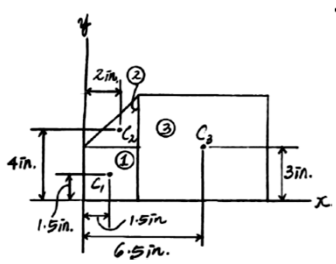
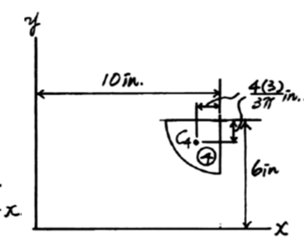
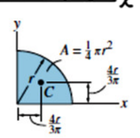
Example 2

Locate the centroid (*x, y*) of the composite area.









Quarter circle area

$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{1.5(3(3)) + 2\left(\frac{1}{2}(3)(3)\right) + 6.5(7(6)) + \left(10 - \frac{4(3)}{3\pi}\right)\left(-\frac{\pi(3^2)}{4}\right)}{3(3) + \frac{1}{2}(3)(3) + 7(6) + \left(-\frac{\pi(3^2)}{4}\right)} = \frac{233.81}{48.43} = 4.83 \text{ in.}$$

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{1.5(3(3)) + 4\left(\frac{1}{2}(3)(3)\right) + 3(7(6)) + \left(6 - \frac{4(3)}{3\pi}\right)\left(-\frac{\pi(3^2)}{4}\right)}{3(3) + \frac{1}{2}(3)(3) + 7(6) + \left(-\frac{\pi(3^2)}{4}\right)} = \frac{124.09}{48.43} = 2.56 \text{ in.}$$

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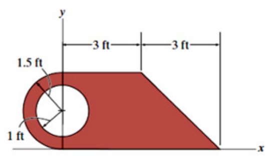
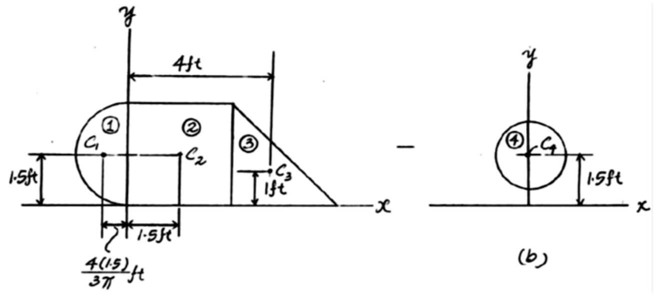
Example 3

Locate the centroid (*x, y*) of the composite area.





$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{\left(-\frac{4(1.5)}{3\pi}\right)\left(\frac{\pi(1.5^2)}{2}\right) + 1.5(3(3)) + 4\left(\frac{1}{2}(3)(3)\right) + 0\left(-\pi(1^2)\right)}{\frac{\pi(1.5^2)}{2} + 3(3) + \frac{1}{2}(3)(3) + \left(-\pi(1^2)\right)} = \frac{29.25}{13.89} = 2.11\text{ ft}$$

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{1.5\left(\frac{\pi(1.5^2)}{2}\right) + 1.5(3(3)) + 1\left(\frac{1}{2}(3)(3)\right) + 1.5\left(-\pi(1^2)\right)}{\frac{\pi(1.5^2)}{2} + 3(3) + \frac{1}{2}(3)(3) + \left(-\pi(1^2)\right)} = \frac{18.59}{13.89} = 1.34\text{ ft}$$

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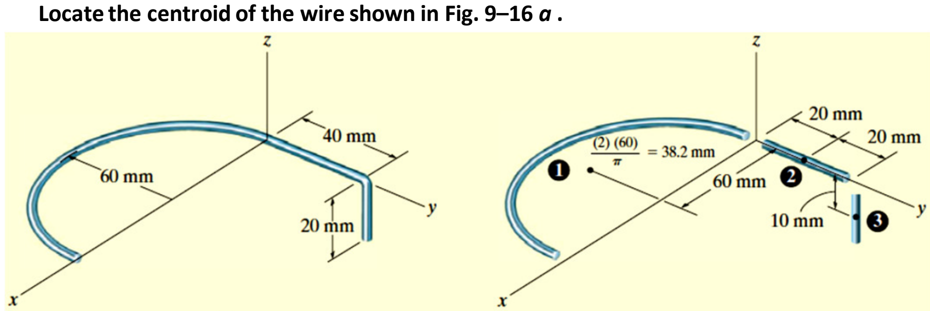
Example 4

Locate the centroid of the wire shown in Fig. 9–16 *a* .

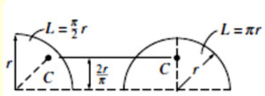


(a) (b)

SOLUTION

Composite Parts. The wire is divided into three segments as shown in Fig. 9–16*b*.

Moment Arms. The location of the centroid for each segment is determined and indicated in the figure. In particular, the centroid of segment (1) is determined either by integration or by using the table on the inside back cover.



Quarter and semicircle arcs

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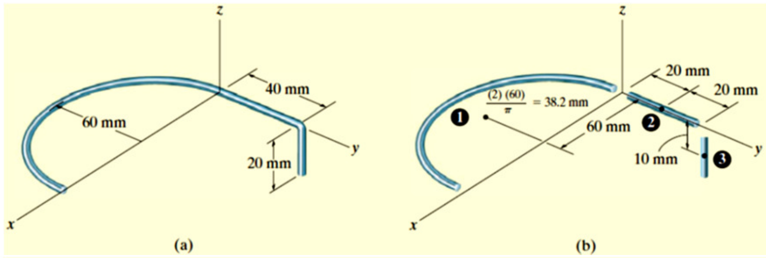
Example 4 Cont..



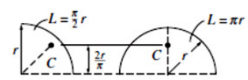
Summations. For convenience, the calculations can be tabulated as follows:

Segment	L (mm)	$\tilde{x}$ (mm)	$\tilde{y}$ (mm)	$\tilde{z}$ (mm)	$\tilde{x}L$ (mm^2)	$\tilde{y}L$ (mm^2)	$\tilde{z}L$ (mm^2)
1	$\pi(60) = 188.5$	60	-38.2	0	11 310	-7200	0
2	40	0	20	0	0	800	0
3	20	0	40	-10	0	800	-200
	$\Sigma L = 248.5$				$\Sigma\tilde{x}L = 11\,310$	$\Sigma\tilde{y}L = -5600$	$\Sigma\tilde{z}L = -200$

$$\bar{x} = \frac{\Sigma\tilde{x}L}{\Sigma L} = \frac{11\,310}{248.5} = 45.5 \text{ mm} \qquad \textit{Ans.}$$

$$\bar{y} = \frac{\Sigma\tilde{y}L}{\Sigma L} = \frac{-5600}{248.5} = -22.5 \text{ mm} \qquad \textit{Ans.}$$

$$\bar{z} = \frac{\Sigma\tilde{z}L}{\Sigma L} = \frac{-200}{248.5} = -0.805 \text{ mm} \qquad \textit{Ans.}$$



Quarter and semicircle arcs

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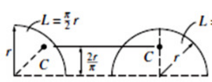
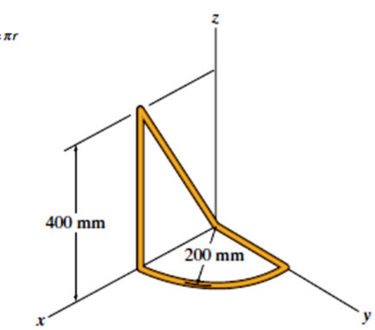
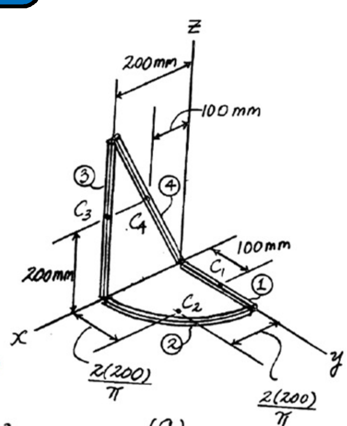
Example 5

Locate the centroid $(\bar{x}, \bar{y}, \bar{z})$ of the wire.



Quarter and semicircle arcs





$$\bar{x} = \frac{\Sigma\tilde{x}L}{\Sigma L} = \frac{0(200) + \frac{2(200)}{\pi}\left(\frac{\pi(200)}{2}\right) + 200(400) + 100\left(\sqrt{200^2 + 400^2}\right)}{200 + \frac{\pi(200)}{2} + 400 + \sqrt{200^2 + 400^2}} = \frac{164.72(10^3)}{1361.37} = 121 \text{ mm}$$

$$\bar{y} = \frac{\Sigma\tilde{y}L}{\Sigma L} = \frac{100(200) + \frac{2(200)}{\pi}\left(\frac{\pi(200)}{2}\right) + 0(400) + 0\left(\sqrt{200^2 + 400^2}\right)}{200 + \frac{\pi(200)}{2} + 400 + \sqrt{200^2 + 400^2}} = \frac{60(10^3)}{1361.37} = 44.1 \text{ mm}$$

$$\bar{z} = \frac{\Sigma\tilde{z}L}{\Sigma L} = \frac{0(200) + 0\left(\frac{\pi(200)}{2}\right) + 200(400) + 200\left(\sqrt{200^2 + 400^2}\right)}{200 + \frac{\pi(200)}{2} + 400 + \sqrt{200^2 + 400^2}} = \frac{169.44(10^3)}{1361.37} = 124 \text{ mm}$$

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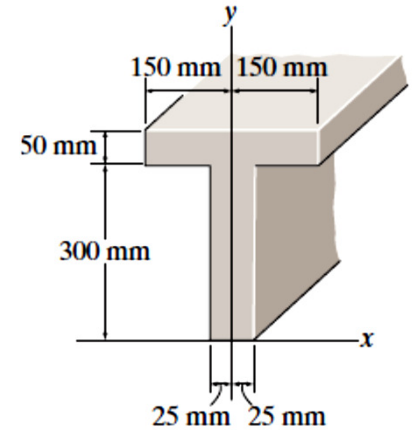
Try it Yourself ☺

Locate the centroid $\bar{y}$ of the beam's cross-sectional area.



$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{150[300(50)] + 325[50(300)]}{300(50) + 50(300)}$$
$$= 237.5 \text{ mm}$$

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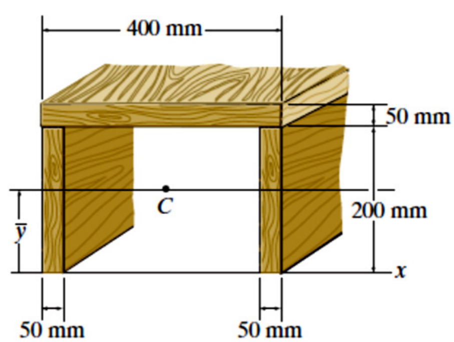
Try it Yourself ☺

Locate the centroid $\bar{y}$ of the beam's cross-sectional area.



$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{100[2(200)(50)] + 225[50(400)]}{2(200)(50) + 50(400)}$$
$$= 162.5 \text{ mm} \qquad \textit{Ans.}$$

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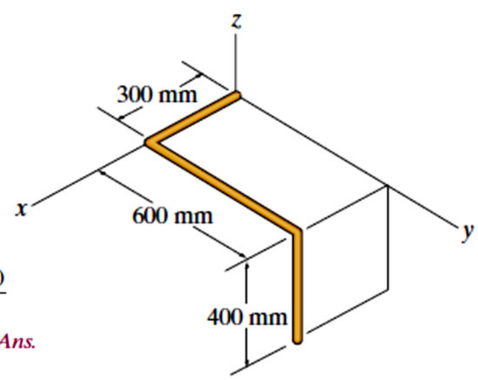
Try it Yourself ☺

Locate the centroid $(\bar{x}, \bar{y}, \bar{z})$ of the wire bent in the shape shown.



$$\bar{x} = \frac{\Sigma \tilde{x} L}{\Sigma L} = \frac{150(300) + 300(600) + 300(400)}{300 + 600 + 400}$$
$$= 265 \text{ mm} \qquad \textit{Ans.}$$

$$\bar{y} = \frac{\Sigma \tilde{y} L}{\Sigma L} = \frac{0(300) + 300(600) + 600(400)}{300 + 600 + 400}$$
$$= 323 \text{ mm} \qquad \textit{Ans.}$$

$$\bar{z} = \frac{\Sigma \tilde{z} L}{\Sigma L} = \frac{0(300) + 0(600) + (-200)(400)}{300 + 600 + 400}$$
$$= -61.5 \text{ mm} \qquad \textit{Ans.}$$

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